THE

PHYSICAL REVIEW

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VOLUME IONIZATION PRODUCED BY LIGHT OF EXTREMELY SHORT WAVE–LENGTH.

By Frederic Palmer, Jr.

INTRODUCTION.

IN 1900 P. Lenard[1] showed that in addition to the well-known Hallwachs effect, or ionization produced at a charged metallic surface when struck by ultra-violet light, there existed also a volume effect which took place in the gas itself, and could be detected at some distance (50 cm.) from the source of illumination. He employed a spark from an induction coil between terminals of aluminum. Since the light from this spark passed through a quartz window and a layer of air more than one centimeter thick it is very unlikely that it contained any effective rays of wave-length less than λ 1800. It seemed worth while, therefore, to investigate the effect of that part of the spectrum discovered by Schumann[2] which lies on the more refrangible side of λ1800. The present research was accordingly begun in February, 1904.

Lenard[1] showed that the ionization observed by him was closely connected with the absorption of ultra-violet light by the gas under investigation. Therefore, since it is known that wave-lengths in the Schumann region are completely absorbed by a thin layer of air at atmospheric pressure, the object of this research was to find out if possible whether this absorption was accompanied by ionization; and if so, whether this result increased as the wave-length of the effective light diminished.

[1]Ann. Phys., 1, p. 486, 1900; 3, p. 298, 1900.
[2]Smiths. Contrib. No. 1413, 1903; T. Lyman, Astroph. Journ., 23, p. 181, 1906.

E. Bloch[1] has shown that volume ionization produced by the light from an aluminum spark is greatly reduced if the gas under investigation is filtered through a plug of cotton wool; and he therefore has ascribed the effect observed by Lenard to photo-electric particles held in suspension in the gas, and not to ionization of the gas itself. More recently J. Stark[2] from other considerations has stated that the Lenard effect is probably due to photo-electric particles in the gas. He has also shown that volume ionization may be produced by the light from a quartz mercury lamp in certain organic vapors. The effective wave-lengths lay in the region between λ1850 and λ3800. He obtained currents through the vapors as large as 5×10^{-9} amp. under a pressure of about 50 mm.

J. J. Thomson and W. C. McKaye[3] have found that the light from a Wehnelt hot lime-covered cathode produces ionization in air, carbon dioxide, and ammonia; but the currents obtained by them were excessively small.

In this paper it will be shown that comparatively large currents may be obtained in air at atmospheric pressure when the extremely short wave-lengths which lie in the Schumann region are used as a source of illumination.

Before undertaking the main part of the research, Lenard's experiments were repeated both with the steam-jet and with a cylindrical condenser as the detecting apparatus. A Dolezalek electrometer instead of an electroscope was used to measure the leak between the cylinders. The chief sources of error were : (1) irregularities in the illumination produced by the aluminum spark; (2) the Hallwachs effect, which was eliminated by the proper use of soap solution; (3) photo-electric fatigue,[4] which resulted in the falling off of deflections obtained successively in the electrometer under apparently similar conditions.

General Arrangement.

Since it is purposed to employ the very short wave-lengths in the Schumann region, the form of the source of light is of the

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[1] Comptes Rendus, 146, 17, p. 892.
[2] Phys. Zeitsch., 10, 18, p. 614.
[3] Proc. Comb. Phil. Soc., 14, p. 417, 1908.
[4] Cf. Hallwachs, Phys. Zeitsch., 21, p. 766, 1906.

utmost importance.   It has been shown[1] that, when the secondary
of a transformer is connected without additional capacity to the
electrodes of a hydrogen-filled discharge tube containing traces of
hydrocarbons, the excitation of the tube gives rise to carbon bands
extending from the ultra-violet down to $\lambda 1700$, and to strong hydro-
gen lines from $\lambda 1650$ to $\lambda 1250$.   It was therefore determined to use
a tube of this type as the source of illumination.   While it is recog-
nized that the carbon bands are capable of producing ionization,
this investigation is chiefly concerned with the properties of the
shorter waves; and therefore in order to get rid, as much as pos-
sible, of the effect produced by the carbon bands the tube was
always carefully washed out with pure dry hydrogen several times
before use.

The apparatus for producing ionization consisted essentially of
three parts: (1) the discharge tube, (2) the screen cell, (3) the
ionization chamber (see Fig.
1).   The discharge tube A was
a duplicate of the one already
described by Lyman.[2]   It could
be exhausted by a mercury
pump and filled with dry hyd-
rogen from a small electrolytic
generator.   The pressure with-
in could be measured by a
McLeod gauge.   The tube was
excited from the secondary of
a small transformer, the pri-
mary of which used about 4
amp. from a 110-volt A.–C.
lighting circuit.   The current
through the tube, as measured

Fig. 1.

Apparatus for producing ionization.

by Pierce's crystal rectifier,[3] proved to be between .03 and .04 amp.

The screen cell B was used as a diaphragm to cut off more or
less of the effective rays according as the pressure in it was varied.
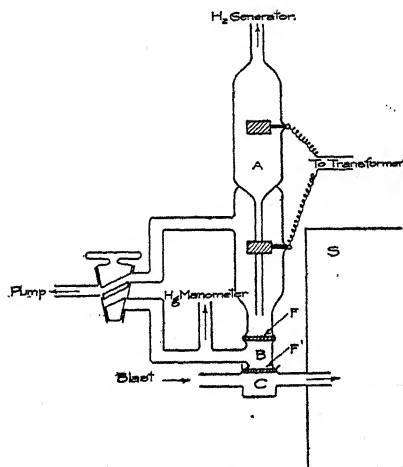
[1]T. Lyman, Astroph. Journ., 23, p. 181, 1906.
[2]*Loc. cit.*
[3]Phys. Rev., 25, p. 31, 1907.

It was 1 cm. in diameter and 1.0 cm. thick, and was separated from the discharge tube by a window of clear white fluorite F 1 mm. thick cemented with Khotinski cement. Ordinarily oxygen was used in the cell as an absorbing gas because its behavior was found to be more regular, and its absorption was better known than that of other gases. Dry oxygen could be admitted to the cell from a small electrolytic generator, and the pressure could be read either upon a closed mercury manometer or the McLeod gauge. A three-way stop-cock connected either the cell or the discharge tube to the mercury pump.

Another clear fluorite window F' 1 mm. thick separated the screen cell from the ionization chamber C, but the latter was arranged to slip over F' before being cemented in place so that the under surface of F' could be brought to a level with the side tubes of C. A blast of the gas under investigation could be maintained through C so that the ions, which were formed in this chamber, could be speedily swept away into the testing cylinders. The volume of the chamber was 1.4 c.c., and the distance between its center and the test cylinders 3 cm. Early in the investigation an ionization chamber was used which was open at the bottom so that a vessel containing mercury covered with soap solution could be raised from below to close the opening, and thus prevent
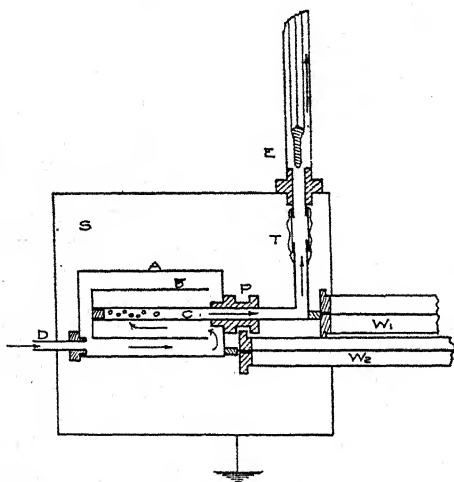
Fig. 2.

Testing apparatus.

surface ionization. However the soap solution was found to be the source of many irregularities in readings obtained, and so the form shown in Fig. 1, made of glass, was finally adopted. A comparison of readings obtained with the two chambers showed almost identical results, which may indicate that the ionization produced by light

of extremely short wave-length is confined to a region very close to the fluorite window F'. The velocity of the gas blast was maintained nearly constant. The value of the velocity was measured by a small open water manometer.

The apparatus for investigating the state of the gas issuing from the ionization chamber consisted of two parts: (1) the test cylinders, (2) the electrometer and its connections (see Fig. 2). The test cylinders of brass consisted of a concentric system of which the outside one A was closed at both ends by brass caps screwed on air-tight. It was 6.0 cm. long and had an inside diameter of 3.5 cm. The inner cylinder B was 5.0 cm. long and 2.0 cm. in diameter outside. The end nearer to the incoming gas was closed with a brass cap. A small brass tube C 0.4 cm. in diameter held B in place. The gas entered the system through the glass tube D which was placed opposite the annular space between A and B. The gas left the cylinders through the holes in the tube C. Thus the gas had to travel down between the cylinders, turn and pass up inside of B, turn again as it entered C, and pass along it for some 15 cm. before finally emerging past a thermometer. It thus seemed probable that even with a small potential gradient between the cylinders and a fairly rapid rate of flow of the gas, it would be possible to catch all of the ions before the gas escaped into the air. This assumption seems to have been justified by the actual performance of the apparatus. The tube C was insulated from A by the vulcanite plug P. A short piece of rubber hose covered with tin foil T connected C to the exit tube E. A stiff wire W_1 connected to C led through a grounded brass tube into the cabinet containing the electrometer and its connections. A similarly screened wire W_2 could be screwed to A, and by means of it A could be charged to any desired potential by a suitable storage battery. The entire cylinder system was enclosed in a large brass cylinder S, 10 cm. in diameter, which could be grounded and thus screen the testing apparatus from all outside influences. The Dolezalek electrometer after being properly adjusted, as described by Rutherford,[1] had a very slight normal leak, and could be made to have a sensitiveness of anything up to 7,500 scale divisions per volt P.D. between

[1] *Radioactivity*, 2d ed., p. 95.

the quadrants. The electrometer needle was suspended by a fine quartz fibre, and was charged from a water-battery by contact, usually to a potential of 40–100 volts. The electrometer was placed inside a cabinet lined with tin foil, and all contacts with it were made without opening the cabinet by means of levers operated by strings from the observer's seat. Either pair of quadrants could be connected (1) to the ground, (2) to a source of known E.M.F., (3) to the test cylinders. Deflections of the electrometer needle were read with a telescope and scale at a distance of about 2 meters. A standard cadmium cell of potential $V = 1.0177$ volts was used for calibrating the electrometer. On account of the sensitiveness of this instrument it was necessary to tap off a known fraction of the E.M.F., and suitable resistances were used in a simple potentiometer device to give one-eleventh the E.M.F. of the standard cell.

In using the apparatus it was desirable to know the pressure in the discharge tube which would produce the maximum effect in the ionization chamber. A preliminary experiment in which the pressure of air in the screen cell was kept at a low constant value (about .15 mm.), while the pressure of hydrogen in the tube was varied, showed a sharp increase in the effect up to a pressure of 1 mm. and a more gradual diminution from that point up to 10 mm. In all subsequent work, therefore, the pressure in the discharge tube was kept at an approximately constant pressure of 1 mm. This was also the pressure giving a maximum brilliancy when the tube was excited, though it did not correspond, as might have been expected, to the passage of maximum current through the tube, as was shown by measuring this current by means of Pierce's crystal rectifier. The maximum for the current took place at a pressure of 0.3 to 0.4 mm. To obtain constant results it was necessary that the tube should be excited for a constant short interval of time. A long excitation, by heating the tube, might set free occluded gases and thus change the pressure and therefore the tube's effectiveness. To this end a contact-maker was employed which illuminated the tube for one and one-fourth seconds. This instrument was tested for constancy of operation by connecting it to a chronograph. The greatest deviation from the mean of seven readings proved to be 2.25 per cent.

Another factor which it was necessary to make constant was the velocity with which the gas to be tested was blown through the ionization chamber. This was done by choosing arbitrarily a value on the water manometer which gave a deflection in the electrometer of the desired amount and then regulating the blast so that the manometer indicated this same reading each time an observation was made. The mean error of setting the blast repeatedly upon an arbitrarily chosen manometer reading was found to be about 2 per cent.

During an experiment it has been noticed repeatedly that apparently identical conditions gave rise to electrometer deflections which differed from one another by about five per cent. It seems that practically all of this may be accounted for by variations in the blast and length of exposure.

MANIPULATION.

Lyman[1] has found that the absorption of light in the Schumann region by oxygen is in the form of a band; and that, as the pressure increases, the absorption spreads much more rapidly toward the less refrangible side than in the other direction. For a column of gas 1 cm. thick at atmospheric pressure the band extends from $\lambda 1268$ to $\lambda 1770$, and for a pressure of .02 atmosphere from $\lambda 1350$ to $\lambda 1500$.

If oxygen is admitted to the screen cell at pressures varying from .0001 to 1 atmosphere, more and more of the effective rays from the discharge tube will be absorbed in passing through it. Thus the ionization produced in the chamber should be reduced with each increase of pressure in the screen cell. The wave-lengths which are available for the production of ionization in the chamber are those which get through the screen cell at any given pressure, and these may be determined when the width of the oxygen absorption band for the given pressure has been found.

The method of procedure in getting the relation between the two quantities—light entering the ionization chamber, and resultant ionization— is as follows: The pressure in the screen cell is reduced very low and measured on the McLeod gauge. A gas is allowed to

[1]Astroph. Journal, 27, p. 87, 1908

blow through the ionization chamber and test cylinders at the
desired constant velocity as indicated by the water manometer.
Before reaching the chamber the gas has to pass through 30 cm.
of glass wool and over 25 cm. of calcium chloride and 25 cm. of
phosphorus pentoxide.   It is therefore assumed to be dust-free and
dry.   While one pair of quadrants in the electrometer is kept in
connection with the earth, the other pair, connected with the inner
of the test cylinders, is now insulated.   The contact-maker is set
in motion and allowed to excite the discharge tube once.   Shortly
after the illumination ceases the stream of gas is shut off, and the
resulting deflection in the electrometer observed.   Both pairs of
quadrants, together with the inner test cylinder, are then connected
to earth, and oxygen from the electrolytic generator carefully dried
over phosphorus pentoxide is admitted to the screen cell changing
the pressure therein by a desired amount, as indicated by the closed
mercury manometer.   The tube is again excited, the resulting de-
flection made, and so on.

Since, as has already been shown, the light of short wave-length
entering the ionization chamber depends upon the pressure of oxygen
in the screen cell, and since the electrometer deflections are very
nearly proportional to the resulting ionization, curves have been
obtained showing the relationship existing between these two quan-
tities for different gases.   None of the gases used produced measure-
able deflections when blown unilluminated by the discharge tube
through the test cylinders.

OXYGEN.

Oxygen, 92 per cent. pure, made by the potassium chlorate black
oxide of manganese process and put up in a cylinder under pressure,
was passed through distilled water, then through two large U-tubes
each arm of which contained 10 cm. of phosphorus pentoxide held
in place by plugs of glass wool.   These were supported vertically
so that the oxygen was forced slowly through the phosphorus pen-
toxide, not over it.   The gas was collected over mercury and finally
blown through the ionization chamber.   Glass tubing only was
used for all connections.   After passing the entrance to the water
manometer the oxygen was again dried over phosphorus pen-

toxide before finally entering the chamber. With oxygen thus dried results were obtained which the curve showing pressures of oxygen in the screen cell, in centimeters of mercury, plotted against electrometer deflections is given in Fig. 3, *A.* Since
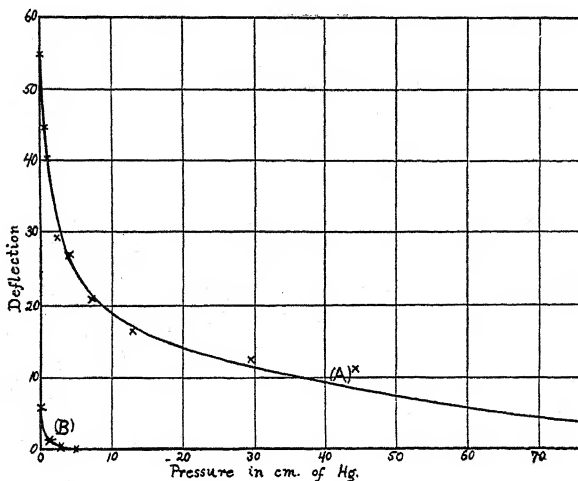
Fig. 3.

(*A*) Negative leak. Oxygen. (*B*) Negative leak. Hydrogen.

the outer of the test cylinders (*A*, Fig. 2) was charged to a potential $V = -20$ volts, this curve represents the negative leak between the two cylinders. A large proportion of the work has been done with the outer test cylinder at a constant potential $V = \pm 20$ volts. The saturation curves (not here shown) which have been obtained, show that for the gas velocities generally used this value of the potential was sufficient. The gas was at a temperature of 24° C., and its velocity through the ionization chamber was 20 cm./sec. The electrometer had a sensitiveness of 165 cm. per volt P.D. between the quadrants, and its capacity, together with that of the test cylinders and connections was 111 cm., or $111/9 \times 10^{-11}$ farads. Taking the maximum deflection in the above curve, it is easy to see that the insulated pair of quadrants has acquired a potential $V = 55/165$ volts, and, since $Q = VC$, $Q = 55/165 \times 111/9/\times/10^{-11}$, or 4.2×10^{-11} coulombs. If it is assumed that the average current may be obtained by dividing this quantity

by the time during which the gas is exposed to ultra-violet light, then

$$i = \frac{4.2 \times 10^{-11}}{1.25},$$

or the average current is 3.3×10^{-11} amperes. Considerably larger currents have been obtained. The normal rate of leak in the electrometer used in these experiments was 0.4 cm. per *minute*, and the maximum leak in the ionized gas was in this case at the average rate of 44 cm. per *second*.

An examination of curve A shows it to be made up of two parts: first, a nearly straight portion, rising slowly as the pressure in the screen cell is changed from 76 to about 19 cm. of mercury; second, a curved part, which rises rapidly as the pressure is reduced from this point down to the lowest value reached. An inspection of Lyman's paper[1] shows that, according to the evidence of the photographic plate, as the pressure in the absorbing cell is reduced from 76 to 19 cm., the width of the absorption band in oxygen becomes only slightly less, *i. e.*, changes from λ1770–1268 at 76 cm. to λ1740–1280 at 19 cm.; while from this point on the band rapidly becomes narrower with decreasing pressure, extending from λ1600 to λ1335 at a pressure of 4 cm., and disappearing entirely below a pressure of 1.5 cm. The absorption band is thus unsymmetrical, the change taking place more rapidly on the less refrangible side of the band. It, therefore, seems justifiable to assume in these experiments that, when the pressure of oxygen in the screen cell is about 19 cm. of mercury, two narrow bands of light pass through into the ionization chamber, in addition to all the light which went through at a pressure of 76 cm. These two bands extend from λ1770 to λ1740, and from λ1280 to λ1268. To them may be due the rise of curve A, which takes place slowly between the above pressures. Then, as the pressure in the screen cell is reduced to its lowest value, more and more of the wave-lengths in the Schumann region pass through into the ionization chamber. To these wave-lengths is due the rapid rise of curve A, which takes place at the lower pressures. It may be mentioned here that there is no reason to believe that the

[1] *Loc. cit.*

smallest amount of energy necessary to produce a chemical effect upon a photographic plate is the same as, or even comparable with the smallest amount necessary to produce ionization detectable by the method employed in this research. It is, therefore, quite probable that the sensitiveness of one method differs considerably from that of the other.

In consideration of these facts the author[1] has already stated that volume ionization is produced by light of wave-length less than $\lambda 1800$, and that this effect increases with decrease in wave-length, at all events in the region between $\lambda 1850$ and $\lambda 1400$.

When the outer test cylinder was charged positively the results were, within experimental error, the same as those given above, showing that in the ionization chamber positive and negative ions are formed in equal numbers.

HYDROGEN.

The gas was made electrolytically, passed through phosphorus pentoxide and collected over mercury in the same manner as the oxygen. In order to obtain deflections large enough to measure accurately the velocity of the gas stream was increased to 30 cm./sec., and the sensitiveness of the electrometer to 190 cm./volt. However, for the sake of comparison with the oxygen curve, the leak in hydrogen when the outer test cylinder was charged to a potential $V = -20$ volts has been plotted on Fig. 3 as curve B, and is on the same scale as curve A.

NITROGEN.

This gas was prepared by passing the nitrogen from a cylinder obtained from the Lindé Air Products Co. of New York through distilled water, over red hot copper, and through phosphorus pentoxide. It was collected in the same gasometer over mercury and used as were the other gases. The manner in which the gas in the cylinder is made by the company permits the presence of small quantities of argon and helium as impurities, but precludes the possibility of carbon dioxide or monoxide. A specimen of this gas analyzed for traces of oxygen and oxides of nitrogen showed:

[1] Nature, 77, p. 582, 1908.

"Oxygen 0.4 per cent., oxides of nitrogen 0.02 per cent. This does not include nitrous oxide or nitrous anhydride. The presence of the latter is, however, almost impossible, and of the former very unlikely." The results obtained with this nitrogen are plotted in Fig. 4. This curve represents the negative leak, since the potential of the outer test cylinder was − 20 volts. Moreover, the gas velocity through the ionization chamber was only 11.5 cm./sec., and the electrometer had a sensitiveness of 75 cm./volt, which corresponds to a capacity of 70 cm.

Nitrogen was also made from potassium nitrite by Gibbs's method. A specimen of this gas which was analyzed gave: "Oxygen none, oxides of nitrogen none." The steep rise of the ionization curve for pressures less than 2 cm. was even greater in this case, which indicates that the sharp rise is not due to the impurity of the gas.

Fig. 4.
Negative leak.   Nitrogen.

Air.

Air from the laboratory was passed through distilled water, and then through the same tubes of phosphorus pentoxide, being collected as before in the mercury gasometer. The results obtained for the negative leak are shown by the curve in Fig. 5. The conditions of this experiment were similar to those for the other gases, though it is not possible to compare the magnitudes of the curves given thus far, owing to differences in gas velocity and sensitiveness of the electrometer.

Comparison.

It has frequently been shown that absorption and ionization in a gas go hand in hand. Therefore since it is known that air and oxygen absorb light of extremely short wave-length very strongly,

while nitrogen is much more transparent and hydrogen hardly absorbs at all, it was to be expected that ionization would be detected in air and in oxygen, but that in nitrogen and in hydrogen, the amount of ionization, if it could be detected at all,
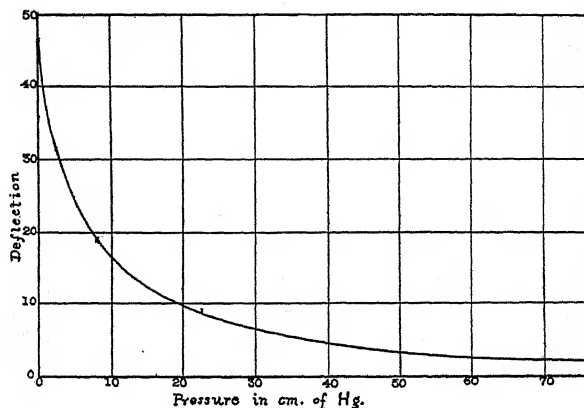
Fig. 5.
Negative leak. Air.

would be extremely small. In Fig. 6 are given the ionization curves obtained all in a single day, and under as nearly the same conditions as possible. The gas velocity was less than that of any of the previous curves and was the same for all. The potential of the outer test cylinder was $V = -16$; and the electrometer had a sensitiveness of 170 cm./volt. Under these conditions the ionization in hydrogen, even when the gas velocity was doubled, was so small as to be unmeasurable.

An inspection of the curves indicates that while air and oxygen show marked ionization, and hydrogen practically none, the ionization which takes place in nitrogen is unexpectedly large, and increases greatly for pressures in the screen cell less than 1 cm. Moreover, since a given thickness of air is more transparent than the same thickness of oxygen, it might be expected that the air curve of ionization would be lower than that for oxygen, but the opposite has always been found to be the case. It may be possible that small amounts of ozone, carbon dioxide, and carbon monoxide, present in the air account for a small portion of the ionization de-

tected, but it seems as if much more of it might be due to nitrogen than had previously been supposed possible.

In all four gases there is unmistakable evidence of a sharp rise in the ionization curve when the pressure of oxygen in the screen cell is reduced below 2 cm. Since it was earlier shown that the shortest
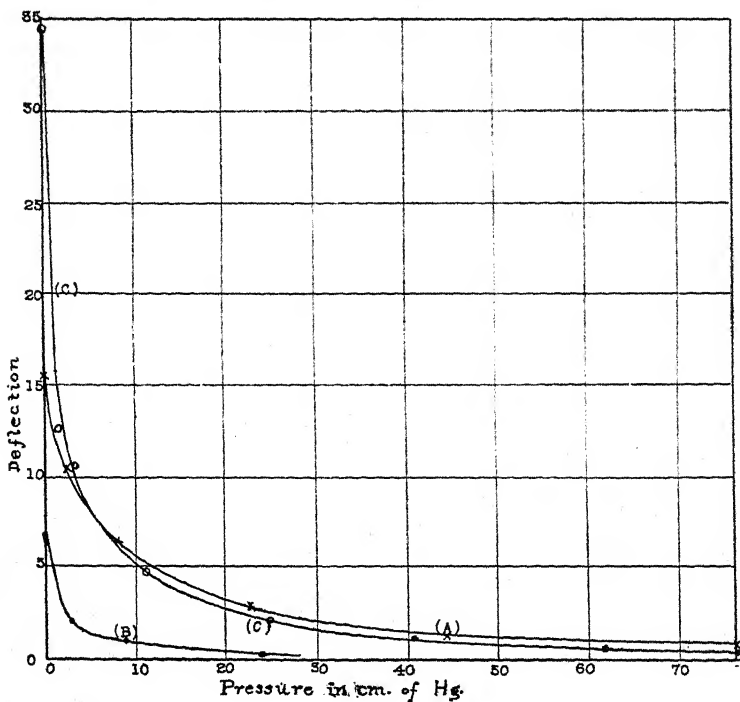
Fig. 6.

Negative leak.

wave-lengths passed through the screen cell only when the pressure in it was as low as 2.5 cm., the above result indicates that ionization increases very greatly as the wave-length decreases, at all events in the region between λ1850 and λ1400.

Although Lyman[1] has found that the absorption of ultra-violet light by nitrogen is small and increases regularly with decreasing wave-length, Schumann[2] states that nitrogen absorbs particular wave-lengths very energetically. It is known that nitrogen absorbs

[1]*Loc. cit.*

[2]*Loc. cit.*

wave-lengths between λ1500 and λ1300, such as pass through the screen cell at the lowest pressures, and if the absorption is particularly energetic in this region it may account for the observed rise in the ionization curve.

Dust.

The work of E. Bloch[1] seems to show that minute particles held in suspension by a gas, when acted upon by ultra-violet light, may produce a photo-electric effect which simulates that of volume ionization. In order to show that the results obtained in these experiments were not due to such a photo-electric effect, a plug of cotton wool 15 cm. long was inserted in the gas blast in addition to the many plugs of glass wool already there. The insertion of the cotton wool produced no detectable effect. It is also to be observed that the ionization in hydrogen is extremely small, yet from the manner of its production, there might be as much dust in hydrogen as in the other gases.

Water Vapor.

If water vapor has any effect, it is of great importance in this work, since it might produce absorption in the oxygen of the screen cell, resulting in a *smaller* electrometer deflection than if the oxygen were dry; or it might produce ionization in the gas being tested, resulting in a *larger* deflection. The following experiments will show that water vapor not only has an effect both in the screen cell and in the ionization chamber, but also that this effect is most pronounced.

Curve *A*, Fig. 7, is the same curve previously shown as *A*, Fig. 3, and represents the negative leak in oxygen when both the oxygen in the screen cell and the oxygen in the ionization chamber are dry. If the oxygen in the screen cell is kept dry, while oxygen which is slightly damp is taken from the previously mentioned oxygen cylinder, and passed through the ionization chamber, the result is curve *B* which falls much less rapidly at low pressure than *A* and does not reach it until the pressure in the screen cell has risen to 54 cm. of mercury. If this same undried oxygen from the cylinder is used in the screen cell as well as in the ionization chamber, the result is curve *C*, which is distinctly lower than *B* even up to atmospheric

[1] *Loc. cit.*

pressure, with the exception of the region below a pressure of 2 cm. of mercury. It was not convenient to saturate the oxygen which was admitted to the screen cell with moisture; so, instead, upon one occasion when the air in the laboratory was exceedingly damp it
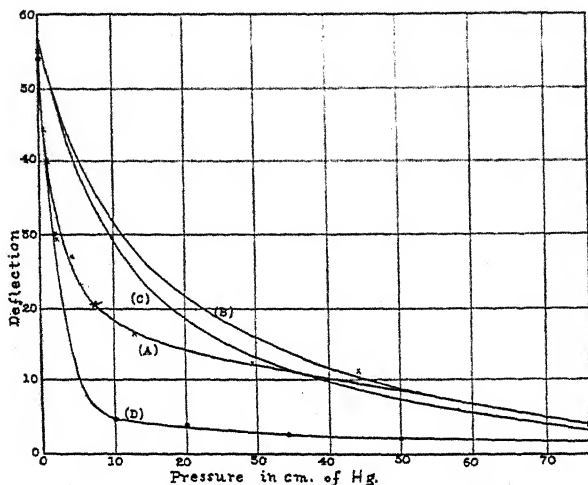
Fig. 7.

Effect of water vapor.

was admitted directly to the cell, while slightly damp oxygen from the cylinder was passed through the ionization chamber. The resulting state of affairs is shown in curve *D* and is conclusive in regard to the possibility of absorption due to water vapor in the screen cell.

MERCURY VAPOR.

W. Steubing[1] has found that mercury vapor is ionized by light in the region of $\lambda 1800$–2400. It is possible that it may be ionized also by light in the region of $\lambda 1250$–1800. If so, since all of the gases examined were collected over mercury, and since the screen cell was in continuous connection with a mercury manometer, it might be possible for mercury vapor to play a part in the phenomena here under investigation. However, since the vapor pressure of mercury is so small at atmospheric pressure, it is not likely that such minute quantities of this vapor as could mix with the gas

[1] J. Stark, Phys. Zeitsch., 10, 18, p. 623.

under investigation in the gasometer would affect the amount of ionization observed, even if the vapor were capable of producing any effect at all. No effect ascribable to mercury vapor in the ionized gas has been detected in these experiments.

Since the oxygen admitted to the screen cell is subjected to pressures all the way from atmospheric down to .01 mm. of mercury, it might very well be the case that at the lowest pressures used mercury vapor might constitute nearly one per cent. of the gas in the cell, and might produce a much greater percentage of absorption. Evidence of such an effect has been observed with all the gases examined at very low pressures of oxygen in the screen cell; but this is especially noticeable with nitrogen, since such a large proportion of the observed ionization in this gas is produced with low pressures in the screen cell.

CHEMICAL CHANGES.

Many investigators have shown that ordinary ultra-violet light changes oxygen into ozone, and Lyman[1] has found that the light of extremely short wave-length used in this work does so to a very marked degree. It is therefore undoubtedly true that some of the oxygen used in the screen cell in these experiments is turned into ozone. However, since it has also been shown[1] that the absorption due to ozone does not differ appreciably from that due to oxygen this fact alone should not be a source of error here; unless, indeed, the method of ionization should prove to be superior to the photographic plate as a detector of absorption. If, on the other hand, the ozone when formed produces or facilitates other chemical changes, it might be possible that the resulting products of such changes would form a very considerable source of error.

Two effects have been observed which might be accounted for on the assumption that the ozone facilitates the formation of a layer of gas very close to the fluorite window, like that described by Hallwachs[2] in his explanation of photo-electric fatigue. One of the observed effects is the falling off of deflections in the electrometer taken successively under apparently the same conditions, when

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[1] *Loc. cit.*

[2] Phys. Zeitsch., 21, p. 766, 1906.

the pressure of oxygen in the screen cell is anything from 1 to .0002 atmosphere. This effect is very similar to that ascribed to the presence of mercury vapor, but differs from it in that the deflection can be brought up to its initial maximum value either by waiting for ten minutes between readings, or by admitting fresh oxygen and reëxhausting to the same pressure as before. The first operation might give the ozone time to break up into oxygen, and the gas layer to become dissipated; while the second might be effective through breaking up of the gas layer by the inrush of fresh gas, followed by the loss of some of the ozone by reëxhaustion as well as by the natural process of breaking up into oxygen.
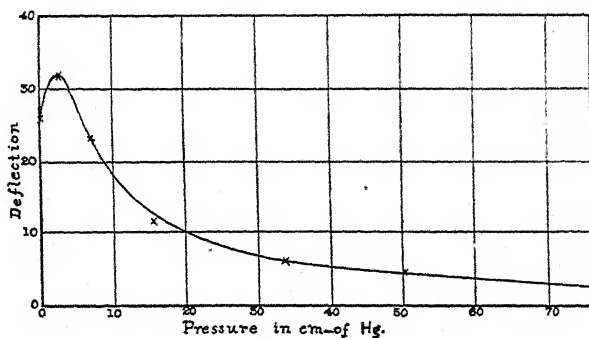
Fig. 8.

Effect of chemical action on fluorite.

The other effect has been observed not infrequently after the apparatus, as shown in Fig. 1, has been in use for some time. It consists of a complete change in the character of both positive and negative ionization curves for pressures less than 2.5 cm. of mercury in the screen cell. Instead of increasing as the pressure in the cell is reduced to its lowest value the curve passes through a maximum at a pressure of about 2 cm., falling off sometimes to only 80 per cent. of its maximum value when the lowest pressures are reached. Fig. 8 represents such a case when $V = -20$ volts and the ionized gas was oxygen. Upon taking apart the apparatus no film was found on the side of the fluorite window F (Fig. 1) next the discharge tube, where one might have been formed from a metallic deposit from the aluminum electrodes; but a film was formed on

the side of the window F next the screen cell. No film has ever been found on the window F' (Fig. 1). The film has always been observed whenever the character of the ionization curve has undergone change, as in Fig. 8. Upon one occasion when the film was carefully examined, it was found to consist of two distinct parts; one was composed of fine, white, microscopic dust, which could be brushed off easily by a touch from some cotton wool; the other, when viewed in reflected light, appeared like a very thin film of whitish metal, which could be removed only after soaking in alcohol, and then rubbing briskly with cotton wool. The layer of dust occupied the exact central portion of the window and was in the form of a disc of diameter very little larger than the end of the capillary in the discharge tube, exactly opposite to which it was situated. The other film was also circular in shape, though its outline was not so clearly defined as that of the disc of dust, and its diameter was about three times as great, the two being concentric. On one side a notch in the circumference of the outer film was noted where no deposit existed. In this instance the fluorite window was 2.5 mm. thick, and a close examination showed a flaw on the side next the discharge tube which proved to be in a straight line from the end of the capillary to the notch noted above, thus indicating that the flaw had cast a shadow, in the line of which no chemical action on the fluorite had taken place. This shows that the action must take place extremely close to the window; that the dust layer must be formed by the more intense light which shines down some length of the capillary; and that the larger film must be produced by the weaker illumination which comes from close to the end of the capillary. Furthermore, since no effect has ever been found on the window F', it shows that the effective rays here are the very shortest. When carefully dried oxygen is used in the screen cell the apparatus may run for weeks without the formation of the film; but if oxygen from the storage cylinder is used, or if air is admitted from the room even if partially dried, the film forms quite quickly, sometimes in a day. The formation of the film, then, seems to be due to the action on the fluorite of ozone in the presence of a small amount of water vapor. This suggests that hydrogen peroxide may be the effective agent.

Since the curves shown represent the ionization produced by the sum total of all the light passing through the screen cell at any given pressure, it is not easy to see how a maximum in such a curve could be produced. It is probable that the dust layer described above plays a double role in this process: (1) absorbing much of the light of shortest wave-length, (2) facilitating the formation of a gas layer on the fluorite by presenting a roughened instead of a smooth surface. If it is assumed that this gas layer forms more readily on the roughened surface at low pressures, or that its absorption is abnormally large for such pressures, and that it absorbs the shorter wave-lengths more easily than pure oxygen, it can be seen how starting with a very low pressure in the cell practically all the absorption taking place would be due to the dust film and to the gas layer; but if a little more oxygen were admitted to the cell the gas layer might be disturbed, while the absorption due to the oxygen itself would increase.

If the increase in absorption due to the introduction of the oxygen was not as great as the gain in transparency due to the breaking up of the gas layer, the curve of ionization would rise. As more oxygen was admitted, the increased absorption due to the oxygen would overbalance the decrease due to the change in the gas layer, and therefore the ionization curve would begin to fall again, and would continue to do so up to atmospheric pressure.

It is possible that the effect here described is the same as that noted by C. T. R. Wilson[1] in his experiments on the formation of clouds by ultra-violet light. He has suggested that the formation of hydrogen peroxide by ultra-violet light would explain the phenomena observed by him. Although his source of illumination (zinc spark in air) gave rise to no such extremely short wave-lengths as those effective here, nevertheless it was a much more intense source than the hydrogen tube of these experiments, and might have produced hydrogen peroxide on that account. Lenard[2] has also found that ultra-violet light produces cloud nuclei which are non-electric in character, though he has not suggested what their real nature might be. It is quite possible that these, too, may be minute

[1] Phil. Trans., 192, p. 403, 1899.
[2] Ann. Phys., 3, p. 298, 1900.

particles of hydrogen peroxide. It may be also that the roughening of a surface under the action of ultra-violet light noted by many investigators is due to the same cause as that which produced the dust layer described above.

Whatever may be the explanation of the phenomenon, it is certain that the maximum in the ionization curve exists only when the film on the fluorite window is present.

TEMPERATURE.

The gases examined were all at nearly the same temperature, that is from 15° C. to 25° C. Between these limits there was no detectable temperature effect.

CONCLUSIONS.

In conclusion it should be noted that the objection raised by Bloch to Lenard's work (dust) does not apply to the present investigation, since the insertion of a long plug of cotton wool in the gas stream was without effect. Furthermore, since it has been shown that the ionization produced in hydrogen is *extremely* small, this fact proves conclusively that the results obtained were not due to photo-electric action at the inner surface of the ionization chamber. This point meets the objection recently raised by Stark.

Finally, although the hydrogen tube used emitted light of low intensity, it was possible to obtain rather large currents through air, oxygen and nitrogen, owing to the great power of ionization possessed by the very short wave-lengths. Therefore, it seems clear that the reason why J. J. Thomson and McKaye, and others have obtained only extremely small currents in oxygen is because they have not employed a source of illumination emitting light of sufficiently short wave-lengths. The large currents measured by Stark were obtained in vapors having a very complex molecular structure, and not in oxygen. During the course of the present investigation it was found that, if alcohol or ether vapor were mixed with the gas under examination, the electrometer deflection in one second was so large as to be unmeasurable. This shows that for vapors of complex molecular structure the shortest wave-lengths also are capable of producing very large currents.

SUMMARY.

1. Using a discharge tube filled with hydrogen as a source of light and a screen cell filled with oxygen at various pressures, ionization curves have been obtained in air, oxygen, hydrogen and nitrogen.

2. The ionization in air, oxygen and nitrogen is considerable. That in hydrogen is exceedingly small. The currents obtained for all gases are large compared with the values of most other observers.

3. The power of ionization increases greatly with decrease in wave-length of light, at all events in the region below $\lambda 1850$.

I take this opportunity of thanking Professor G. P. Baxter for valuable suggestions as to the production of pure nitrogen and also for superintending the analysis of the specimens of that gas.

The investigation was carried on in the Jefferson Physical Laboratory at Harvard University.

HAVERFORD COLLEGE, HAVERFORD, PA.,
    May, 1910.

A METHOD FOR THE QUANTITATIVE ANALYSIS OF MUSICAL TONE.

By Preston Hampton Edwards.

I. Introductory.

THE present investigation was undertaken with the purpose of making an analysis of the musical tones actually emitted from a violin or other musical instrument. The problem of the quality of tones has engaged the attention of many investigators, yet no systematic quantitative results have been obtained. Helmholtz[1] did a great deal in the analysis of tones, both from consideration of the method of their production, as in the case of the vibration of a violin string and that of the air used in an organ pipe; and by his resonators, which gave an indication of what was actually going on in the air. Koenig[2] attacked the problem with resonators and manometric flames, while Scripture[3] studied enlarged phonograph records. Dayton Miller[4] has recently used an improved oscillograph method.

As has been said, none of these methods has as yet furnished quantitative results, and they can hardly be expected to do so unless greatly improved in the future. The use of resonators by Helmholtz and Koenig depends on the human ear in one case and on manometric flames in the second, and either can only give qualitative results. The other methods are dependent on the analysis of irregular curves, which is a difficult and not always certain process. There is also in many cases the complication introduced by the membrane or other parts of the recording device having free periods of their own. So the first step of this work was to look for a better method of analysis.

[1]Tonempfindungen, Abs. 3, 4, 5.
[2]Expériences d'Acoustique, p. 70 ff.
[3]Elements of Experimental Phonetics.
[4]Reported at Baltimore meeting of A. A. A. S., Dec., 1908.

II. The Method.

The first device tried was one used by M. Wien,[1] who cut away a portion of the wall of a Helmholtz resonator and replaced it by a metallic diaphragm tuned to the tone of the resonator. The vibrations of this diaphragm were communicated by a stylus to a light steel spring extending over the diaphragm from a support at the side, and on the end of the spring was mounted a small mirror forming an image of a slit on a screen, to indicate the amplitude of the vibrations. In my tests a cylindrical resonator was used, with the usual small orifice at one end, while at the other end was a larger opening, over which membranes of various sorts could be placed and held fast by a flat brass ring screwed on. It was soon found, however, that the system formed by the coupling of these two units, instead of having a single response, answered to two notes several tones apart, even though the two were tuned to the same frequency before coupling. This fact makes its indications doubtful, thus rendering it useless for the purposes of this investigation. Following this some tests were made with Rayleigh's disk,[2] which finally led to the adoption of a system composed of a mica disk hung close in front of the opening of a resonator, at an angle of 45° with the plane of the opening.

FIG. 1. RESONATOR WITH RAYLEIGH'S DISK.

SEEN FROM ABOVE.

B. Brass Rod.   R. Glass Rod.
Q. Quartz Fiber.   G. Glass Vane.
D. Mica Disk.   C. Oil Cup.
M. Concave Mirror.

FRONT VIEW
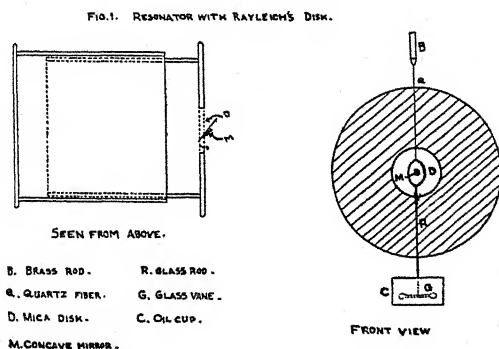
Fig. 1.

The arrangement of this system may be readily understood from the accompanying sketches (Fig. 1). The disks were made of mica,

[1] Wiedemann's Annalen, 36, p. 835, 1889.
[2] Rayleigh, Theory of Sound, II., p. 44.

on account of its lightness and ease of handling. The circles were cut out with a pair of bow dividers with sharpened points. The concave mirrors were made by silvering, or better, platinizing the surface of a concave eyeglass lens such as can be obtained from opticians, having by reflection a focal length of about 50 cm. After silvering the lens it was cut into strips and chips struck off from the surface by hitting the edge with a hammer. Mirrors can be made in this way of 1 milligram or less in weight, which will give a good image of a Nernst filament at one meter's distance. The quartz fiber could be made of any fineness, according to sensibility desired. At the bottom of the disk was attached a fine glass rod, and at the lower end of this was a little glass dumbbell hanging in a cup of kerosene for damping. The motion of air in and out of the mouth of the resonator when it is "speaking," exerts a couple on the disk, tending to set it in the plane of the opening, and the deflection of the image thrown by the mirror on a screen measures the angle through which it turns.

The cylindrical form of resonator made of two pieces of telescoping brass tubing was chosen on account of ease of tuning. The larger ones were made by Koenig, of Paris, and were kindly put at my disposal by the psychological laboratory of this university. In use the nipples made for insertion in the ear were closed with small corks. The smaller ones were made in the workshop of this laboratory, and had no nipples (see Fig. 1).

A series of 17 resonators was set up in this manner, for the analysis of the tones g and c'. (In this paper Helmholtz's designations will be used for the notes, and the partials will be designated by their relative frequency numbers, *i. e.*, the fundamental will be called the first partial, the octave the second, etc.) The whole set up comprised the following:

Partials of g.	Partials of c'.
1. g	1. c'
2. g'	2. c''
3. d''	* 3. g''
* 4. g''	* 4. c'''
5. b''	5. e'''
6. d'''	* 6. g'''
* 8. g'''	7. bb''' (app.)
*12. d^{IV}	8. c^{IV}

Partials of g.	Partials of c′.
*16. g^{IV}	* 9. d^{IV}
	10. e^{IV}
	*12. g^{IV}
	16. c^{V}

Those resonators marked with a star were common to the two sets. The whole set was placed on a table and covered by a box of light paper pasted on a skeleton frame, with a window in front made of very thin, transparent celluloid. The object of this cover was to protect from air currents while allowing free passage for sound. The resonators were inclined to the front of the box so as to allow the disks to face the source of light and the screen, both of which were placed at about a meter's distance in front of the array of resonators. The source of light was a Nernst glower and the screen was made of tracing paper stretched on a wooden frame. Its dimensions were about 40 by 75 cm.

FIG. 2 SHOWING ARRANGEMENT OF SET.
(SEEN FROM ABOVE)

S- SCREEN.
N- NERNST GLOWER.
O- OBSERVER.
P- PERFORMER.
R. SET OF RESONATORS UNDER COVER.
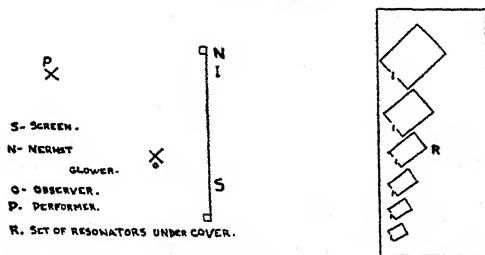
Fig. 2.

As one of the leading objects of this paper is to establish the value of this method of sound analysis, it will be advisable to discuss the qualities which differentiate it from other methods and contribute to, or limit, its usefulness.

III. FEATURES OF THE METHOD.

1. *Sensibility.*—This is one of the most valuable features of the method. Systems set up without unusual care showed deflections of several millimeters for the gentlest possible sound of the voice of the observer, or for a piccolo blown softly at the far end of a neighboring large room. Naturally the sensibility of a system depends on the following:

(a) *Multiplying Factor of the Resonator*, which has been shown by Helmholtz to be expressed by the formula $dp/dP = \sqrt{2R^3/\pi^2 S}$ (see page 29).

This gives a comparatively low value for the resonators corresponding to the higher partials, since their openings are in general larger in comparison to their volumes than for the lower resonators.

(b) *Size and Length of the Quartz Fiber.*—We can always increase the sensibility in this direction, at the expense of stability and ease of handling. A proper balance between these two desiderata must be found for any one investigation; in the present work no attempt was made to push the sensibility to extreme values.
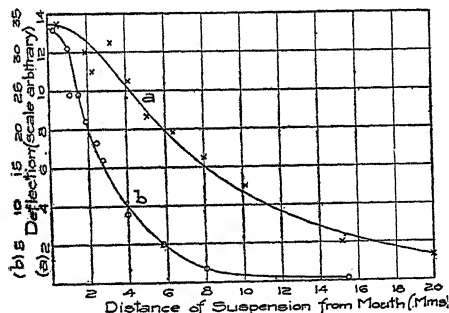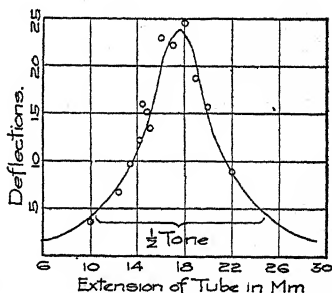
Fig. 3.   Fig. 4.

(c) *Size of the Disk.*—Lord Rayleigh's formula for the couple is: $M = \frac{4}{3}\rho a^3 W^2 \sin 2\theta$ (where a = radius of disk, W = mean velocity of vibrating particles, ρ = density of air, and θ = inclination of disk to plane of mouth), which shows that the effect can be increased by increasing the diameter of the disk, in proportion to the cube of this diameter. The size of the disk, however, must be kept well under the size of the resonator's mouth. It was thought that the presence of the disk might have the same effect as reducing the size of the mouth, *i. e.*, might lower the pitch of the resonator, but a test with several persons of good musical ear did not indicate such an effect in the slightest degree.

(d) *Distance of Suspension from Plane of the Mouth.*—In Fig. 3 are shown curves between these distances and the deflections produced by a constant source of sound (an organ pipe blown at uniform pressure). This would afford an easy way of varying the sensibility

if so desired.   In this work the distance was kept constant, and as
small as practicable (about 1 mm.).

2. *Selectiveness.*—Fig. 4 shows a curve of deflection obtained when
the tuning was varied by drawing out or forcing together the two
telescoping tubes forming the resonator, the source of sound being
kept constant.   The extension corresponding to a semitone change
of pitch is indicated in the figure.   Quite similar curves were ob-
tained when the source of sound was tuned (*e. g.*, an organ pipe
tuned by turning down a portion of the top margin, and compared
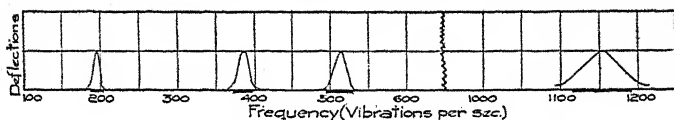
Fig. 5.

by beats with a tuning fork).   In Fig. 5 four of these curves are
plotted on a line of abscissas which represents total frequencies.
The heavy line beneath each curve indicates a range of a semitone.
This sketch shows us that neighboring harmonics will not interfere
until we reach the neighborhood of the sixteenth, where they are
about a semitone apart.   It will be noticed that the g′ and d‴
resonators, which were cylindrical, were less selective than the g
and c″, which were of the Helmholtz pattern.   Whether this is due
to their cylindrical shape, or to some relation between volume and
size of mouth, has not yet been investigated.

3. *Damping.*—It was found possible by the arrangement of a glass
rod dipping in oil as shown in Fig. 1, to make the vibrations of the
disk practically dead beat, so that in a fraction of a second the image
on the scale, on the sounding of its tone, will move to a position
which will be steady so long as the pitch and intensity are constant.
Of any variation in either, however, it is a very delicate indicator,
and it often warns the observer that a note sung is not holding to its
pitch when his ear does not notice the fact.

4. *Indications.*—It will be noticed from the formula for Rayleigh's
disk, that so long as the deflections are small, *i. e.*, θ very near 45°,
the turning couple, and therefore the deflection, is proportional to
the kinetic energy of vibration of the air at the resonator's mouth.

If therefore we know the "multiplying factor" of each resonator
and the moment of torsion of each suspension, we can pass at once
to the absolute energy belonging to each partial, in the open air.
It seems more to the point to describe the quality of a tone by the
energy of its components than by their amplitudes, and in the
graphical representations of tone quality in this paper, the strength
of each partial will be indicated by ordinates proportional to its
energy of vibration, obtained immediately from the deflection of
the corresponding disk, by the use of the two constants just men-
tioned.   Helmholtz's formula for the multiplying factor of a reso-
nator is

$$dp/dP = \sqrt{2R^3/\pi^2 S},$$

where dp represents the variation of pressure in the outer air, dP
that in the resonator, R = radius of mouth, S = volume of reso-
nator.   Where we have to do with energy values, however, we must
use the square of this ratio.   Wien[1] experimentally found values in
the neighborhood of 30 per cent. below the theoretical values as
given by the above formula.   His method however is not one which
may be considered as accurate, and it is highly desirable that some
means be found of experimentally arriving at a value of this ratio.
Much time and effort were expended in this direction in connection
with the present work, but no results worth recording were obtained.
The writer hopes to continue these experiments in the near future.
Meanwhile the theoretical value will be used in this paper.

To find the reducing factor for the suspended system, a system
similar to those in use with the resonators, was suspended in a
vacuum and its period observed.   It was then removed and a piece
of copper wire about as long as the diameter of the disk, was
attached, by a bit of wax, along a horizontal diameter of the disk.
The period was again observed in a vacuum, and since the moment
of inertia of the piece of wire could be calculated, the moment of
torsion of the quartz fiber was thus found.   Each suspension was
then directly compared with this standard by suspending the two
successively in front of the same resonator and taking deflection
produced when the note of the resonator was sounded by an organ

[1] M. Wien, *l. c.*

pipe at a definite air pressure. This comparison gives a factor by means of which the deflection of any suspension may be at once reduced to the equivalent deflection of the standard. Multiplying this deflection by its known moment of torsion gives M in the formula

$$M = \tfrac{4}{3}\rho a^3 W^2 \sin 2\theta.$$

Measuring a, the radius of the standard disk, and taking $\sin 2\theta = 1$, since θ does not vary much from 45°, the deflections being generally small, we have a means of calculating $\tfrac{1}{2}\rho W^2$, the mean kinetic energy per c.c. of the vibrating air at the resonator's mouth, and multiplying this by the factor $2R^3/\pi^2 S$ we find the mean kinetic energy per c.c. in the air, belonging to the corresponding partial. For the present purpose the deflections were reduced to the equivalent deflections of the standard and then multiplied by the factor $2R^3/\pi^2 S$, thus giving quantities *proportional* to the energy of the various partials.

It would be interesting to use this method to compare the total input and output of energy for various musical instruments, thus giving their efficiency as sound producers. Webster[1] has arrived at values of this efficiency, using only a single partial (the fundamental). The present work indicates that for many instruments his values would be very largely increased by taking account of all the partials.

IV. Room Used for the Tests.

In the preliminary trials it was found that the quality of a tone might vary greatly as the instrument producing it was moved about the room. This effect is doubtless due to standing waves produced by reflection from the walls of the room, and depends on the position of the source and of the receiver, and on the wave-lengths of the sound. The difference of wave-length of the different partials causes some to be weakened by interference at certain points, while others are strengthened, and so on. The curves in Fig. 6 show the effect on the relative strength of the first three partials of a c' reed pipe, when the pipe was moved perpendicularly away from the

[1] A. G. Webster, On the Mechanical Efficiency of the Production of Sound, Leipzig, 1904.

wall of the room, its height from the floor being kept constant.
The room was an octagonal one about 4.5 meters across. The
resonators were near the opposite wall of the room. It is interesting
to note that the maxima (or minima) of the first and third partials
are found at points separated by distances nearly equal to their
respective wave-lengths (for first partial $\lambda = 130$ cm., for third,
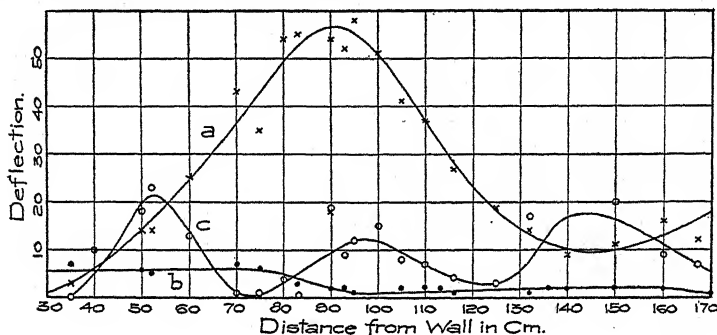
Fig. 6.

$\lambda = 43$ cm.). It would be easier to explain this if the distance
were $\frac{1}{2}\lambda$. The second partial does not show any such marked
effect as the other two.

In order to eliminate complications arising from this effect, a
space was curtained off by burlap lined with cotton batting, forming
a chamber about 10 feet by 6 feet by 7 feet, in one end of which
was placed on a table the set of resonators protected by the paper
covered box. The performer with his instrument stood at the other
end of this chamber. The floor, as well as the sides and top, was
covered with cotton batting about an inch thick. It is probable
that even with all the precautions taken there were still complications
present due to shadows and reflections, since the observer and the
screen were between the performer and the resonators, and the table
and other solid articles offered a considerable area of reflecting sur-
face. In the more extended experiments which it is hoped will
follow, a larger room must be prepared in which the observer and
screen may be to one side, and solid materials reduced to a minimum.
The table, for instance, might be made of slender steel rods with a
top made of iron gauze. It would be advantageous to carry on

experiments in an open space out of doors, if such were available where the air would be sufficiently still.

V. TAKING THE OBSERVATIONS.

The manner of doing this was the simplest possible. The suspensions were so adjusted that each mirror threw an image of a Nernst glower on a screen, and the observer sitting in front of the screen soon became familiar with the position of the image corresponding to each partial. The performer stood behind the observer and produced a given tone half a dozen or a dozen times while
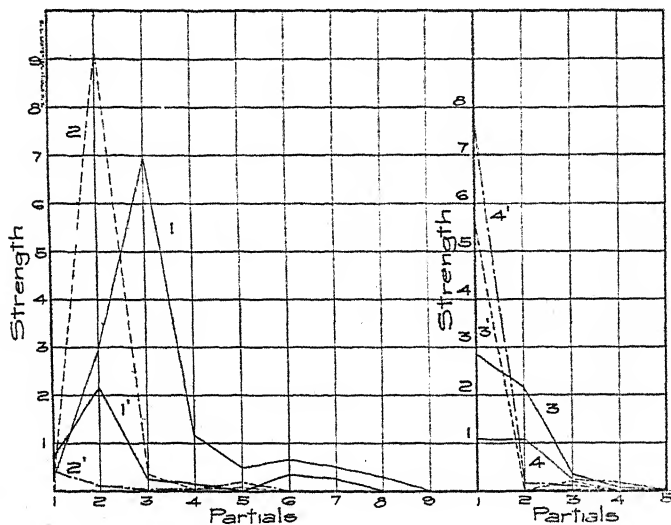
Fig. 7.

the latter noted the deflections of the images. There were often differences of as much as 100 per cent. in the deflections of the same image for successive productions of the same note, so an average value was taken. The following set of readings is given as an illustration. The column of readings headed by *m* under each tone was taken with the violin muted with a metal mute. An asterisk (*) indicates that there was no resonator to respond to that partial.

The figures must be modified as explained under III., 4, to show the actual proportions of the total energy of the tone, furnished

by the several components. This has been done to obtain the ordinates used in Fig. 7. It must be borne in mind that in this and similar graphical representations of tone quality only the ordinates have a meaning, the lines joining the summits of these ordinates being merely a means of showing that these ordinates belong to the same tone, in a way to appeal easily to the eye.

Record of P. H. E.'s Violin, played by A. H. P.

Partials.	Open g.		c'		g' Harmonic.		g' on d' String.		c''		g''	
		m		m		m		m		m		m
1.	.4	1.0	.7	2.0	6.5	13.0	2.5	18.0	30.0	.9	6.9	3.0
2.	3.4	2.5	24.0	.3	3.0	.0	1.5	.2	.8	.4	.3	.5
3.	9.0	.3	2.5	.2	.3	.2	.2	.1	.2	.0	.0	.0
4.	.8	.1	.2	.1	.1	.0	.2	.0	.5	.0	.0	.0
5.	.8	.0	.2	.5	*				.0	.0		
6.	.3	.2	.2	.0	.0	.0	.0	.0	.0	.0		
7.	.2	.1	.0	.0	*				*			
8.	.2	.0	.0	.0	.0	.0	.0	.0	.0	.0		
12.	.0	.0	.0	.0					*			
16.	.0	.0	.0	.0								

A more refined method of taking the observations would be, by means of auxiliary mirrors, to bring all the images into one vertical line, also to so adjust the sensibility of the systems that their deflections would all bear the same proportion to the energy of the corresponding partial in the open air. Then a film could be exposed while the tone was sounding and while not sounding, and the displacements of the images would give a picture of the instantaneous quality of the tone.

VI. DISCUSSION OF THE RESULTS.

The readings given in the table above, and (in part) represented graphically in Fig. 7, bring out at once two remarkable and important facts, (1) that the lower tones of the violin are extremely weak in fundamental, a very large proportion of the energy being contained in the overtones, and (2) that the effect of muting these tones is to increase, not only relatively but absolutely, the strength of the fundamental, while damping the upper partials. Muting the higher tones diminishes all of the partials together.

Fig. 8 shows the effect of different manners of bowing, in a some-what exaggerated way. Record (1) is from a good English-made Amati model, about 150 years old, bowed by a good violinist to bring out its best effect; (1') is from the same violin bowed with great force by another performer, to bring out its full power. It
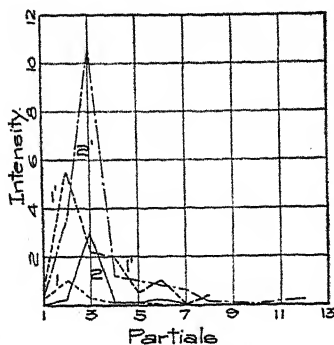
Fig. 8.

will be noted that while all the par-tials are increased in intensity by the stronger bowing, some higher ones are brought out which were not before present in measurable amount. Rec-ords (2) and (2') are similarly taken from a somewhat ''scratchy'' violin of not very powerful tone, age and make unknown. The same sort of effect from strong bowing is shown as in the first case. It is interesting to note that among the differences intro-duced by different bowing, the individuality of each instrument still appears very clearly.
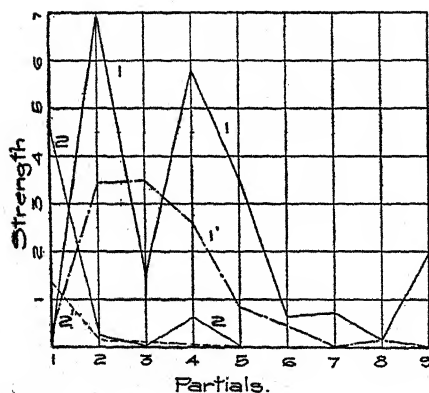
Fig. 9.
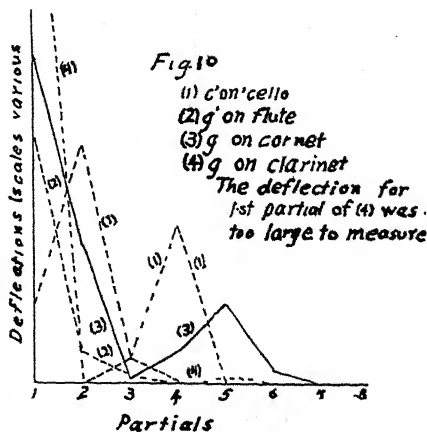
Fig. 10.

Fig. 9 shows the effect of varnishing. Through the kindness of Mr. Della Torre of this city a violin was obtained and strung ''in the white,'' *i. e.*, without any varnishing at all. After making a record with it thus, he applied a varnish of his own and another record was taken.[1]

The three modifications of tone just mentioned, *i. e.*, muting, difference of bowing, and varnishing, are the only ones yet sufficiently tested to be reported on. It is hoped that the test may be continued long enough to get valuable data as to the effects of various other factors which easily suggest themselves, such as setting of sound post, position, shape and height of bridge, graduation of back and belly, etc.

Fig. 10 shows records of several other instruments which need not be here discussed. Each one of these instruments, however, ought to be studied in the same complete way as has been suggested for the violin.

VII. VIOLIN TONE.

The question put before us is, what constitutes good violin tone? Some suggestions for an answer may be found in Fig. 11,
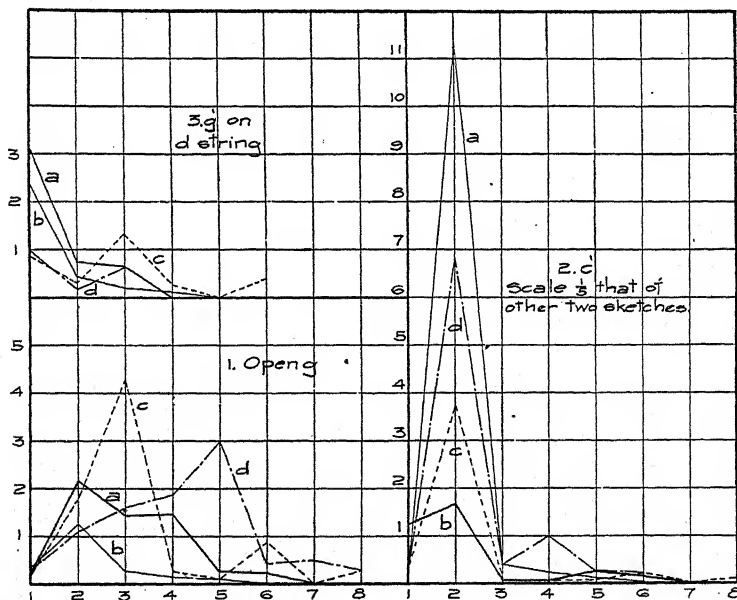
Fig. 11.

[1] It may be added that after varnishing this seemed to be an extremely good violin, as indeed will appear from the similarity of its curves to those of the best instrument shown in Fig. 11. It remains of course to see the effect of aging on its tone-quality. All the parts were adjusted and graduated with the greatest skill and care by Mr. Della Torre, who is a master in this line, and the result leads us to hope that good violins will not vanish from the world when the Cremonas wear out.

which shows the records of several instruments tested. So far as the writer can judge at present, the nearest approach to the ideal is represented in record (*a*), which is from a very valuable violin owned, and very kindly brought to the laboratory and played for me, by Professor Van Hulsteyn, of the Peabody Conservatory of Music. A rough attempt has been made to arrange the other records in order of decreasing merit of the instruments, but no great weight should be given to this order now. First a considerable number of the very best instruments should be very carefully compared in order to arrive at an idea of what ideal violin tone would be, and then it will be easier to classify various specimens by their deviations from this standard. It will be found, of course, that a great many violins will approach the standard in some parts of their range, while they will fall away from it in others.

VIII. The Vowels.

Our method ought to be useful in attacking the very interesting and unsettled question of the composition of the vowels. Figure
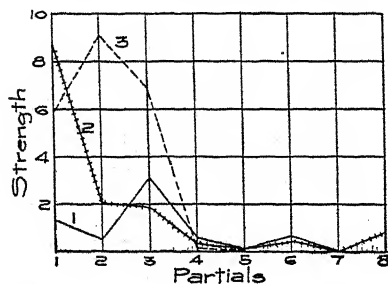
Fig. 12.

12 shows specimen records of a few vowels,[1] but the question is far too large to be discussed on the basis of the limited number of experiments we have so far been able to carry out in this line.

There seems no reason why some such device as this might not be used to test voices for training, first to find whether the voice has possibilities, and then to test the effects of a given method of training in bringing the voice up to what might be settled on, as indicated above for the violin, as a standard of quality for a voice of that range. Of course it is not meant that we must subject voices to an absolutely mechanical criterion, but a physical test ought to prove of great assistance.

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[1] Only a very few of a number of observations are here figured, as it seems useless to give a large number of undigested results.

Summary.

1. The method devised gives *quantitative* records of the composition of sounds *actually emitted* from instruments, in terms of the energy of the partials.

2. The quality of a sound heard in a room with reflecting walls depends on the position of the instrument producing the tone, and of the hearer.

3. The lowest tones of a violin are almost lacking in fundamental.

4. Muting increases fundamental and decreases overtones in the lower tones of a violin; in the higher ones it decreases all the constituents.

5. In a good violin the lower tones are weak in fundamental and strong in the first two or three overtones, the energy rapidly disappearing towards the sixth or eighth partial. For higher tones fundamental increases and overtones die out, so that on the *a* string there is scarcely any overtone above the second partial. The seventh partial rarely appears in noticeable amount in any tone.

6. Some records were made of other instruments and of vowel qualities, but are not discussed.

STUDIES IN LUMINESCENCE.

BY EDWARD L. NICHOLS AND ERNEST MERRITT.

XV. ON FLUORESCENCE AND PHOSPHORESCENCE BETWEEN + 20° AND − 190°.[1]

AMONG the earlier investigations made by the use of liquid air were the observations of Dewar[2] upon the effect of cooling various phosphorescent substances. He showed that many bodies which are phosphorescent at ordinary temperatures cease to respond to excitation at the temperature of liquid air and that numerous organic substances begin to show phosphorescence only at very low temperatures.

In 1904 the present writers[3] published an extended list of such substances with notes concerning intensity, duration, color, and the easily noted changes observed during cooling.

More recently Lenard has stated the general principle, based upon the study of numerous phosphorescent sulphides, that for each band in the spectrum of a body excited to phosphorescence there is a certain range of temperatures within which it is phosphorescent and above and below which no phosphorescence of long duration occurs, although fluorescence may be present both at higher and lower temperatures.[4]

Some Preliminary Observations of the Effects of Temperature on Phosphorescence.

A study of the rates of decay of various phosphorescent solids, including some of the sulphides investigated by Lenard, shows a more complicated set of phenomena than can be directly accounted for by his hypotheses.

[1]A portion of the apparatus used in this investigation was purchased under a grant from the Carnegie Institution.

[2]Dewar, Proc. Royal Soc., 55, p. 340.

[3]Nichols and Merritt, PHYS. REV., XVIII., p. 356.

[4]Lenard, Annalen der Physik (4), XXXI., p. 663, 1910.

For the observation of the effect of temperature upon the bright-
ness and duration of phosphorescence down to the temperature of
liquid air a layer of the substance to be studied was placed in a
capsule formed by plugging one end of a brass tube about 20 cm.
long and 2 cm. in diameter with a metal disk that fitted the bore
of the tube snugly and was counter sunk about 4 mm., leaving
between its upper face and the end of the tube a shallow cylindrical
opening.   This tube t, Fig. 102, of
which only the upper end is shown,
was threaded to fit within a mas-
sive cylinder of copper, c, which in
turn fitted into the bottom of a
tube of vulcanized fiber, f.   The
tube f was inserted in a circular
opening in a box of the same ma-
terial.   Only a portion of the bot-
tom and of one of the walls is

Fig. 102.

shown in the diagram.   The surface of the phosphorescent sub-
stance was viewed through a tube, s, inserted through the wall of
the box and carrying a converging lens and rectangular prism at its
inner end.
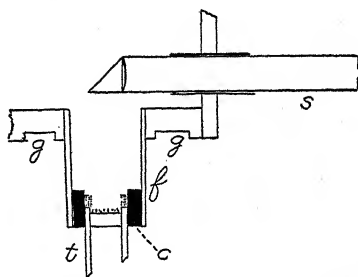
The substance was excited to phosphorescence by projecting upon
its surface by means of a system of quartz lenses and a mirror of
speculum metal, not shown in the figure, the light from a flaming
arc rich in ultra-violet rays.   In the path of the exciting light, but
outside of the box, was mounted a shutter, the opening and closing
of which were automatically recorded on a chronograph, thus mark-
ing the times at which excitation began and ended.

The brightness of phosphorescence from moment to moment after
the close of excitation was measured by means of a simple form of
photometer especially constructed for these experiments.   It con-
sisted of a broad T-tube, as shown in Fig. 103, within which a vertical
sheet of glass was mounted which carried at its center a small
right-angled prism.   This prism received light from the ground
glass screen g and reflected it to the eye at l.

The straight arm of the photometer was in line with the tube
(s, Fig. 102) so that the observer at l looking past the small prism

received light also directly from the phosphorescent surface. His field of view appeared as a disk of phosphorescent light p, Fig. 2, in the middle of which was a square patch of light due to the prism.

The ground glass screen g (Fig. 103) was illuminated from without by a small tungsten lamp with frosted bulb, and by moving this lamp along a photometer bar in the prolongation of the axis of the transverse tube c (Fig. 103) the contrast field just described could be adjusted to equality. Suitable color screens in front of the lamp adapted its light in quality to that of the phosphorescent substance. The color match thus obtained, while not perfect, was sufficiently close to permit of satisfactory readings with the photometer.
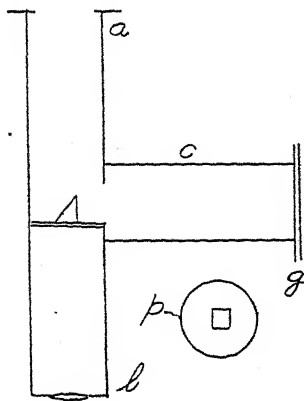
With the assistance of Mr. J. F. Putnam the phosphorescence of a number of substances was studied. To obtain the curve of decay, the comparison lamp was placed at a measured distance from the ground glass screen, the substance was excited to phosphorescence by opening the shutter for a time more than sufficient to ensure saturation, $i.\ e.$, ten seconds,[1] the appearance of the contrast field of the photometer was watched as the phosphorescence died away and the instant of balanced brightness was recorded on the same chronograph sheet on which the opening and closing of the shutter had been automatically registered. This operation was repeated several times, and similar sets with the lamp at various distances were made.

Fig. 103.

To obtain the corresponding curves at lower temperatures the bottom of the tube t (Fig. 102) was submerged in liquid air. The temperature of the phosphorescent substance was determined from the electrical resistance of a coil of fine copper wire which surrounded the top of the tube t, close to the substance and within the walls of the hollow copper cylinder c. The circular groove gg, Fig. 102, was cut in the bottom of the fiber box to receive the lip of the Dewar flask during the operation of cooling.

[1] It had been found by trial that with the powerful source of ultra-violet light used in these experiments saturation was attained in about six seconds.

As will be seen from Fig. 104, which contains the curves of decay of phosphorescence of a sample of zinc sulphide (Sidot blende) studied by the method just described at $+ 20°$ and $- 125°$, the effect of cooling this substance was to greatly hasten the rate of decay.

Both of these curves are of the well known type common to all cases of phosphorescence that have thus far been quantitatively investigated, as may be shown by plotting the square root of the reciprocal of the intensity as a function of the time, and the first and second processes of decay are distinct and well marked.

The intercepts of the curves for $I^{-\frac{1}{2}}$ show that the initial intensity at $- 125°$ was considerably greater than at $+ 20°$ although decay of phosphorescence was more rapid.

This result appears at first sight to be in agreement with the recent dictum of Lenard that for each substance there is a temperature of maximum phosphorescence below which the activity diminishes and finally disappears. Further study however develops more complicated relations. If, for example, instead of finding the curves of decay at two or more fixed temperatures the substance is allowed to cool slowly to the temperature of liquid air and the length of time required at various temperatures, after exciting to saturation, for the phosphorescence to fall to a given brightness is recorded, the curve shown in Fig. 105 is obtained. From these measurements it is evident that there is a maximum of duration of phosphorescence at some
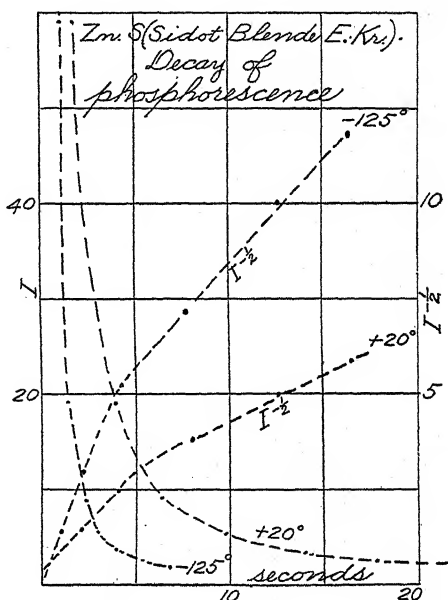
Fig. 104.

temperature above + 20° C., another, very pronounced, at − 40° and third below − 160°.

Similar studies of two phosphorescent sulphides of barium, prepared by the method of Lenard and Klatt, gave the curves shown in Figs. 106 and 107. The first of these sulphides, which had a trace of copper as the active metal with a lithium salt as flux, gave the characteristic yellow-red phosphorescence at + 20° with long duration; but between − 30° and − 140° its action was either too weak or too fleeting to permit of measurement with our apparatus. Upon further cooling a slight increase in the duration occurred which reached a maximum at − 160° and fell off again to scarcely measurable values at the temperature of liquid air.
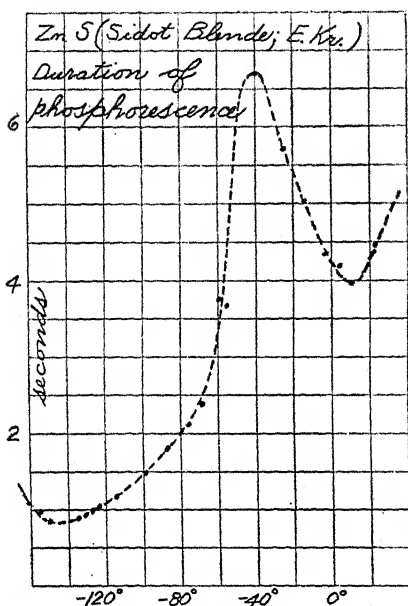
Fig. 105.

The other sulphide, the curve for which is given in Fig. 107, has bismuth as its active metal with a potassium salt as flux.

This substance shows a minimum of phosphorescent activity at − 60°, although still of measurable duration and this is followed on further cooling by a condition of greatly increased brilliancy. The crest of the curve lies at − 135° at which temperature the duration exceeds that at + 20°. Below − 135° the diminution is exceedingly rapid and before − 160° is reached no phosphorescence, measurable by this method, remains. This substance after having been observed while cooling throughout the range from + 20° to − 160° was allowed to warm very slowly and measurements were made at intervals to see whether the curve of cooling would be repeated. It will be seen that these observations, which are indicated by the symbol *x*, all lie close to the curve.

In addition to these changes in the duration of phosphorescence there are numerous cases in which the color of the emitted light changes with the temperature and others where the color shifts during decay.[1] Furthermore phosphorescence sometimes differs in this respect from the fluorescence of the same substances. Andrews[2] has described several striking instances in which a light pink
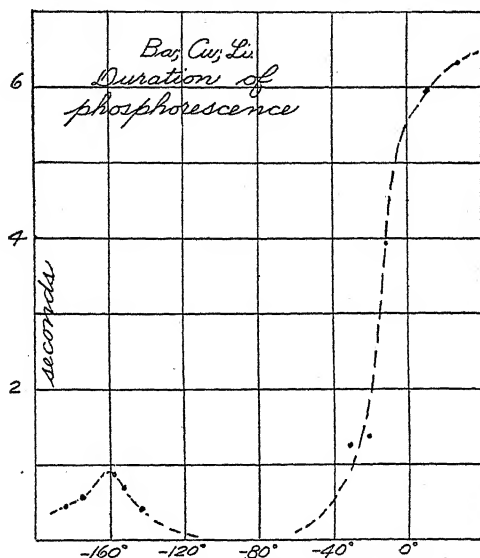
Fig. 106.

fluorescence is followed by an orange-yellow phosphorescence, yellow by light green, etc. On the other hand, the present writers have shown that a single band in the phosphorescence spectrum of Sidot blende dies away as a whole, without change in the distribution of intensities or shift of the wave-length of the maximum. These observations have been verified and extended through a much greater range of time by photographs of the phosphorescence spectrum of this substance made by Dr. C. A. Pierce.[3] If, as seems probable, this is a general characteristic of such spectra it would seem to indicate a more complex structure in many fluorescence spectra than has been commonly assumed, and in particular that many broad

[1]Nichols and Merritt, PHYS. REV., XVIII., p. 355.
[2]W. S. Andrews, Science, XIX., 1904, p. 435.
[3]Pierce, PHYSICAL REVIEW, XXX., p. 663.

bands which are usually described as single really consist of two or more overlapping components.

If the various components of such a composite band die away at very different rates, the changes of color often observed during decay would be accounted for, and if differently affected by temperature the remarkable variations in the quality of the phosphorescent light when substances are cooled by liquid air would be explained. The presence of superimposed bands, each having a
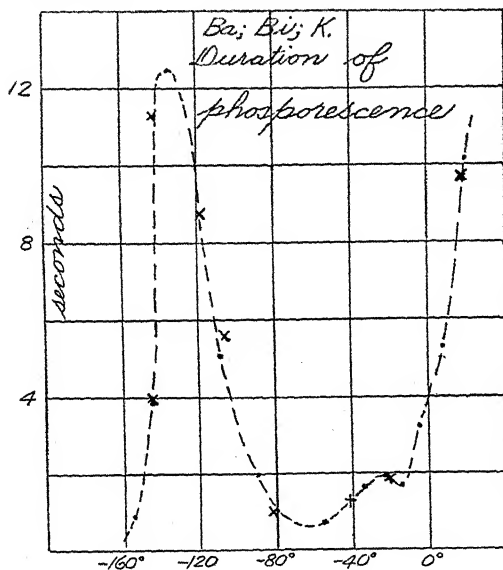
Fig. 107.

maximum of brightness at some given temperature and a maximum of duration, while the maxima for the various components of a broad band occur at widely different temperatures, would doubtless suffice to account for the phenomena already described in this paper.

For these reasons it seemed important to study the nature of these spectra more precisely than had hitherto been done and to observe in particular the effects of temperature upon their intensity, wave-length, and structure.

Some Effects of Temperature on Fluorescence Spectra.

Fluorescence lends itself more readily to precise measurements than phosphorescence because of its greater brightness and its sta-

bility, but special precautions are necessary to avoid the blending of the exciting light with that emitted by the fluorescent substance. Even a small percentage of stray light from the exciting source will greatly modify or may entirely mask the fluorescence of a body. Many fluorescent solids are not recognizable as such when illuminated by daylight on this account, and it is only when they are exposed to ultra-violet radiation or to cathode rays that their fluorescence appears. It is necessary in such cases to excite the substance by radiation of wave-lengths readily separated by dispersion from the fluorescence bands.

In the measurements to be described the apparatus was similar to
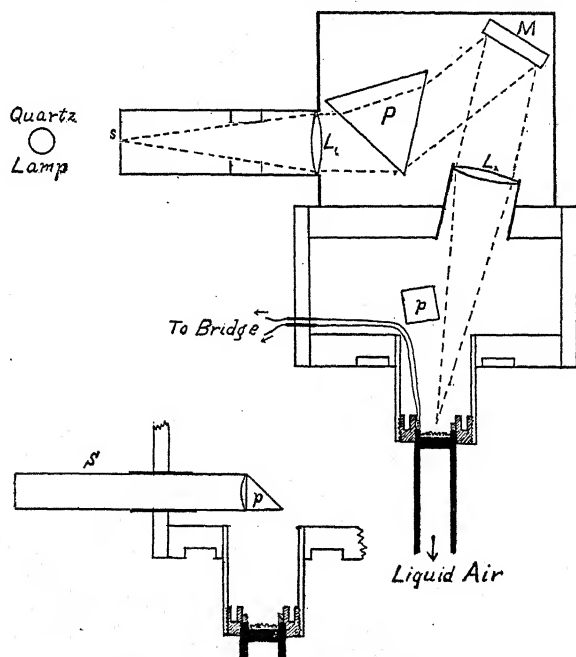
Fig. 108.

that shown in Fig. 102, but the source for excitation was a mercury arc in quartz, the light from which was dispersed by the quartz system $L_1P_1L_2$ (Fig. 108). By turning the mirror M any desired portion of the mercury spectrum could be thrown upon the surface of the substance under observation. The arrangements for cooling by liquid air and for measuring the temperature were those already described.

Instead of the photometer, however, one collimator of a Lummer-Brodhun spectrophotometer was placed in line with the sighting tube, *S*, and received the fluorescence light from the specimen. As a comparison source an acetylene flame was used and the illumination of the comparison slit was varied by moving this flame, properly housed and screened, along a photometer bar. In this way the troublesome corrections for slit-width discussed in a previous communication[1] were avoided.

By this method the brighter bands of several fluorescent liquids and solids were carefully measured, wave-length by wave-length.

In each case as narrow a slit was used as the brightness of the band would permit and care was taken to locate the crest by means of readings made alternately on either side at wavelengths successively nearer together. The most convenient and successful method consisted in locating the comparison flame at some point on the bar such that the intensity of its spectrum in the region of the crest would be considerably less than that of the brightest portion of the band, and exploring the fluorescence spectrum for regions in which it and the comparison spectrum were of equal intensity. A repetition of this process for various intensities of the comparison flame affords data from which a complete map of the band or group of bands may be plotted.
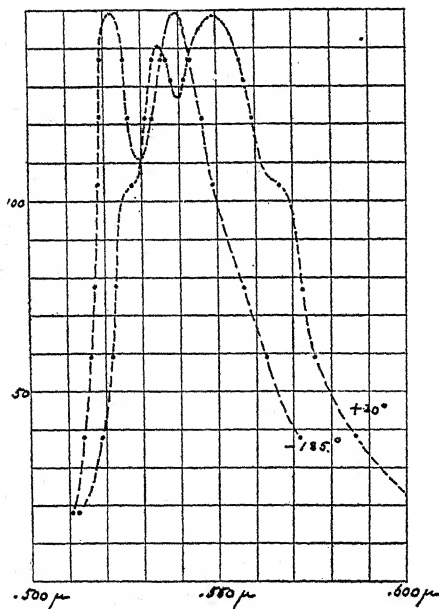
Fig. 109.

Fig. 109 shows the results of measurements of an alcoholic solution of fluorescein which we studied in the manner described when excited at $+ 20°$ and $- 185°$. What appears by ordinary spectro-

[1]Nichols and Merritt, PHYS. REV., Vol. XXXI., p. 500.

scopic inspection to be a single broad band extending from about
.500μ to beyond .600μ is found at $+ 20°$ to consist of at least four
overlapping bands with crests at .528μ, .534μ, .550μ and .567μ.
Two of these bands appear merely as shoulders upon the sides of
the curve. Upon cooling to $- 185°$ there is a marked shift towards
the violet, a narrowing of the composite bands and complete dis-
appearance of the two shoulders. The crest towards the violet is
increased in brightness and the yoke between the crests is much
deeper. At this temperature one might under favorable conditions
recognize the doubleness of the band by inspection with an ordinary
spectroscope.

That these changes are not entirely due to the freezing of the
solution may be seen from Fig. 110, in which the form of the band is
given at $+ 20°$ and at $- 80°$. The shift at the latter temperature
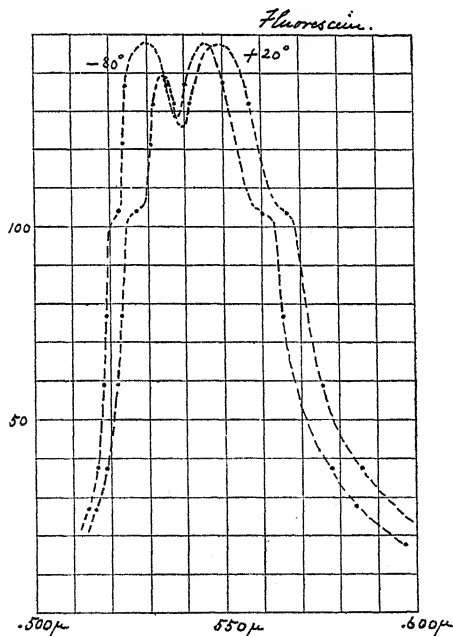
Fig. 110.

is only .004μ as compared with .012μ at the temperature of liquid
air and there is no deepening of the yoke. The shoulders on either
side are still present, and while there is a slight narrowing owing

to a greater shift of the red side than of the violet side of the band below the shoulders there is a widening of the upper part of the curve on account of the growth of the crest at .530μ, which at $-80°$ is broader than that towards the red.

The most interesting interval in the study of this substance, however, lies between $-120°$ and $-160°$. At $-120°$ (see Fig. 111) the fluorescence band has been greatly narrowed by loss on the side towards the red. Both peaks are narrower than at higher tem-
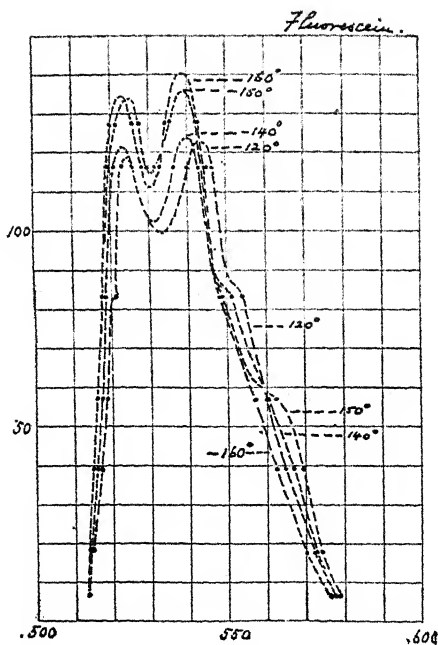
Fig. 111.

peratures, particularly the one towards the violet, which has shifted further towards the violet. The height of the crests is lower than at any other temperature at which measurements were made. The shoulders are still present, although that towards the violet, .620μ, has nearly disappeared. Between $-120°$ and $-140°$, the shoulder at .555μ gradually disappears while at $-140°$ a new shoulder between .560μ and .565μ begins to develop. This has become very prominent at $-150°$ but is gone again at $-160°$.

These, or any similar set of movements and changes in fluorescence

bands can undoubtedly be accounted for, by assuming a sufficient number of superimposed components each having its fixed position in the spectrum and each changing in intensity independently with the temperature. These elementary bands, on the other hand, may be supposed to shift with temperature and to change in width as well as in brightness.

That in the obviously complex band of fluorescein under discussion there are regions very differently affected by temperature is shown in Fig. 112 in which the variation of certain wave-lengths
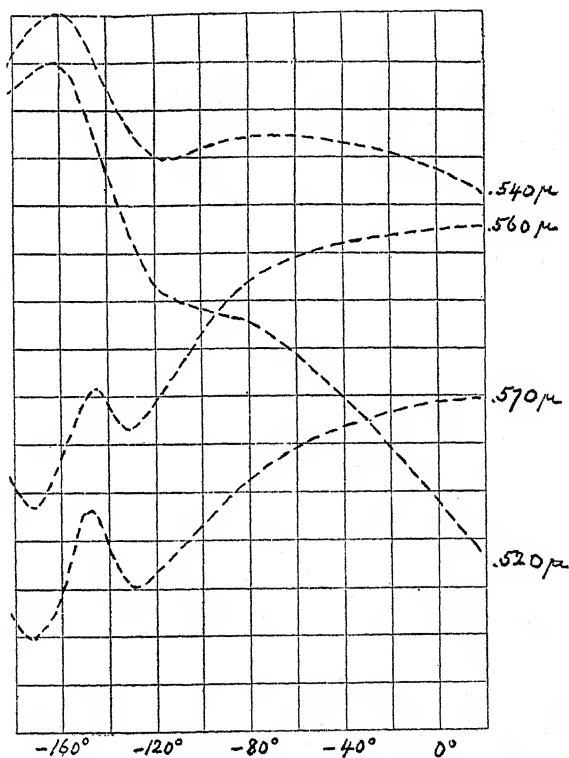
Fig. 112.

is traced throughout the range of temperature from $+ 20°$ to $- 180°$.

If wave-lengths on the violet side of the band are selected curves are obtained of which those for .520μ and .540μ in the figure are typical, whereas on the side towards the red, from .560μ to .570μ,

quite a different type occurs showing on the whole decrease of intensity on cooling.

Several other examples of independent variations with temperature of different parts of a fluorescence band have been observed in the course of our work. An interesting case is that of Sidot blende (Fig. 113) in which the regions about .514μ and .452μ were compared.
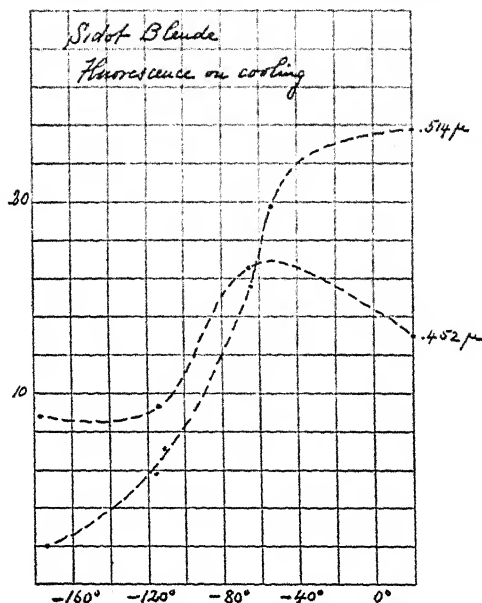
Fig. 113.

At + 20° the ratio of intensities .514μ/.452μ, the corresponding regions in the spectrum of the acetylene flame being as usual the basis of comparison, is 1.83. At − 63° it is unity and at − 175° it is 0.23. This denotes a marked change in the color of the fluorescence of this substance when cooled to the temperature of liquid air.

Two crests of the rather complicated fluorescence band of the phosphorescent sulphide designated as Ba, Bi, K in an earlier paragraph of this paper were studied in the same way and the results are shown in Fig. 114.

Not all fluorescence bands are so complex as that of fluorescein. A sample of natural willemite, mapped at various temperatures, gave the set of curves in Fig. 115. The only sign of complexity is

the slight shoulder on the side towards the violet in the curve for
$+ 20°$. At $- 90°$ this has vanished and that side has moved
towards the longer wave-lengths, but there is no appreciable shift of
the crest and the side towards the red is coincident with that of the
curve for $+ 20°$. At $- 165°$ there is a decided shift of the entire
curve towards the red, narrowing of the band, and a considerable
reduction of intensity.

The very interesting behavior of the fluorescence band of a dilute
solution of resorufin in alcohol is illustrated in Figs. 116 and 117.
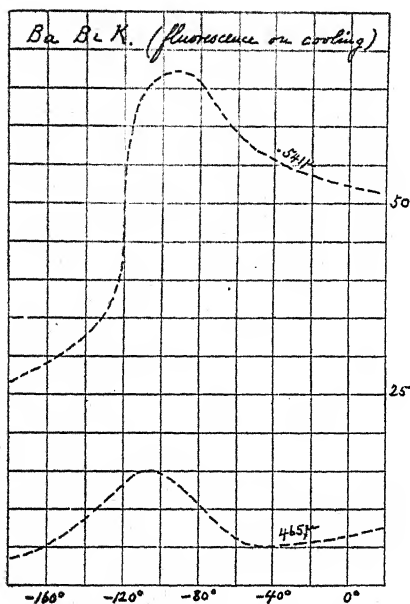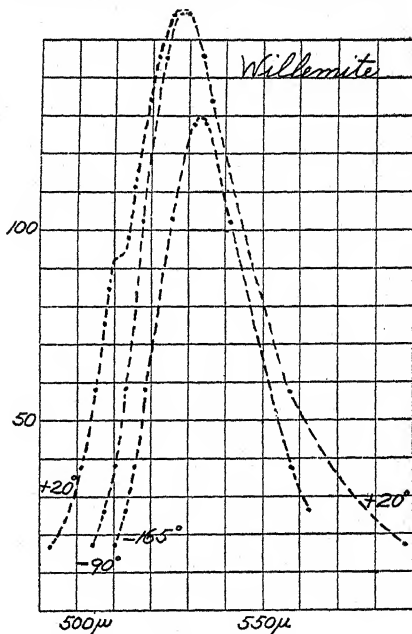
Fig. 114.   Fig. 115.

This band is typical of what we have regarded in all our previous
work as a single simple band. It retains its form accurately at all
four of the temperatures at which it was measured. Upon cooling
to $- 90°$ there was diminution of intensity at the crest to about
one-half, but no appreciable shift. On further cooling to $- 165°$,
during which process the liquid was solidified, there was shift from
$.593\mu$ to $.596\mu$ and when the temperature was reduced to $- 185°$
there was great increase in temperature without further shift.
From this case alone one might be inclined to connect the shift with

the change of state but for the fact that a very similar movement occurs in solids, such as willemite, just described, anthracene, etc.

Resorufin is an exceptional case in its simplicity. In most instances the study of fluorescence and phosphorescence by the methods described in this paper is rendered almost futile on account of the complexity of the phenomena and the fact that one is compelled to be content to observe the resultant of movements which
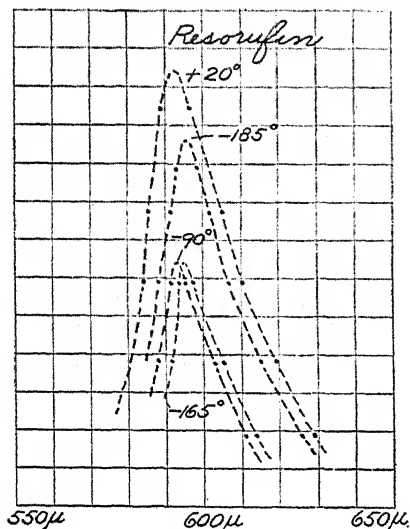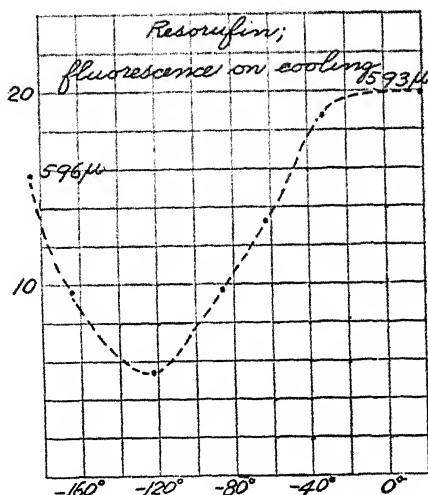
Fig. 116.   Fig. 117.

we can hardly hope to analyze and the components of which we have not as yet succeeded in separating.

One very interesting class of substances in particular, the phosphorescent sulphides of Lenard and Klatt, yield as the result of investigation with the spectrophotometer only evidence of the extreme complexity of their bands and of the complicated changes when the temperature is modified.

One example out of many will serve to illustrate the character of the phenomena. In Fig. 118 are the curves for one of these substances, a strontium sulphide with barium as the active metal and a sodium salt as flux. The curves of its band are given from measurements at $+ 20°$ and $- 190°$.

Since we cannot isolate the numerous components of such a group

we must seek simpler cases for our preliminary studies, and these are to be found in the double band .534μ and .502μ of commerical anthracene, the parts of which can be measured separately, and the
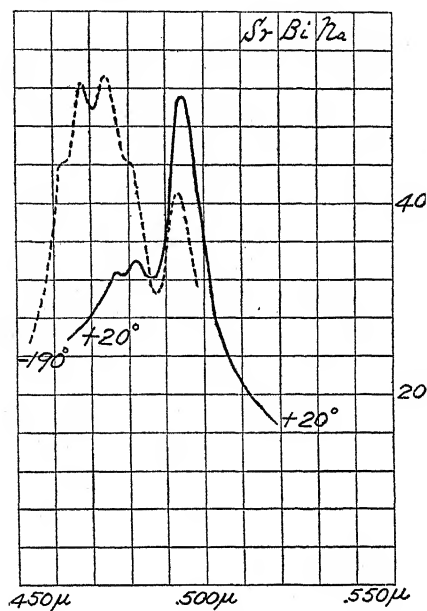
Fig. 118.

narrow single line like bands of the uranyl salts recently studied by photographic methods at various temperatures by Becquerel.[1] Of these we are now making a detailed spectrophotometric study, the results of which we reserve for a separate communication.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
    May, 1910.

[1]Becquerel, Onnes, and Pauli, communications of the Physical Laboratory of the University of Leiden, III, 1909.

PREPARATION AND PROPERTIES OF THE HEUSLER ALLOYS.

By A. A. Knowlton.

SINCE the discovery by Heusler in 1903 that it is possible to prepare strongly ferro-magnetic alloys of the non-magnetic metals copper, manganese and aluminum important papers concerning the properties of these alloys have been published by Heusler,[1] Starck, Take, Gumlich, Guthe and Austin, McLenan, and others. The most important facts as developed by these investigators may be briefly summarized as follows.

1. The magnetic permeability of the best specimens is about that of cast iron. Hill reports one specimen having an induction of 11,000 in a field of 8 c.g.s. units. This value is however considerably above that obtained by other observers.

2. The permeability is often greatly increased by annealing for some time at a temperature a little above 100° C. At higher temperatures the alloys become non-magnetic and this transformation may be irreversible so that upon cooling to room temperature the specimen is sometimes found to have suffered a partial or, in some cases, even a total loss of magnetic quality.

3. The most strongly magnetic alloys were found by Heusler and Starck to be those containing manganese and aluminum in atomic proportions; *i. e.*, in the ratio of two parts by weight of manganese to one of aluminum. Largely upon this ground it has been quite generally assumed that the copper plays no part in the alloy except as a solvent for a magnetic manganese-aluminum alloy which would otherwise be too hard and brittle for experimental use.

4. Other trivalent metals as arsenic, antimony, or tin may be substituted for the aluminum but the alloys thus obtained are much inferior to those containing aluminum.

5. Zahm and Schmidt have found a Hall effect in these alloys of

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[1] For bibliography of references to the papers cited see an article by Stephenson PHYS. REV., vol. 31, p. 252, 1910.

the same sign as in iron but larger than in any substance previously studied.

6. Ingersoll has found that these alloys do not show the Kerr effect. This is of special importance both on account of the theoretical importance of the phenomenon itself and because it is the only way in which the magnetic properties of these alloys are strikingly different from those of iron.

As nearly all observers had found occasion to complain of the poor mechanical quality of their specimens it seemed best at the outset to undertake such a study of their preparation as should, if possible, overcome this difficulty. An account of the methods finally adopted and results obtained constitutes the first part of this paper. Later fourteen specimens prepared in the winter of 1908 were tested with respect to their electrical conductivity, magnetic properties, crystalline structure, and changes under heat treatment.

The first stage of the preparation consisted in dissolving manganese in molten copper. On account of the great affinity of manganese for silicon a graphite crucible must be used and it is even then impossible to employ any flux without causing the metal to eat rapidly into the crucible. To secure a uniform mixture the melt must be kept hot for several hours and stirred frequently;—or the same result may be obtained with less loss by oxidation if the bath is cast and remelted several times. The materials used were ordinary electrolytic copper and, at first, chemically pure manganese. Later commercial manganese obtained from the Goldschmidt Thermit Company was used with apparently equally good results. To the copper-manganese melt aluminum in slightly more than the calculated amount was added and the bath cast and remelted several times to ensure a uniform mixture. Stirring should be avoided during this part of the operation because of rapid oxidation of the aluminum. Even under the best conditions of working with small quantities of material the loss in this way is considerable and uncertain so that the proportions by analysis may vary widely from those of mixing. After several remeltings the alloy was cast in graphite molds made by boring a one eighth inch hole in an arc furnace electrode. To provide for a uniform rate of cooling in all parts of the casting these molds were heated before use to a tempera-

ture slightly above the melting point of the alloys which was about 850° C. The very heavy slag which forms instantly over any exposed surface makes it essential that the metal be poured rapidly in an unbroken stream. The molds containing the still molten alloy were covered over with a layer of asbestos wool of such thickness as to allow the temperature to fall to 250° C. in about an hour. When this temperature was reached, as indicated by a thermocouple in contact with the molds, the casting was quenched by immersion in cold water. Specimens prepared in this way were of excellent mechanical quality, free from flaws and blow holes and, in many cases malleable to a considerable degree. As will appear later the magnetic test appears to indicate that a somewhat slower rate of cooling and a slightly higher temperature of quenching might have been advantageous. From a considerable number of specimens thus prepared fourteen were selected at random for examination. These were wet ground with emery to a uniform diameter of six millimeters and tested as hereafter described.

The following table (I.) gives the chemical composition and density of each specimen.

TABLE I.

Specimen.	Cu.	Mn.	Al.	Si.	Density.
1	66.6	25.9	6.8	.73	6.72
2	67.2	22.5	10.0	.25	6.77
3	64.9	22.0	12.6	.49	6.83
4	66.9	22.3	10.4	.35	6.85
5	65.0	25.8	8.8	.40	6.86
6	62.4	26.2	12.2	.24	6.88
7	62.0	29.6	8.2	.24	6.89
8	56.4	29.6	13.1	.89	6.94
9	69.6	22.1	9.1	.20	7.08
10	65.8	26.0	7.7	.49	7.12
11	70.4	19.0	9.4	1.22	7.13
12	69.2	22.1	8.4	.32	7.14
13	72.6	20.1	7.05	.23	7.16
14	72.2	17.4	10.3	.08	7.17

In general the density was found to increase slightly on annealing at 120° C. This increase never exceeded one third of one per cent. although it appeared to take place in all cases. Hill has reported variations in density of much greater magnitude under similar

treatment. From his description it seems likely that this was a result of working with a poor mixture due to insufficiently prolonged heating. The variation in the per cent. of silica is probably caused by variations in the condition of the crucible used; a new crucible giving up more silica than one which has been in use for some time. The hardness of the various samples bears a very evident relation to the amount of silica present, No. 11 for example being very hard and brittle while No. 9 and No. 14 are soft and malleable. The use of silica-free crucibles would doubtless be advantageous. A trace of iron was present in all cases. The observed density is greater than that calculated from the composition for specimens having less than 22 per cent. of Mn while for those of higher Mn content the observed density is usually less than the calculated. The greatest difference shown by any specimen is in the case of No. 11 with an observed density 5 per cent. greater than that calculated. A considerable shrinkage frequently occurs in the specimens of good magnetic quality, although it was not possible to show any relation between the two phenomena.

ELECTRICAL RESISTANCE AND TEMPERATURE COEFFICIENT OF RESISTANCE.

McTaggart and Robertson[1] measured the electrical resistance of eight specimens of varying composition and obtained results ranging from 19.31×10^{-6} to 68.36×10^{-6}. These observers also found that the resistance-temperature curves for several of their specimens showed strongly marked minima which were not at the same temperature for the different samples. Both on account of the very wide difference in resistance found for specimens not very different in composition and because the presence of minima seems at variance with theory it appeared of interest to determine the resistance and temperature coefficient in the case of alloys prepared under different conditions. For the measuring of resistance a Thomson double bridge was used. As it was necessary to carry on magnetic measurements at the same time the apparatus was arranged as shown in Fig. 1.

A solenoid S 33.1 cm. in length and 4.5 cm. internal diameter

[1]Bul. Roy. Soc., Canada, 5, 1908, p. 31.

was wound with 1,380 turns giving a field of 49.4 lines per ampere. A cylindrical tube of soapstone 3.3 cm. external diameter was double threaded and wound non-inductively with high resistance wire. This heating coil was covered with asbestos and placed in the axis of the solenoid S which was protected from excessive heating by forcing a blast of air through the space between the heating coil and the solenoid. The specimen to be tested was supported in the axis of this system by means of the clamps used for connection to the double bridge. Temperatures were measured with a mercury thermometer since convenience was of more importance than great accuracy in this part of the work. The correctness of thermometer readings was checked before and after the observations. In the
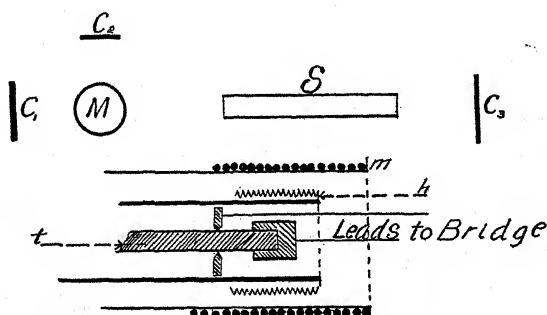
Fig. 1.

magnetic tests the magnetometer deflection method was used, the effect of the solenoid S being compensated for by the method described by Ross and Gray,[1] and proper corrections made for the end effects of the specimen itself. Table II. shows the values ob-

TABLE II.

No.	$\sigma x\, 10^6$	$a x\, 10^4$	I	No.	$\sigma x\, 10^6$	$a x\, 10^4$	I
1	47.9	23.5	81	8	62.7	15.7	112
2	53.0	23.0	251	9	60.6	15.5	246
3	59.3	13.9	284	10	64.2	13.5	208
4	45.9	27.3	331	11	56.0	13.4	152
5	45.4	22.7	331	12	60.8	11.8	160
6	46.4	24.6	332	13	54.9	15.3	172
7	48.9	18.4	212	14	55.9	9.1	4.8

[1]Proc. Roy. Soc. Ed., 29, 1908, p. 182.

tained by electrical and magnetic measurements upon the specimens in their original condition. The external field H' was constant and equal to 80 lines/cm.² during these tests. No correction is here made for end effects.

The variation in the value of the specific resistance (σ) for different specimens, while much less than that found by McTaggart and Robertson, is still larger than can be accounted for by differences in composition. Later work indicates that a slower rate of cooling would have given more uniform results. The variation of σ with temperature is shown in Fig. 2 which is a set of curves for specimen 3 but wholly typical of what happens in every case. The curves are straight lines differing from those for a pure metal only in slope and in that at certain temperatures the resistance undergoes a permanent decrease with time; a process of artificial aging frequently met in alloys. The numbers given in the diagram indicate the time which elapsed between the lettered readings; *e. g.,* in curve II. the reading at C was taken two hours after that at B. The microscopic examination which was carried on in connection with this part of the work showed that each decrease in resistivity accompanied an improvement in crystalline structure and that any treatment, such as quenching from a comparatively high temperature which impaired the crystallization was accompanied by a corresponding increase in resistivity, an increase which in one case amounted to doubling the value before quenching. There was no indication that the resistance depended at all upon the kind of crystals. The temperature coefficient (α) which was originally very low and lowest for the specimens of highest resistivity increased as the resistance at zero decreased and in nearly the same ratio as is shown by the fact that the successive curves are parallel. The fact that the descending branch of curve II. does not fall exactly upon the ascending branch of curves III., IV. and V. is probably due to an error in measuring the length between the clamps of the double bridge. It will be noticed that after five hours at 140° the succeeding heatings represented by III. and IV. (both of which began at 28°) produced no effect, but that above 160° a second rapid decrease in resistivity took place until a second condition of stability was reached which persisted until the temperature was raised above

225°. In some later work it has been found that these three temperatures are important transition points for many specimens. The only way in which these alloys are different from many others in respect to their conductivity is in the degree of their sensibility to
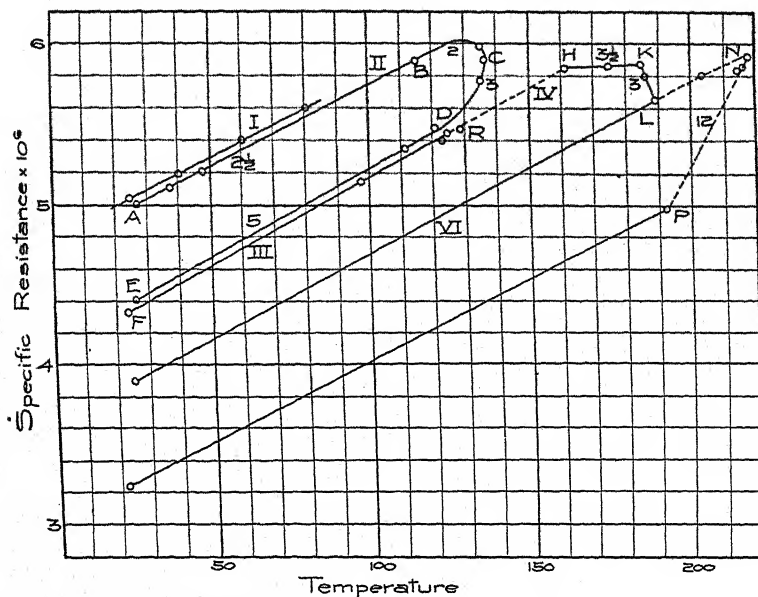
Fig. 2.

annealing processes and this is altogether due to the ease with which their crystalline structure undergoes modification, as is shown in a later part of this paper.

RELATION OF MAGNETIC QUALITY TO CRYSTALLINE STRUCTURE.

During this part of the work several of the specimens were subjected to extensive cycles of heating in the furnace previously described, magnetic measurements being made before and after each cycle and at such other temperatures as seemed desirable. Before and after each cycle one end of the specimen was polished and etched with a mixture of hydrochloric acid and ferric chloride. This mixture was adopted after experiments with many etching agents because it was found to give more uniform results than any other, and also because the coloring action of the ferric chloride

greatly increased the contrast between the different crystals. In this way very complete studies were made of specimens 2, 3, 7 and 10, and work enough done on all others to determine their transformation points and to show that their general behavior was similar to that of the specimens more extensively studied. An outline of the treatment to which a single specimen (No. 7) was subjected, together with some account of the results obtained, is given in tabular form in Table III. The total time of annealing in this case was somewhat more than a hundred hours, and the value of the magnetization reached varied from about 400 c.g.s. units to zero with corresponding variations in crystalline structure, as is shown in the accompanying microphotographs.

TABLE III.

Temp. Max.	Hrs.	I	σx 10^6	Photo.	Remarks
——	——	298	50	30	16 months after casting.
100	10	351	42.1	31	
133	3.75	376	41.7	32	Curve I.
160	4	385	38.9	—	
225	5	270	35.4	35	
290	2	350	35.4	36	Curve II.
100	5	——	——	—	Heating continued without cooling.
206	2	350	——	—	No marked change.
225	1	253	——	—	Curve IX.
225	3.5	153	35.6	36	Curve XIV.
300	5	206	——	38	
250	9.5	118	——		Curve XIX.
350	.25	145	——	39	Cooled rapidly.
290	5.75	123	——	40	Cooled slowly.
350	2.5	219	——	41	Curve XXIV.
275	5.5	80	39.1	42	
350	5	——	60.6	43	Quenched. Non-magnetic.
250	3.5	——	——	—	No effect.
400	4	——	——	—	No effect.

A comparison of the value of I in Table III. with that for the same specimen (No. 7) in Table II. shows an increase from 212 to 298 in the intensity of magnetization for the same field strength. This increase, which amounts to 40 per cent. of the original value, took place at ordinary temperatures during a period of sixteen months. Heating to temperatures up to 160° apparently completed

in something less than 20 hours the transformation which had been going on slowly at room temperatures. Although the maximum intensity of magnetization was slightly increased by the last heating (160°) the specimen was at the same time rendered much harder magnetically by this treatment. Heating at 225° for five hours caused a 25 per cent. decrease in the value of I while two hours at 290° restored the specimen nearly to its best condition (curve II.). Subsequent heatings at temperatures below 225° produced no effect, curve V. which was taken after two hours at 100° and an equal time at 206° being identical with curve II. Even a short time at 225°, however, produced a marked change as is shown by curve IX.
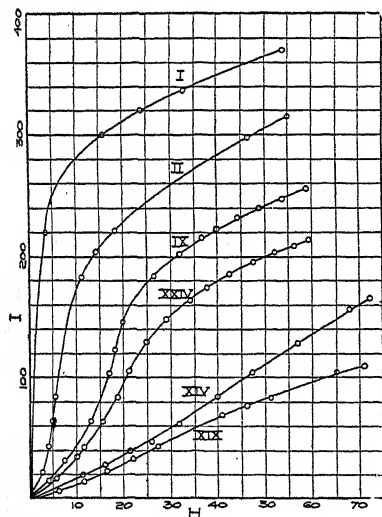
Fig. 3.

and the loss of magnetic quality continued on further heating at 225° and 250° (curve XIV.). Partial recovery again followed heating to 300°. Nine and one half hours between 225° and 250° left the specimen very poor magnetically (curve XIX.) with the expected recovery at 300° or slightly above (curve XXIV.). After this heating to 350° it was noticed that the alloy reacted to the etching agent (HCl and FeCl₃) very differently from its previous behavior. Up to this point there had been no tendency to deposit copper even when the etching was continued for from five to eight minutes, but on attempting to etch following this heating a coating of copper so thick as to entirely conceal the structure was deposited in less than a half minute. This of course indicates a breaking up of some of the crystals. Chilling from this temperature left the alloy non-magnetic and it was not found possible to render it magnetic again by any thermal treatment although after some months at ordinary temperatures it was found to have become appreciably magnetic again.

These results are in good accord with those of Asteroth who

states that he found it possible to control hysteresis losses by the rate at which his specimens were cooled through the transformation temperature. The writer's interpretation of the phenomena is, however, somewhat different. Transformation from the magnetic to the non-magnetic state takes place over a range of about 50°, the lower limit for this particular specimen being 225° where the loss of magnetic quality begins to take place rapidly and the upper limit being 275° where the specimen is totally non-magnetic. Annealing at the lower limit leaves the alloy very hard and with a low maximum induction while annealing at the upper limit followed by moderately rapid cooling softens the specimen and increases the maximum induction. This was found to be the case with all specimens examined.

So far as the permanent changes which take place on heating are concerned they may be explained if we assume that the magnetic properties belong to molecular groups which are constituents of a certain type of crystals which form at temperatures in the neighborhood of 300° and are stable at ordinary temperatures but pass through a meta-stable condition between 225° and 275°, while the non-magnetic crystals which constitute the stable form within this range are meta-stable both at 300° and at room temperatures. An examination of the micro-photographs taken in connection with the curves of Fig. 3 as indicated in the table appears to verify this assumption since a certain type of crystal is found to increase on annealing at either 100° or 300° but disappears, to a considerable extent, on annealing at 225°.

The micro-photographic examination which was carried on in connection with the magnetic testing showed that the alloys contain crystalline masses of three distinct sorts as seen in the photographs of Fig. 4. The etching agent was in all cases the hydrochloric acid-ferric chloride mixture previously referred to and a magnification of about 200 diameters was used in all cases.

Photograph no. 24 is a copper-manganese alloy (Cu 70 per cent.) such as was used as a basis for the preparation of the Heusler alloys. It is included to show the striking change in structure produced by the addition of aluminum.   Photographs 36–37–38–41–44 are of No. 7 and show the structure corresponding to the curves previously de-
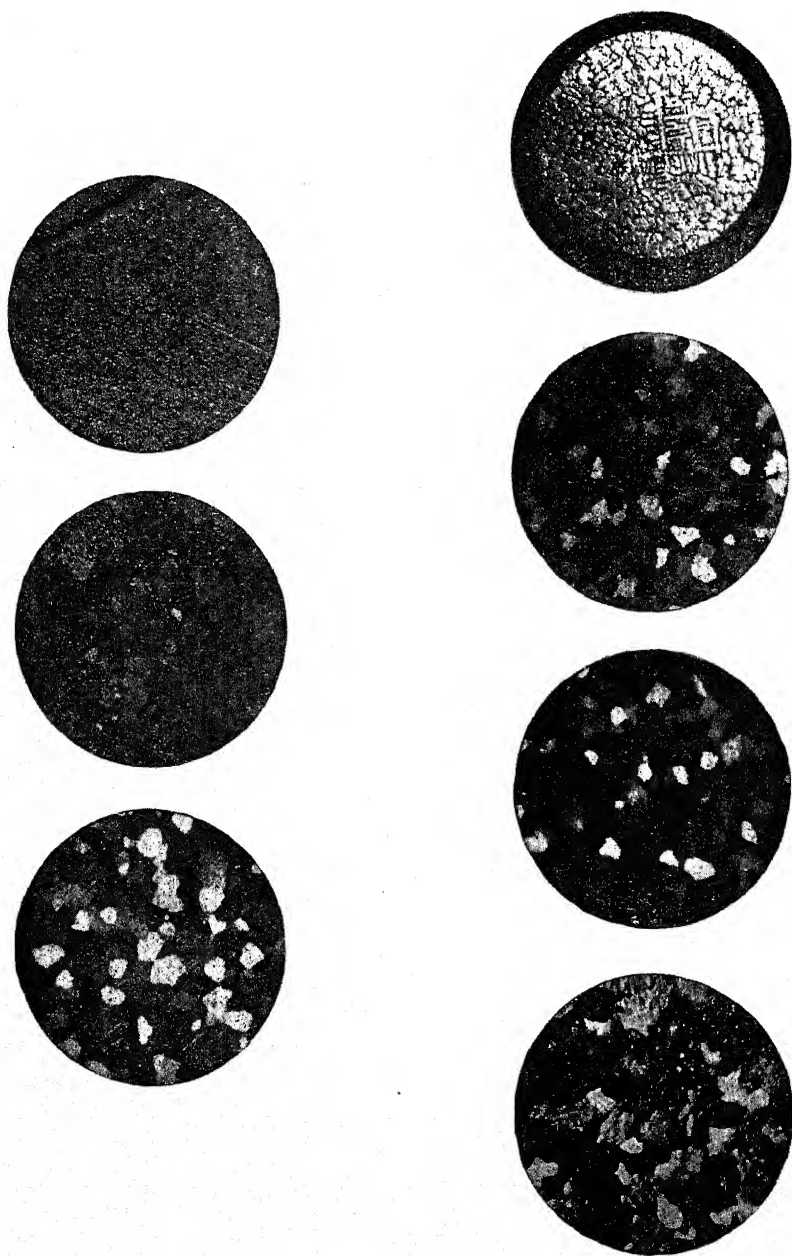
Fig. 4.

scribed as indicated in Table III. Of the three sorts of crystalline masses (white, dark gray and black) those which are unaffected by the reagent and consequently appear white in the prints are by far the most striking and show the widest range of variation. It will be noticed that the area of the photograph covered by these crystals is large in all cases where the specimen was highly magnetic and decreases with loss of magnetic quality (compare 36 and 38, strongly magnetic; with 37, weakly magnetic; and with 44, non-magnetic). No. 41 was taken after the first heating to 350° and amply confirms the supposition as to the breaking up of the crystals already discussed. On account of the rapid deposition of copper in this case it was necessary to etch in relief by polishing with rouge on a soft cloth moistened with the etching agent. The spidery matrix, not present in any previous photograph, is left in relief while the crystal-

line masses are etched out to a considerable depth and hence are less perfectly differentiated than in cases where a more even illumination was obtained. To the eye the matrix appears yellow and evidently consists of nearly pure copper. Of the crystals some are much pitted and roughened while others retain a polished surface. In 44 every trace of the characteristic structure has disappeared along with the magnetic properties. The specific resistance in this condition was 70 per cent. greater

Fig. 5.

than in the condition represented by 38.

So marked was the relation between structure and magnetic quality that after some experience it was found possible to make a fairly accurate estimate of the value of I at saturation from a visual examination of the structure. The areas covered by the white crystals on a number of photographs both of this and other specimens were estimated by means of squared paper, and within
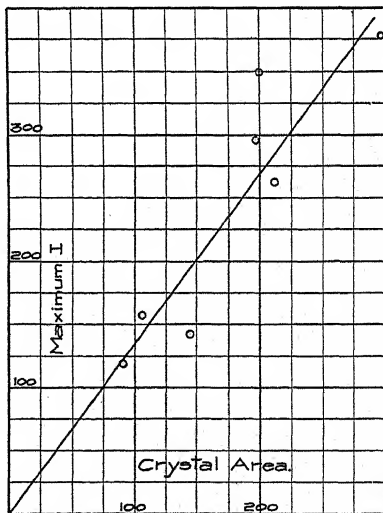
the limits of error which are naturally large, these areas appear to be proportional to the values of I at saturation. Fig. 5 shows this relation for a number of cases.

A striking confirmation of the hypothesis that the white crystals contain the magnetic units was afforded by specimen No. 14 which was originally supposed to be non-magnetic and was included in the lot under test to see if it could be rendered magnetic by heat treatment. When photographed the specimen showed the white crystals which had in all other cases been found associated with good magnetic quality (photo. 54). At the time the contradictory results were unexplainable. On determining the transformation temperatures the specimens were found to divide into two groups. In one
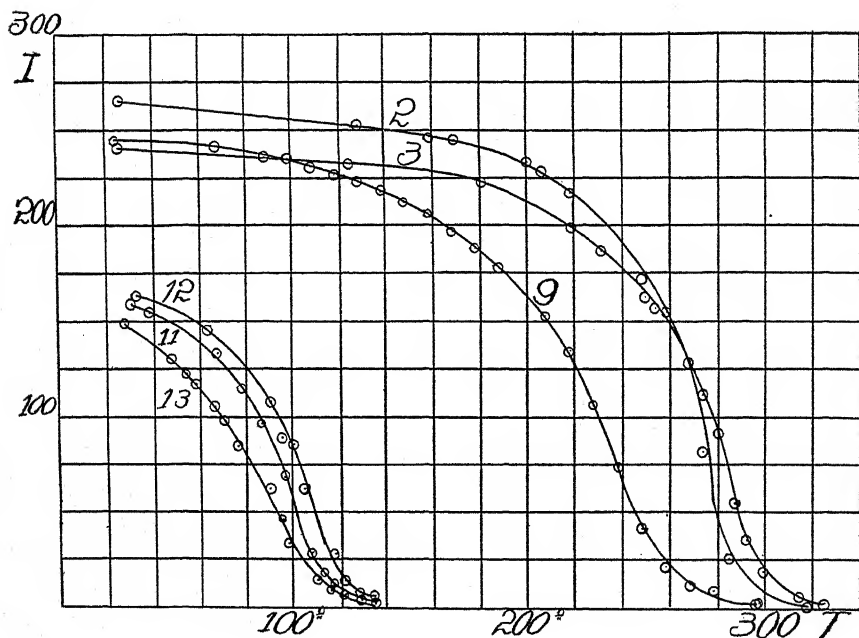
Fig. 6.

group transformation to the non-magnetic state was complete at 275°, in the other at 135° as shown in Fig. 6.

This suggested that No. 14 might have a still lower transformation point and trial showed this to be the case, a small telephone magnet lifting it readily from a cake of ice only to have it drop off after a minute or two as the temperature rose. Fig. 7 is an $I–H$ curve for

this specimen at zero, the heating coil of Fig. 1 being replaced by an ice bath.   The extreme softness is characteristic of all specimens near the upper limit of transformation.   The value of I at saturation would have been considerably greater at a still lower temperature as shown by the transformation curve which was still rising rapidly at zero.

Although the differences in chemical composition did not appear sufficient to account for the observed differences in the transforma-
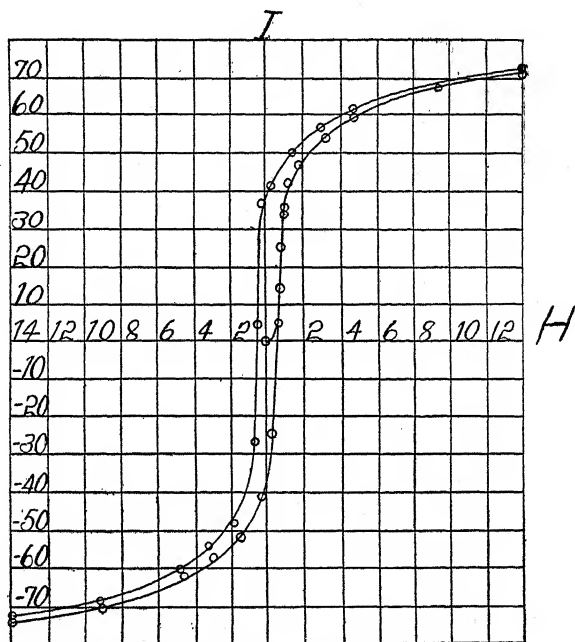
Fig. 7.

tion temperature, no other cause could be assigned since in no case did the heat treatment appear to produce any change in this regard. An attempt to reproduce the alloys of low transformation temperature by making specimens having a high ratio of copper to manganese (Cu 72, Mn 18, Al 10) was successful.   This result is in line with the work of Ross and Gray in indicating that the copper plays a part of much greater importance than a mere solvent for a Mn-Al alloy as has frequently been assumed to be the case.

Since the white crystals are always present in magnetic specimens and are never found in specimens which are not, under proper

conditions, magnetic and since there seems to be a simple relation between the quantity of crystals of this kind and the magnetic quality it seems fair to assume that they contain the magnetic constituents of the alloy. Attempts to isolate these crystals were unsuccessful. The fact that these crystals may, as is seen in the photograph of No. 14, exist when the specimen is above its transformation point would seem to indicate that the magnetic quality depends in some way upon the internal structure of the crystal and that this molecular grouping is, in turn, for an alloy of given composition, dependent upon the temperature. A surprising feature is the apparently slight difference between the magnetic and non-magnetic states as indicated by the fact that careful experiments with an apparatus of great sensitiveness failed to show positive evidence of any evolution of heat on passing through the transformation range.

Since the above was written Stephenson (Phys. Rev., vol. 31, p. 252, 1910) has published curves indicating the evolution of small amounts of heat on passing through the transformation point.

Summary.

1. The precautions to be observed in the preparation of the alloy are long heating, the use of silica-free crucibles and molds, casting in hot molds, annealing to the upper limit of transformation with subsequent quenching.

2. The magnetic examination shows the favorable effect of annealing at the upper limit of transformation, and the unfavorable effect of annealing at the lower limit. The transformation temperature depends upon the composition.

3. Photographic examination shows the close dependence of the magnetic properties upon the crystalline structure, and indicates that the white crystals in the photographs shown probably contain molecular groups which are capable of becoming magnetic under proper conditions of temperature.

In conclusion I wish to extend my very sincere thanks to all members of the teaching staff at the Ryerson Physical Laboratory for many courtesies and numerous helpful suggestions during the progress of these experiments which were carried out there.

University of Utah,
July 20, 1910.

EXPERIMENTS IN IMPACT EXCITATION.

By George W. Nasmyth.

I. The Characteristics of Short Arcs Between Metal Electrodes.

§ 1. *The Development of Impact Excitation and the Short Spark Generators.*

Impact Excitation.—The year 1906 marks the beginning of the history of "impact excitation" (Stosserregung) as a method of producing high frequency electric oscillations of considerable power which are only slightly damped. In October of that year Wien[1] published a brief account of his experiments "On the Intensity of the Two Waves in Coupled Transmitters," in which he found that for small spark lengths of about 0.15 mm., a third maximum appears between the maxima of the two waves usually met with in coupled circuits. This third maximum is higher than the other two, and the shape of the curve indicates that the damping of the corresponding oscillations must be very small. Its frequency corresponds to the natural frequency of the antenna, or secondary of the coupled circuits, whereas the frequencies of the two ordinary component waves are respectively greater and less than this natural frequency. According to the theory of impact excitation, the appearance of the third wave is due to the rapid recovery of the resistance of short gaps, so that the oscillations in the primary circuit are extinguished after they have given up the greater part of their energy to the secondary circuit. The oscillations in the secondary then continue as if it were an isolated circuit, with its own natural frequency and damping factor. The electric oscillations excited by "impact" correspond, therefore, to the vibrations of a pendulum which has been started by a blow, while the ordinary

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[1] Max Wien, "Ueber die Intensitaet der beiden Schwingungen eines gekoppelten Senders,"Verhandlung der Deutschen Physikalischen Gesellschaft, Vol. 8, 20, pp. 486–9, Oct. 30, 1906. See also Science Abstracts, Vol. 9, p. 633, no. 2076, 1906.

oscillations correspond to the vibrations of one of two coupled pendulums.

Lepel's Generator.—In the same year Lepel[1] began his work on the discharge between metal electrodes, and in November and December, 1906, invented a simple apparatus, using large iron electrodes separated by a film of oil, or Venetian turpentine. As early as 1890 and 1892 Elihu Thomson had succeeded in producing a type of discharge intermediate between the spark and the arc, using metal electrodes, but was unable to obtain either constancy of action or a sufficiently powerful oscillatory current for practical purposes. These difficulties were overcome by the two essential features in Lepel's device, the parallelism of the surfaces of the electrodes, thus making it possible for the discharge to take place at different points of the surface without altering the length of the path, and by the exclusion of air. Lepel later replaced the oil between the plates by mica, which was found unsuitable because it became conducting when heated, and then by one or two sheets of paper, which was found to answer every purpose. The iron electrodes were replaced by copper, either electrolytically pure or silver plated, and still later the negative electrode, when direct current was used, was made of brass or an especially dense bronze. By September, 1907, Lepel's apparatus had been so perfected that he was able to transmit messages and musical tones from Slough to Scheveningen, 160 miles, using an E.M.F. of 440 volts and 1.5 amperes of direct current.

Von Arco's "Quenched Spark."—The first public announcement of any of the short spark methods of generating oscillations was made by von Arco[2] in April, 1909. The Telefunken Company had experimented with Lepel's apparatus early in 1907, but finding it unsatisfactory in its form at that time, proceeded to develop its own system. The gap at present used by the Telefunken Company

[1] The best descriptions of Baron von Lepel's system and apparatus are to be found in The Electrician (London), Vol. 63, pp. 142, 157, 174, 345, 376, 1909; Vol. 64, pp. 153, 386, 1909–10. Abstracts from his German patent no. 24,757, filed August 20, 1907, are given in Vol. 63, p. 374, 1909.

[2] The Electrician, Vol. 63, p. 89, "A New System of Wireless Telegraphy used by the Telefunken Company," by Count Arco, April 30, 1909. See also pp. 332, 370, 461 and 720. For the relation and comparison of the Lepel and Arco systems, see also Vol. 63, pp. 228, 325, 617, 661; Vol. 64, pp. 726, 768 and 947.

differs from Lepel's gap in the use of a mica ring instead of paper, a groove being cut in the electrodes at the inner edge of the mica to prevent the discharge occurring at that place. The distance between the electrodes is about 0.25 mm., and several gaps are used in series to increase the power. The electrodes are provided with flanges to assist in radiating the heat. The secondary or antenna circuit is closely coupled to the primary so that the energy of the spark discharge is quickly absorbed. The company has given the name "quenched spark" (Loeschfunken) to this form of excitation, which is identical in principle with Wien's Stosserregung. Using an alternating current at a frequency of about 2,000 cycles the sparks may be made to follow each other so equally and regularly that they give out a clear musical tone, and this system is known as the "singing spark" (toenende Funken) system.

The practical and technical advantages of the new system over the older spark methods as well as over the undamped arc oscillations have been given by von Arco under twelve headings. Since most of these advantages are common to the other short spark systems, they are briefly summarized below:

1. Considerably smaller antenna may be used. This is due to the speed with which the sparks follow each other, so that greater power may be transmitted from the antenna at a smaller maximum voltage, and to the excellent characteristics of the short spark as a generator of short waves.

2. High overall efficiency. It is claimed that with the singing spark from 50 per cent. to 75 per cent. of the machine output can be changed into energy at the antenna, as contrasted with 20 per cent. for the older spark system and 10 per cent. for the arc lamp generators.

3. Adaptability for high power sending stations. No such limitations in the wave-length or difficulties in keeping the oscillations constant as arise in older arc methods present themselves.

4. Noiseless operation.

5. One wave is transmitted instead of two.

6. Small damping of wave sent out, from 0.08 to 0.025.

7. Constancy of the oscillations.

8. Large range of frequencies available, and the possibilities

of using overtones whose frequencies are four, five or even six times the fundamental.

9. Military advantages of using short waves which are absorbed before they travel a longer distance than is intended.

10. Sharp tuning due to constancy of oscillations and small damping of radiated wave. The freedom from disturbances is from 2 per cent. to 5 per cent., as contrasted with 20 per cent. to 30 per cent., which may in exceptional cases be reduced to 5 per cent. or 6 per cent., in arc lamp practice.

11. Elimination of atmospheric disturbances and individuality of musical tones.

12. Acoustic tuning of receivers which enables relays to be used for calling, etc.

Peukert's Rotating Disk Generator.—Recently Professor W. Peukert[1] has announced the invention of a short spark generator in 1906, for which a patent application was filed February, 1907, prior to either the Lepel or Telefunken Company applications. Peukert's apparatus consists essentially of two carefully surfaced metal disks held together by a spring pressing against one of them, with a filling of oil between. One disk is rotated at a constant speed of about 800 r.p.m., and oil is fed regularly at the center of the fixed disk and distributes itself in a uniform capillary layer between the plates. The frequency of the discharges depends upon the constants of the circuits, but in one test it was found to be 48,900 per second, as measured by the radiation to a revolving photographic plate from a needle point connected to a circuit coupled to the secondary. The gap efficiency was found to be as high as 60.2 per cent. in one test, with an applied E.M.F. of 615 volts; but the overall efficiency was not given. The generator may be made to give a definite pulsation to the oscillatory energy, thus producing a musical note in the telephone receiver, by dividing the disks into sectors, each alternate one consisting of insulating material. In this form it resembles Marconi's studded disk discharger. In its origin,

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[1] The Electrician, Vol. 64, p. 361. See also p. 550 for an abstract of a dissertation presented to the Brunswick Technische Hochschule by Dr. A. Wasmus on Peukert's High Frequency Generator for Wireless Telegraphy on the Quenched Spark Method, and Peukert's English patent 4762 (1909).

Peukert's generator goes back to the results of Steidles' work[1] on contact between a spring and a rotating oil-covered disk against which it presses.

These three short spark generators constitute the chief means at present available for producing high frequency oscillations of the impact excitation type. The principle has opened up a new region and given the investigator of electric waves and oscillations a new tool, and its practical applications are so important in radio-telegraphy that new forms of generators and improved modifications of these present forms will undoubtedly result. Indeed, Wien[2] has recently announced that he has been able to obtain impact excitation effects with a spark gap of the usual length by including a vacuum tube in the condenser circuit. A wide tube having a pressure of 0.5 to 0.1 mm. of mercury, and two aluminium electrodes 2 to 3 cm. apart, was used. With a degree of coupling from 6 to 12 per cent. only one wave was obtained, the primary oscillations being damped out in a short time by the rapid recovery of resistance of the vacuum tube.

In principle the three spark generators have much in common, and experiments and theory dealing with any one will have a measure of value for each of the others. The author's experiments have been made on the Lepel generator. This is the simplest of the three, since it uses direct current instead of alternating, as in the case of the Telefunken generator, and it has no moving parts, as in the case of Peukert's apparatus.

Arc or Spark.—The discharge between the electrodes of all three generators is intermediate in character between the ordinary arc and spark discharges, and in the absence of exact definitions for the two terms, it is difficult to decide in which class they should be placed. From the arc point of view, Lepel's generator is a d.c. arc between metal electrodes, the Telefunken arrangement is an a.c. arc, and Peukert's generator is an arc between relatively rotating electrodes. But in general when the capacity circuit is connected the discharge of all three generators becomes discontinuous, and

[1]Elektrotechnische Zeitschrift, Vol. 26, p. 679, 1905.

[2]Max Wien, Ueber ein neue Form der Stosserregung elektrischer Schwingungen; Physikalische Zeitschrift, Vol. 11, p. 76, February 1, 1910.

corresponds to a spark gap. This condition also arises in the Poulson arc, although to a less extent. Von Arco has suggested the following definitions: "By arc excitation one might understand that method of producing oscillations by which the rate of condenser discharges is equal to or about the frequency of the oscillations; by spark excitations, that method by which this rate is considerably less." This definition makes the type of discharge depend upon the constants of the circuit, and the same generator might be an arc or a spark gap under different conditions. The author's experiments on the Lepel generator have demonstrated that in many of its general characteristics it bears a marked similarity to the Poulson arc, and for the present it will be called an arc whenever the applied voltage would be sufficient to maintain the current through the gap if the condenser circuit were disconnected.

§ 2. *Three Kinds of Oscillations.*

Besides the familiar damped discharge of the ordinary spark gap, three distinct types of arc oscillations have been recognized quite generally by investigators[1] in this field. For convenience the author has named these types after the experimenters who first produced them:

1. *Duddell Oscillations.*—The characteristic of these oscillations is that the maximum value of the condenser current is less than that of the direct current through the arc, so that the arc remains lighted. These are the true "singing arc" or "musical arc" oscillations, since the frequency is always within the audible range. The efficiency of these oscillations falls off rapidly as the frequency is increased, the upper limit being about 10,000. Although these oscillations are not available for radio-telegraphy on account of their low frequency, their discovery by Duddell[2] in 1900 marks the beginning of the development of undamped oscillations, and led to the discovery of the second type.

2. *Poulsen Oscillations.*—In these oscillations the maximum value of the condenser current is greater than that of the direct

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[1] See K. Vollmer, Jahrbuch der Drahtlosen Telegraphie und Telephonie, Vol. 3, p. 119, December, 1909.

[2] W. Duddell, Journ. Inst. Elec. Engineers, Vol. 30, p. 232, 1900.

current through the arc, so that the arc is extinguished and relighted at each oscillation.   In the Poulsen[1] arc, which is the Duddell arc immersed in an atmosphere of hydrogen and having one electrode of copper, water-cooled, oscillations having a frequency of a million per second or more may be obtained with good efficiency.   These oscillations may also be produced by connecting several arcs in series, as Simon has demonstrated.   Oscillations of this second or Poulsen type have been used extensively in radio-telephony. Since these oscillations are usually above the audible range, they can be called "singing arc oscillations" only by analogy.

3. *Wien Oscillations.*—Here the maximum value of the condenser current is so great that the arc is not only extinguished, but is relighted in the opposite sense.   Then follows a series of damped oscillations through the arc of the well-known kind, until the potential difference is no longer sufficient to relight the arc.   The condenser is then charged by the direct current until the arc resistance breaks down again, and another train of damped oscillations appears and disappears.   These oscillations are easily produced if both electrodes are of metal and the arc length is short.   These are the oscillations usually known as "Stosserregung" or "impact excitation" oscillations.[2]

It is with this third type, or Wien oscillations, that this investigation is chiefly concerned.   The Duddell and Poulsen oscillations have been the subject of many interesting and important researches, but up to the present very few details of quantitative experiments have been published on the subject of the Wien oscillations or the characteristics of the new type of generators.

§ 3. *Arrangement of Experimental Apparatus.*

Besides the general working characteristics of an impact excitation generator, three characteristics are of especial interest: the form of the voltage current curves, on account of their importance in the theory of the action of apparatus; the intensity of the oscillations as a function of the arc current and the constants of the circuits, on account of its relation to the efficiency of the generator;

[1] V. Poulsen, British Patent Specification 15599 of 1903. See also Science Abstracts, Vol. 8A, p. 521.

[2] Max Wien, *l. c.*

and the influence of the discharger upon the frequency of the oscillations which it produces. In the author's experiments, on the Lepel arc, the arrangement of apparatus, as shown in Fig. 1, was designed with special reference to these three characteristics.
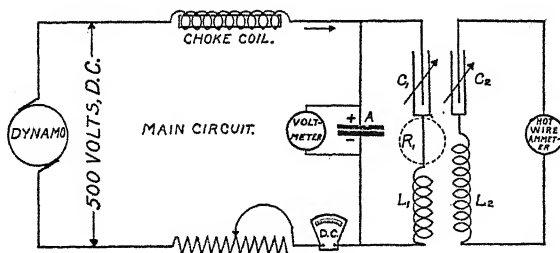
Fig. 1.

Diagram of connections.

The source of E.M.F. was a 500-volt shunt wound dynamo, direct connected to a two-phase, 900 r.p.m. induction motor. At first a Tirrell regulator was used on the dynamo, but later it was disconnected for fear that the pulsating current which resulted might introduce irregularities into the experiments. Without the regulator the voltage of the generator was found to be very steady over short intervals of a few minutes, although over intervals of a half hour or more it was subject to slow drifts of $\pm$ 20 volts. A large water rheostat made from an old oil barrel, together with two Cutler-Hammer rheostats arranged for connection in series or parallel, each having a maximum resistance of 200 ohms and a current capacity of 1.5 to 2.5 amperes, constituted the resistance in the circuit. The choke coils were two solenoids 47.5 cm. long, wound with seven layers of 192 turns each of double cotton covered copper wire 2.05 mm. in diameter. The inside diameter was 8.2 cm. and the outside 11.4 cm. The coefficient of self induction, measured by comparison with an Ayrton-Perry standard, was 0.0296 henry for each coil. In the condenser circuit, a variable number of Leyden jars, having a capacity of 0.00188 microfarad each, were connected in parallel. The inductance in the primary circuit was also varied but for most of the experiments a solenoid 20.55 cm. long, having a mean diameter of 12.85 cm. and consisting of 15 turns of stranded wire insulated and wound on a glass jar, was used. The diameter

of the wire, which was made up of 36 strands, was 0.65 cm.; the coefficient of self induction of this solenoid was 14,300 cm., and the resistance of the condenser circuit with this solenoid connected and the capacity short circuited, was only 0.15 ohms, as measured by a Wheatstone bridge.

A Weston d.c. voltmeter of the d'Arsonval galvanometer type, with a 600-volt and 150-volt scales, was connected across the arc, and a Weston millivoltmeter, with a 5-ampere shunt, was connected in series with the main circuit, as shown.   The intensity of the oscillations was measured by an unshunted hot wire ammeter containing no iron and giving a maximum deflection at 7 amperes of direct current.   The position of the ammeter when used for measuring the oscillations is shown by the dotted circle in the primary circuit. The expanding wire was of copper, 0.22 mm. in diameter and 9 cm. long, and the resistance of the instrument is included in the 0.15 ohms mentioned above.   The hot wire ammeter was calibrated before and after reading by short circuiting the capacity and causing a direct current to flow through the hot wire ammeter and the main circuit ammeter in series.

§ 4. *The Lepel Arc.*

After experimenting with various forms of the Lepel apparatus, it was found that with flat copper plates water cooling of the electrodes was unnecessary, as the radiation to the air was large, and the generator seemed to work better, if anything, as it became heated by the discharge.

The Electrodes.—The author's first experiments were made with brass and copper plates.   The results obtained when either metal was used as cathode were fairly satisfactory, but the pitting of the brass plate, due possibly to the volatilization of the zinc, was found to be a source of great annoyance and to make the oscillations inconstant by changing the arc length.   Two electrodes of sheet copper ¼ inch thick and 6 inches in diameter were then obtained. These were carefully surfaced in a lathe with a sharp tool set at an acute angle, and were then rubbed with sandpaper and oil.   The connecting wires were at first soldered to the copper plates, but after they had been melted off two or three times, they were fastened

to screws set into the edges of the plates. These electrodes were found very satisfactory, and Fig. 2 is from a photograph showing the appearance of the electrodes and insulating paper after they had been used almost continuously for six hours. Although the paper was burned brown and the copper discolored by the heat they were working as well at the end of the run as at the beginning, and after the bluish deposit of carbon from the burnt paper had been rubbed off with emery cloth the electrodes were ready for another test.
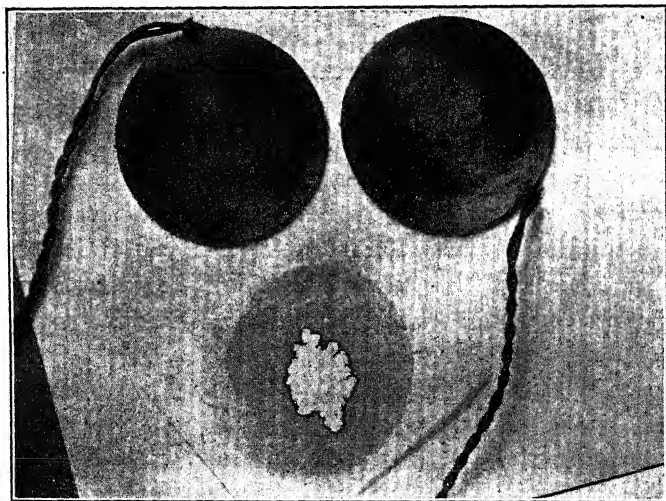
Fig. 2.

Photograph showing appearance of copper plates and paper after a six-hour run.

The oscillations are more irregular when the surfaces of the electrodes have been rubbed with emery cloth than when oil and a sand-paper block is used. When the emery is used it is probable that little points remain sticking in the copper. On the other hand, the vapor of the oil (ordinary machine lubricating oil was used) seemed to have a marked effect in steadying the arc conditions.

Very good electrodes may be made from the hard rolled copper plate used in the photo-engraving processes. The surface of this copper is very smooth and flat, and it holds its shape well on account of its hardness. If this copper sheeting is used, it is unnecessary to turn the plates to a flat surface in a lathe, an operation requiring

some skill. The plates need only be rubbed with fine pumice stone and oil, and then with a piece of wood carbon and oil, and the surface is almost perfect. Contact with the connecting wires may be made by means of a piece of tinfoil held down by a small weight.

The Insulator.—Any kind of paper seems to work well as an insulator and as a source, when disintegrated by the discharge, of the hydrogen and carbon atmosphere which the arc requires for satisfactory working. For the sake of uniformity in conditions, W.S. & B. Paragon typewriter paper in legal size sheets was used in all the quantitative experiments. In thickness this paper is very closely 0.05 mm. per sheet and the arc length was varied by increments of this amount by using a varying number of thicknesses. Mica rings were tried without success, as the discharge occurred either at the edge of the mica, or, if this were prevented by cuting a sufficiently deep groove in the plates, the discharge took place through the mica instead of across the air gap. Since there is no hydrogen atmosphere present when mica is used, the potential difference required for sparking is higher. Moreover, the insulating properties of mica disappear in large measure with rise of temperature. Water cooling is probably essential for the successful use of mica, and in addition to the groove at the edge the plates should be planed off outside the groove, so that the distance between the plates through the mica is greater than the distance through air gap.

Starting the Arc.—If a potential difference of 500 volts is applied to the electrodes of the Lepel arc when they are separated by two or three sheets of paper having a small hole in the center, the discharge will not in general take place. If the insulation of the gap can be broken down, however, the discharge will continue, the average voltmeter reading across the arc being from 37 v. upwards, according to the current, arc length, etc. To start the arc various devices may be used, the most convenient one being to short circuit the arc and then suddenly break the short circuit, thus bringing the full pressure of the inductive kick of the choke coils to bear on the gap. To break the short circuit suddenly, the author has used the following simple form of oil switch. One electrode of the arc is connected to a brass binding post having a flat top. Insulating oil poured on

this will be held by surface tension to a depth of a millimeter or more. Then a piece of brass having a wooden handle and connected to the other electrode is touched to the top of the binding post through the oil, making a short circuit across the gap. The circuit is then broken in the oil, resulting in a high potential difference across the gap which is usually sufficient to start the discharge. Theory shows that the inductive kick is greatest when the resistance in the main circuit is least, *i. e.,* when a large current flowing through the short circuit is suddenly interrupted.

If the edge of the hole in the paper has been charred, by a match or some other means, the discharge will start as soon as the potential difference is applied. After the discharge has been stopped and the electrodes taken apart, it will usually start automatically if the same paper is used when the apparatus is put together again. If the paper is charred too much, either a short circuit will result, giving a voltmeter reading of from 20 to 40 volts, or an inactive stage of the arc sets in which does not produce oscillations although the voltmeter gives a steady deflection of about 325 volts and a current of two or more amperes flows through the arc. To start the oscillations again after this inactive stage has set in, either the plates must be rubbed clean or the charred part of the paper must be moved to a new place on the plates.

Arcs in Series and Parallel.—As in the case of the Poulsen arc the Lepel generator may be made to give increased power by connecting several arcs in series. By starting the arcs one at a time separately and then connecting them together in series, the author has been able to make as many as three arcs run on 500 volts impressed E.M.F. This was only possible at relatively large currents, however; at smaller currents the arcs would go out and it would be necessary to start from the beginning again. The power obtained in the oscillations with three arcs in series was considerably less than three times the power delivered by one arc.

A better way to obtain increased power at a given impressed E.M.F. is to connect two arcs in parallel, each with its own choke coil and resistance in series. If the two cathodes (or anodes) are connected together, and the capacity and inductance of the condenser circuit connected to the two anodes (or cathodes) the same

potential difference will be required as for one arc alone and the two arcs will add their characteristics, doubling the power available for the oscillations.   The critical resistance in the condenser circuit at which oscillations can be obtained is equal to the sum of the absolute values of the negative slopes of the volts-amperes character- istic curves for the two arcs, and is therefore about double the resis- tance which can be used with a single arc.

The method of generating oscillations by connecting arcs in parallel in this way was discovered by the author in 1907 in attempt- ing to apply the principle of Vreeland's mercury vapor magnetic field oscillator to the arc, and the advantages of the connection were set forth under the title "An Improved Form of the Duddell Singing Arc."[1]   Later the attention of the author was called by Mr. Duddell to a lecture given in England before the Institution of Electrical Engineers in 1900, in which practically the same principle had been used.   The author had overlooked the reference to the earlier work, and is glad to take this opportunity of acknowledging the priority of Duddell in discovering the method of generating oscil- lations by connecting arcs in parallel.

§ 5. *The Volts-Ampères Characteristic Curves.*

The form of the characteristic curves between the voltage as measured by a d.c. voltmeter across the arc and the current through the arc is of considerable theoretical and practical importance. According to the theory of the Duddell oscillations, in order that oscillations may be produced, the curve must have a negative slope and its value, $-dV/dA$, must be greater than the resistance of the shunt circuit.   From its similarity to the Duddell and Poulsen arcs, it might be inferred that a similar law would hold for the Lepel arc.   Moreover, the shape of the characteristic curve has an important bearing upon the question of the variation of frequency in the oscillations produced by the arc.   One of the first problems in the author's experiments, therefore, was to de- termine the form of these curves.

The characteristic curve for an arc length of 0.15 mm. when the arc is generating oscillations is shown in Fig. 3.   Three sheets

[1] PHYS. REV., Vol. 28, p. 459, 1909.

of paper were used, and the capacity and inductance of the oscil-
lation circuit are given. The points plotted are from tests made
on two successive days, each extending over the whole range of
the curve. For currents much larger than 4 amperes the arc be-
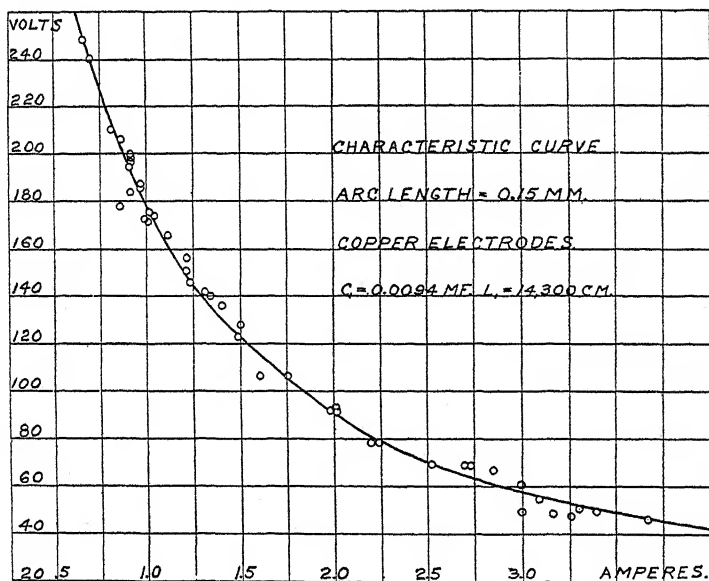
Fig. 3.

Characteristic volts-amperes curve for the Lepel arc with oscillations.

comes inactive and the oscillations cease, probably on account of
the flatness of the curve for large currents. For currents less than
0.7 ampere it is difficult to obtain steady conditions on account of
the steepness of the curve. Fig. 3 shows therefore the range over
which steady oscillations can be obtained with an arc length of
0.15 mm.

The readings of the d.c. instruments represented in Fig. 3 are of
course average or integrated values. Theory shows that with
the large inductance of the choke coils in the main circuit the
current varies but little during the charge and discharge of the
condensers. The fluctuations of the voltage may be very great,
however, and the voltmeter reading, expressed mathematically, is

$$V = \frac{1}{t} \int_0^t v \, dt,$$

where v is the instantaneous voltage and is integrated over the complete cycle of charge and discharge.   To obtain the complete history of the arc current and voltage during the cycle it would be necessary to employ an oscillograph.
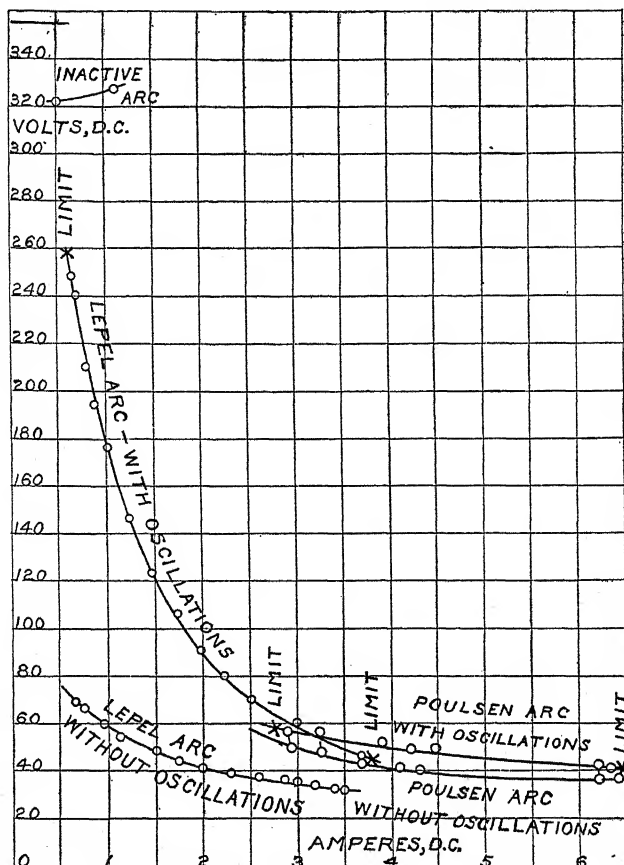
Fig. 4.

Comparison of Lepel and Poulsen arc characteristics.

The relation of the Lepel arc characteristics to those of the Poulsen arc are shown in Fig. 4.   The upper and lower curves for the Poulsen arc are taken from an earlier paper[1] by the author and represent the characteristics for an arc between a water-cooled

[1] The Frequency of the Singing Arc, PHYS. REV., Vol. 27, p. 126, 1908.

copper anode and a graphite cathode in an atmosphere of illumin-
ating gas when the switch in the oscillation circuit is open and closed,
respectively. The points for the corresponding curve for the Lepel
arc "with oscillations" are taken from Fig. 3; the lower curve for
the Lepel arc gives the corresponding characteristic when the con-
denser circuit is disconnected.

One reason for the greater effectiveness of the Lepel arc is due to
the steepness of the characteristic in the region of small currents
in which it operates. Even at larger currents, however, where
the two generators can be directly compared, the Lepel charac-
teristic seems to be much steeper than that for the Poulsen arc.
The separation of the two curves for the Lepel arc is also seen to
be much greater than that for the Poulsen arc. In experiments
with the Poulsen arc, the author has found that the oscillations
cannot be started unless the resistance of the condenser circuit is
less than the smaller of the negative slopes of the characteristics,
but that when once started the resistance can be increased until
it is but slightly less than the negative slope of the steeper charac-
teristic. · If the same conditions hold for the Lepel arc, the resis-
tance would still have to be quite small in order to have the oscil-
lations start, on account of the small value of the slope of the lower
characteristic, but once started, it should be possible to increase
the resistance through a large range before it becomes equal to
the negative slope of the characteristic "with oscillations."

The limits of the current between which oscillations could be
obtained in the author's experiments are indicated by crosses in
Fig. 3. At small currents the characteristic of the Lepel arc be-
comes so steep that the oscillations are very irregular, and although
oscillations have been obtained at currents less than 0.5 amperes,
they were transient or very unsteady. At larger currents the
characteristic becomes too flat, but by decreasing the resistance of
the shunt circuit still further oscillations could be obtained with
currents larger than 4 amperes. The product of the difference of
the voltages of the two characteristics by the corresponding amperes
may be taken as a measure of the power converted into oscillations.
Since the characteristic "without oscillations" varies but little for
the different arc lengths, a comparison of the characteristics "with

oscillations" will furnish a means of finding the arc length and current which will transform most power into oscillations and give the greater efficiency. It will be found from the three following figures that with with the author's apparatus the maximum power and efficiency are obtained at an arc length of 0.2 mm. and with a current between 1.0 and 1.5 amperes.

An inactive state of the arc which results sometimes when it is extinguished and started again, or when the paper is charred too much, is represented by the short curve between 320 and 330 volts. When this inactive stage sets in, the plates must be taken apart and the surfaces cleaned before steady oscillations can be again obtained. A condition which sometimes exists when the oscillation circuit is opened is represented by the horizontal line at 356 volts. This
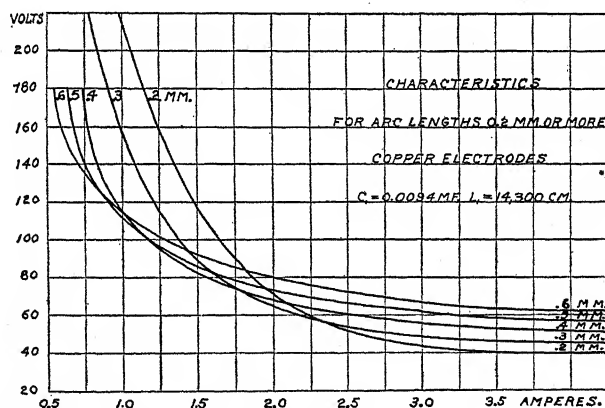
Fig. 5.

corresponds to the minimum sparking potential between copper electrodes, and seems to be accompanied by a small leakage current. When the oscillation circuit is again connected the oscillations do not always start, but the voltage remains at 356 no matter how small a resistance is placed in series with the arc.

In attempting to find the change in the characteristic curve when the arc length was varied, apparently discordant results were obtained. For large currents the voltage increased with increasing arc length, as in the case of the Poulsen arc, but for small currents the voltage sometimes increased and sometimes decreased with

increase of arc length.   It was then decided to make a systematic
exploration of the whole characteristic field, taking a hundred or
more observations at each arc length under different conditions
and at different times, and drawing a smooth curve through the
observations, as in Fig. 3.   The results of these experiments are
shown in Figs. 5 and 6, representing more than a thousand single
observations.   The observations for each arc length were drawn
on a separate sheet, and when a transparent sheet of cross-section
paper was placed over these and the curves traced one after another,
the cause of the apparent discrepancies became clear.
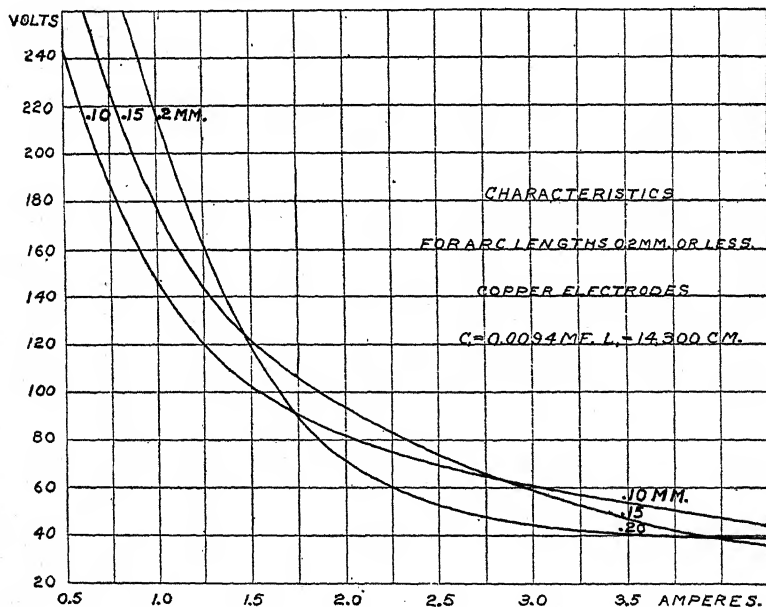
Fig. 6.

Lepel arc characteristics for different arc-lengths.

Although at large currents the curves tend to lie one above the
other in the order of the corresponding arc length, at low currents
the case is far different.   To avoid confusion the characteristic
curves have been plotted in two groups, the common characteristic
for the 0.2 mm. arc length serving as a standard of reference.   It is
evident from these curves that for small currents (e.g., one ampere) as
the arc length is increased from 0.1 to 0.2 mm., the voltage increases;

then as the arc length is increased to 0.5 mm. the voltage falls off rapidly at first, then more slowly, and begins to rise again as the length is increased to 0.6 mm. If cross-sections of the curves are taken in this way at constant currents, and the potential difference across the arc plotted against the arc length, the family of curves shown in Fig. 7 is obtained.
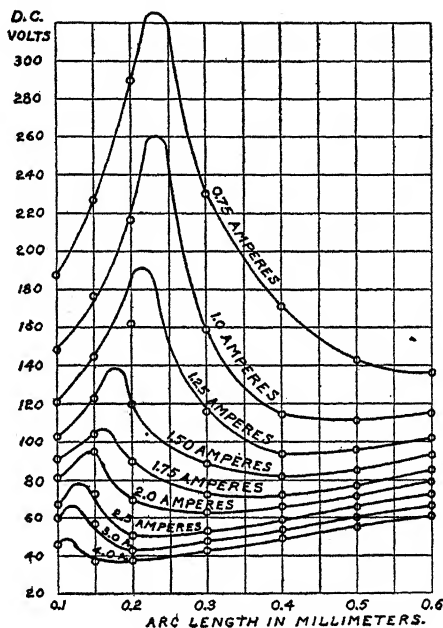
Fig. 7.

P.d. across arc for constant current and varying arc length.

The form of the curves suggested a resonance phenomenon, and tests were at once made to determine whether the effect was a true one corresponding to something inherent in the short spark form of excitation, or a false effect, due to some peculiarity in the measuring instruments. The substitution of another voltmeter, which gave the same readings at all points which were tried, showed that the phenomenon was not due to any resonance with the natural period of the voltmeter pointer. Moreover, the high resistance of the voltmeter, amounting to about 60,000 ohms on the 600-volt scale, precluded the possibility of resonant oscillations through the voltmeter.

The measurement of the resistance in series with the arc at various currents and arc lengths furnished another test which seemed to be conclusive.   In all the tests that were made, it was found that, within the limits of the errors of observation, the resistance of the circuit was the same, whether it was found from the ratio of the open circuit, E.M.F., E, to the current I, through the main circuit when the arc was short circuited, or from the formula,

$$(1)   E - V = RI,$$

where V is the voltmeter reading across the arc.   The resistance in the main circuit corresponding to any current and arc length or voltage can be determined, therefore, to a close approximation from the curves of Fig. 7, if E is taken as 500 volts.

The characteristics from which the curves of Fig. 7 are taken were drawn as smooth curves in Fig. 5 and 6, all small deviations being taken as due to errors of observation.   From an inspection of the points plotted in Fig. 3, however, it is apparent that there are systematic deviations in the neighborhood of 3 amperes, which would accentuate the maxima and minima in Fig. 7 slightly.   A reexamination of all the original curves plotted showed that a similar effect ran through all the observations.   In general these deviations are less than those shown at about 2.8 and 3.2 amperes in Fig. 3, and if they had been taken into account in plotting the final result would have been changed but little.   At the longer arc lengths some small deviations gave rise to the suspicion that the minor maxima which are apparently just beyond the range of Fig. 7 came into this range at larger currents, but the effects were too small to make this certain.

The first clue to the cause of the phenomenon shown in Fig. 7 was obtained accidentally.   Part of the variable resistance in the main circuit consisted of german silver coils, 61 in all, each about 61 cm. long, 1.5 cm. mean diameter and having about 2 turns per cm. of wire 1.13 mm. in diameter.   The inductance of this resistance was about 330,000 cm., and it was found that the voltage reading for the same current and arc length varied when this resistance was replaced by the non-inductive rheostat.   Thus, with an arc length of 0.2 mm. and arc current of 2.25 amperes, the voltmeter reading

rose from 59 volts to 64 volts when the inductive resistance was replaced by the rheostat, the intensity of the oscillations increasing with the voltage.

§ 6. *Variation of Voltage with the Capacity of the Arc.*

In seeking the reason for the variation of the arc voltage with the inductance of the main circuit, the relative times of charge and discharge of the condensers seemed to offer the most satisfactory basis for explanation. But the relative times of charge and discharge depend even more directly upon the capacity of the condensers, and experiments were made at once to determine if the capacity had any effect upon the arc voltage. It was found that increasing the capacity of the shunt circuit by adding more Leyden jars increased the intensity of oscillations, while the voltage sometimes increased and sometimes decreased. The d.c. ammeter reading would increase as the capacity was added, usually, and the resistance of the main circuit would have to be increased in order to bring it back to the same reading, but sometimes this effect would be reversed also.

These observations led to the suspicion that the curious variations in the arc voltage as the arc-length was increased might be due to the change in the capacity of the arc as the distance between the plates was varied. A rough computation, using 1 as the dielectric constant of the paper and gas between the plates, showed that the capacity of the arc varied between rather wide limits, from 0.0016 mfd. at 0.10 mm. arc-length to 0.00023 mfd. at 0.70 mm. Accordingly, a variable air condenser was connected directly across the arc, and the values of the arc voltage and oscillation current were read as the capacity in parallel with the arc was varied. The arc-length used was 0.25 mm., and the d.c. ammeter reading was maintained constant at 2.5 amperes by varying the resistance in the main circuit. The results are given in Table I. and graphically in Fig. 8.

In Fig. 8 the variable capacity in parallel with the arc is plotted as abscissa, with the d.c. voltage across the arc and the corresponding intensity of the oscillations indicated by a hot wire ammeter as ordinates. The capacities corresponding to arc-lengths of 0.05,

TABLE I.

Change of Arc Voltage and Oscillation Intensity with Arc Capacity.

Capacity in Parallel.	Volts, d. c.	Oscillation Current.	Resistance in Series.
0.000015 mfd.	64.5	12.4	176 ohms.
0.000075	68	12.8	174
0.000156	69.5	12.8	171
0.000195	73	12.5	171
0.000240	74	12.4	171
0.000285	77	11.1	166
0.000300	79	11.2	166
0.000320	80	11.2	166
0.000345	79	10.9	166
0.000375	77	11.0	168.5
0.000420	72	11.5	168.5
0.000560	70	12.6	174
0.00110	77	13.1	166
0.00176	83	13.2	158
0.00190	87	13.2	158
0.00248	96	13.4	155
0.00353	98	14.0	154

D.c. ammeter reading in main circuit maintained constant at 2.50 amperes by changing resistance. 500 volts open circuit.

Capacity across arc.
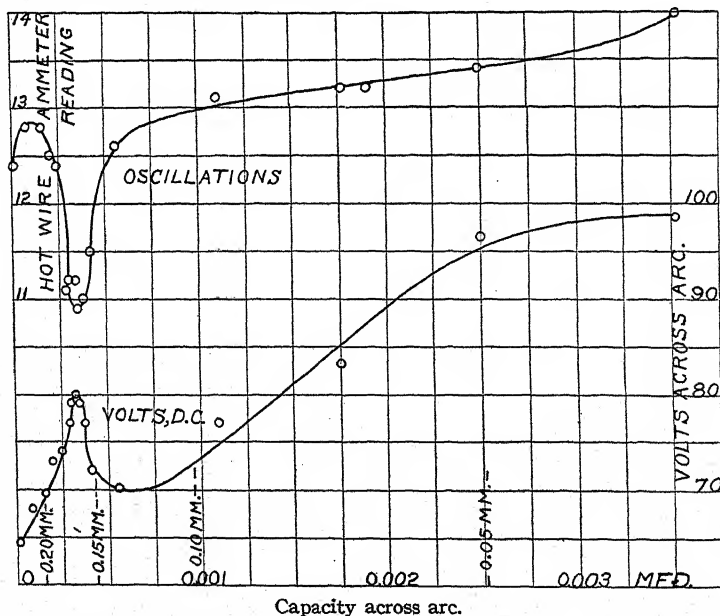
Fig. 8.

Effect of arc capacity on arc voltage and oscillations.

0.10, 0.15 and 0.20 mm. have been marked, and the left-hand boundary of the figure, with no capacity in parallel, corresponds of course to the arc-length of 0.25 mm. actually used. The resistances in series with the arc have not been plotted, but it is evident from Table I. that the resistance curve is approximately the reciprocal of the voltage curve. At the voltage maximum the resistance is a minimum, agreeing with the oscillations curve. At larger capacities, however, the resistance decreases with increasing voltage, whereas the oscillations increase. In all cases the value of the resistance observed agrees closely with the value obtained by subtracting the potential difference across the arc from 500 volts and dividing by 2.5 amperes, according to equation (1).

It is apparent that the voltage curve with varying arc capacity corresponds qualitatively at least with the voltage curve for varying arc length at 2.5 amperes shown in Fig. 7. The maximum in the voltage in Fig. 7 occurs at an arc-length less than 0.15 mm., while in Fig. 8 the maximum occurs at a capacity equivalent to an arc-length slightly more than 0.15 mm., as calculated. The discrepancy is due in part to the manner in which the plates forming the arc electrodes were arranged in the two experiments. In the earlier experiments which gave the data for the curves of Fig. 7 the plates could not be adjusted so as to completely overlap, on account of the projection of the screws which held the connecting wires. In the later experiments when the importance of the capacity of the arc had become apparent they were made to overlap more completely, thus increasing the capacity, and making an arc length of about 0.25 mm. in the later experiments equivalent to an arc length of about 0.20 mm. in Fig. 7. This would shift the maximum in Fig. 7 into the region of shorter arc lengths and tend to bring the two sets of curves into agreement.

The inductance of the wires which connected the variable air condensers in parallel with the arc would decrease the effect of their capacity and give a correction which would also shift the maximum in the right direction. This inductance was small, however, as the area enclosed by the connecting wires was not more than 3×25 sq. cm. The small correction due to a dielectric constant greater than one for the paper and gas between the plates

would make the equivalent capacity for each arc length slightly greater and would tend to neutralize in part the effects of the other two corrections.

The curve for the intensity of the oscillations in Fig. 8 shows that the oscillations are a minimum when the voltage is a maximum, *i. e.*, when the equivalent arc length is about 0.15 mm. The readings of the hot wire ammeter given in the table and shown graphically in the curve are in scale divisions of the instrument. The divisions were unequally spaced so as to give a simple multiplying factor when the ammeter was used unshunted, so that the readings may be taken as proportional to the oscillation current, even when used with a shunt, if, as in this experiment, the frequency is practically constant. In attempting to calibrate the instrument later by comparison with a less sensitive unshunted ammeter, the expanding wire was burned out. From the readings obtained however, and from observations of the intensity of the oscillations made under similar conditions later, the multiplying factor may be taken as about 0.4, so that a reading of 14 scale divisions indicated about 5.6 amperes.

As a further test of the effect of capacity on the arc voltage, a run was made with a constant d.c. ammeter reading of 2.0 amperes. The large upper electrode was replaced in this experiment with a small copper disc 3.85 cm. in diameter and an arc length of 0.15 mm. was used, giving a capacity of only 0.000069 mfd. as compared with 0.00108 mfd. for the larger plate with the same arc-length. On varying the capacity of the air condensers in parallel with the arc, it was found that the maximum in the voltage had shifted to a smaller capacity, corresponding to a longer arc length of slightly more than 0.2 mm. This is in agreement with the shift of the maxima to longer arc lengths with smaller currents shown in Fig. 7.

In this experiment it was possible to reduce the capacity of the arc to smaller values than had been obtained before, and it was found that the voltage decreased from a maximum of 100 volts at a capacity equivalent to about 0.2 mm. to a minimum of 80 volts at 0.4 mm. equivalent, and then increased to another maximum of 85 volts at a capacity 0.9 mm. equivalent, falling off to 75 volts

when there was no capacity in parallel with the small electrode. The second maximum observed evidently lies just beyond the range of Fig. 7. A minimum of 4.7 amperes was observed in the oscillations when the voltage was at the first maximum corresponding to 0.2 mm., although the minimum in the curve for the oscillations was not so sharp as that shown in Fig. 8. There was no minimum in the oscillations corresponding to the smaller second maximum in the voltage at 0.9 mm. equivalent, but instead a maximum of 5.3 amperes, the reason for which is unknown.

At 1.5 amperes the maximum in the voltage shifted over to the region of still smaller capacities, corresponding to a longer arc length between 0.20 and 0.25 mm., and the maximum increased to 113 volts. Both the shift and the increase in the maximum are in agreement with Fig. 7.

The experiments with capacity in parallel with the arc show therefore that the maxima observed in the voltage in Fig. 7 are due chiefly to the changing capacity of the arc itself, as the arc length is varied, and that corresponding to these maxima in the voltage there are minima in the oscillations produced by the arc. This is in agreement with the hypothesis advanced to account for the maxima in the voltage-arc length curves. When the time required for the main current to charge the condensers is long compared with the time of discharge, the effective value of the oscillations should be small and the average value of the d.c. voltmeter reading large. The reason for the minima in the oscillations and the corresponding maxima in the voltage together with the importance of these minima in the production of impact excitation effects will be considered more fully in the later sections which deal especially with the intensity of the oscillations. A brief summary of the explanation, however, is given in section 7.

From the connection which has been established between the maxima in the voltages and the minima in the oscillations it is possible to predict what would be the effect upon the curves in Fig. 7 if the capacity or inductance of the condenser circuit were varied. It will be shown in the sections dealing with the intensity of the oscillations that the d.c. ammeter reading at which the minima in the oscillations occur increases with increasing capacity

and with decreasing inductance in the condenser circuit. This is the same effect as shown in Fig. 7, where the corresponding maxima in the voltage moves into the region of smaller arc-length and larger arc capacity as the d.c. ammeter reading is increased. We may consider, then, that the capacities of the condensers and of the arc are in parallel, and that the effective capacity of the condensers is decreased by the inductance in series with them. If the curves shown in Fig. 7 were determined with a larger capacity or a smaller inductance in the shunt circuit, therefore, the capacity of the arc would have to be reduced in order to have the same capacity for the whole system, and the maxima would move into the region of longer arc-lengths.

The reason for the increase of voltage observed when the inductance of the main circuit was decreased becomes clear when we consider the effect upon the relative times of charge and discharge. With a smaller inductance in the main circuit the generator current falls off more during the condenser charge, and the time of charge is made longer. But this is the same effect as that produced by an increase of capacity. Decreasing the inductance of the main circuit should have the same effect as increasing the capacity or decreasing the arc length, therefore, and from Fig. 7 it is apparent that at 0.2 mm. and about 2.25 amperes a decrease of arc-length should raise the voltage. This is in agreement with the effect observed. A change of inductance in the main circuit should produce an increase of voltage if made on one side of the maximum and a decrease if made on the other, and this effect has also been observed in the author's experiments, always in the direction which agrees with the theory.

§7. *Explanation of Maxima and Minima.*

At small capacities the condensers will charge up quickly, the number of discharges per second will be great, and the sparking potential will be low, since the gas will remain ionized to a large extent. The intensity of the oscillations, as measured by a hot wire ammeter, will be large, and the average d.c. voltmeter reading low, partly on account of the low sparking potential, and partly because the charging time, when the voltmeter reading is obtained,

is relatively small compared with the discharging time, when the voltmeter indication should be relatively small.

At large capacities, on the other hand, the quantity of energy liberated at each discharge is large, the trains of oscillations will last a long time, and their effective value will be large even though the number of discharges per second is relatively small. A large reading of the hot wire ammeter should therefore be obtained. But the resistance of the arc will be small while the large current is flowing through it and the time of each discharge is large compared with the time of charge. In this case also, then, the average voltmeter reading will be small.

Between these limiting cases a low value of the oscillations may result from a low value of the group frequency, due to high sparking potential, and a low value of effective oscillation current, due to small capacity. The time of charge will be relatively great and the time of discharge relatively small, giving a large value of the average voltmeter reading. The minimum in the oscillations should correspond with a maximum in the voltage, therefore, and this effect is observed in Fig. 8.

The reason for the shift of the maxima into the region of larger arc capacities as the current is increased is clear from this explanation. If the direct current from the generator is increased, the time required to charge the condensers will be decreased. This is the same effect as would be produced by a decrease of capacity. To neutralize this effect the actual capacity would have to be increased, thus restoring things to the same condition. In other words as the charging current is increased the maxima should move over into the region of larger capacities or shorter arc-lengths, and this is the effect found experimentally.

The combination of all the independent and dependent variables, capacity, sparking potential, times of charge and discharge and group frequency, may give rise to several maxima and minima when the capacity is varied and the current is maintained constant. A complete discussion of these can only be obtained with the aid of the equations of charge and discharge after the form of the sparking potential as a function of the effective arc current, the capacity and the other constants of the circuit are known.

The rapidity with which the oscillations are damped out will evidently be a function of the maximum amplitude with which they start, depending upon the primary capacity and inductance, and of the sparking potential and group frequency. The minimum value of the oscillations corresponds both to relatively small initial amplitude and high sparking potential and therefore to strong quenching. The points of the maximum voltage readings in Fig. 7, which correspond to minima and strong quenching in the oscillations, represent conditions especially favorable for the production of impact excitation effects.

§ 8. *Variation of Sparking Potential with Arc Current.*

In order to test the assumption that the sparking potential required to break down the insulation increased as the arc current was diminished, a Braun cathode ray oscillograph was connected across the arc, as shown in Fig. 9, so that the maximum deflection of the ray by the electromagnetic field measured the maximum instantaneous or sparking potential across the arc.
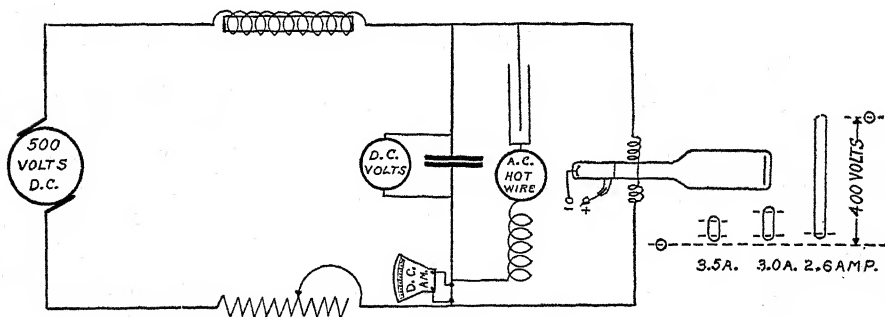
Fig. 9.

Determination of sparking potential by Braun oscillograph.

The oscillograph was calibrated by noting the deflection produced by different applied voltages before the arc was started, and it was found that the potential difference measured by the voltmeter across the arc was directly proportional to the deflection of the fluorescent spot on the cross-sectioned screen. It was found that a good deflection of 21 scale divisions could be produced by one half an ampere, and a resistance of 450 ohms was placed in

series with the deflecting field before it was connected across the arc. On account of the appreciable current taken by the deflecting field, the five ampere shunt which converted the millivoltmeter into a d.c. ammeter was connected so that only the arc current passed through it.

At the right of Fig. 9 is shown the appearance of the fluorescent spot under the different conditions of the experiment. With no potential difference across the arc the fluorescent spot was a small circle, as shown at the left. When the current of 3.5 amperes d.c. was sent through the arc and the oscillations started, the circle spread out into a band of light with bright spots at the ends and a less bright region between. The lowest position of the spot of light corresponded to 29 volts across the arc, and highest position to a maximum potential of 68 volts. The voltmeter reading was between these limits, 52.5 volts, and the intensity of the oscillations was about 5.4 amperes.

When the d.c. ammeter reading was reduced to 3.0 amperes, the bright discs spread still farther apart, the lower one rising slightly to 31 volts, and upper limit increasing to a maximum potential of 104 volts. The voltmeter reading was 68 volts, and the intensity of the oscillations 5.6 amperes.

When the current was reduced still further to 2.6 amperes d.c., bringing it into the region of the oscillation minimum and voltage maximum, the lower disk was raised to a point corresponding to 34 volts, but the well defined upper disk disappeared. A bright region extended up to something more than 100 volts, and beyond this frequent long streamers or tails reached up to 400 volts or more. The voltmeter reading was about 76 volts, occasionally running up to 80 volts, and the intensity of the oscillations fell off to 5.0 amperes. Finally the arc went out, and the impressed potential difference across the plates was found to be 400 volts.

No oscillations could be obtained at smaller currents, nor could the arc be kept in operation. Apparently the shunt of 450 ohms across the arc prevented the high sparking potentials which were necessary at smaller currents from being attained. An arc-length of 0.15 mm. was used in all cases, and the capacity and inductance of the oscillatory circuit were 0.0169 mfd. and 14,400 cm. respect-

ively. The experiments were repeated with a capacity of 0.0132 mfd. and gave very nearly the same results for the large currents, while at 2.7 amperes the maximum voltage as shown by the streamers on the fluorescent screen was about 300 volts.

The experiment shows therefore, that the maximum or sparking potential increases as the arc current is decreased and is accordingly in agreement with the theory advanced. Apparently at the point where the oscillations are a minimum there is a cumulative effect which increases the sparking potential more for a given decrease of direct arc current than at other points. This is due undoubtedly to the effect of the oscillations themselves upon the sparking potential. The lowest potential across the arc also increases slowly as the direct current is increased, and apparently agrees fairly well with the curve marked "without oscillations" in Fig. 4, the voltages being slightly lower in this case because the oscillations lower the resistance of the arc.

All the phenomena observed with the cathode ray oscillograph pointed to a form of voltage oscillation curves very similar to those observed by Barkhausen[1] using the Poulsen arc. He employed an electric field to give a deflection proportional to the arc voltage, and a magnetic field to give a deflection at right angles proportional to the condenser current. His curves show large increases of the sparking potential as the arc current is decreased, and smaller increases of the lower limit of potential, in agreement with the author's experiments. Unfortunately, no plates for making an electric deflecting field were available inside the Braun tube used by the author. It is evident, however, that the phenomena observed in one dimension in the author's experiments on the Lepel arc are very closely related to the boot shaped diagrams drawn out in two dimensions in Barkhausen's curves.

§9. *Variation of Arc Current with Arc Length.*

If horizontal sections are taken through the family of curves shown in Figs. 5 and 6 the variation of the current through the arc, as read by the d.c. ammeter, is found to have maximum and mini-

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[1] H. Barkhausen, "Die Erzeugung dauernder Schwingungen durch den Lichtbogen," Jahrbuch der drahtlosen Telegraphie und Telephonie, Vol. I., p. 243, 1907.

mum values as the arc length is increased while the voltage across
the arc is maintained constant. This is chiefly an effect due to the
capacity of the arc, and is explained in the same way as the variation
in voltage at constant current was explained, the two phenomena
being complementary. The curves obtained in this way are shown
in Fig. 10.
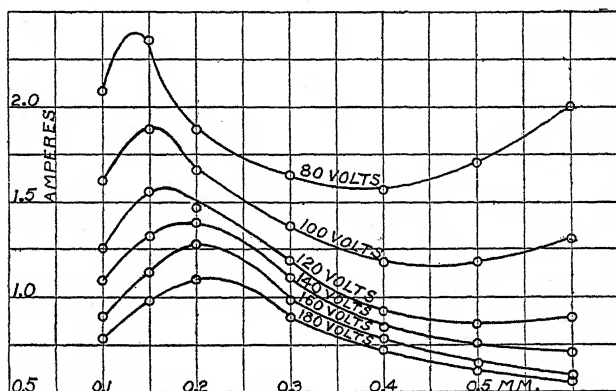
Fig. 10.

Variation of arc current with arc length at constant voltage.

It is apparent from a comparison of Figs. 7 and 10 that the maxima
for the volts at constant amperes correspond to the maxima for
the amperes at constant volts, all the quantities being measured
with d.c. instrument. In both sets of curves the maxima move
into the regions of longer arc-lengths as the voltage is increased
or the current reduced.

An alternative interpretation may be given to the curves in
Fig. 7 and 10 if we assume that the voltage across the arc depends
not merely upon the direct current through the arc, but rather
upon the effective arc current, or the square root of the sum of the
squares of the direct current and the oscillatory current. Ac-
cording to this interpretation, when the oscillations are at a mini-
mum, corresponding to a critical value of the capacity of the arc,
the effective current through the arc would be small, and the voltage
across the arc would rise to a maximum even though the d.c.
ammeter reading is kept constant. The maxima in Fig. 10 cor-
respond to the minima oscillations, and it is apparent, from the

point of view of effective current values, why the direct current must be increased in order to keep the voltage constant.

According to this interpretation, if the effective arc current, as measured by a hot wire ammeter, were maintained constant, the voltage across the arc should remain constant when the capacity across the arc is varied, and should change but slightly when the arc length is varied.

§ 10. *Analogous Effects Observed by Duddell and Others.*

Investigators who have used the ordinary spark gap as a means of producing high frequency oscillations have long been familiar with a phenomenon similar to that shown by the curves in Fig. 10. For a given constant applied voltage at the high tension transformer or induction coil, there is a definite spark length which will give a maximum current in all the circuits. It is possible that this effect is due in part to the influence of the capacity of the spark, but it is probable that the variation of the sparking voltage and spark resistance with the spark length plays a much more important than in the case of the short arc between large electrodes.

A more marked effect has been observed by Duddell[1] in the secondary current of an induction coil when a mercury interrupter was used to make and break the primary current. With a small spark gap between two aluminium points about 1.2 mm. apart in the secondary, the current measured by a hot wire ammeter rose to a maximum about double the value observed when the secondary spark gap was short circuited. The maximum in Duddell's curve with currents as ordinates and spark lengths as abscissas is sharper than that shown in any of the curves in Fig. 10, but the general characteristics are the same. Duddell's curve has also the same features of a maximum followed by a minimum and then an increase of current as far as the observations are continued shown by the upper curves in Fig. 10. The influence of a change in the shape of the electrodes in Duddell's experiment and the variations in the oscillations when the capacity of the secondary circuit was changed by touching an insulated metal plate to various points of it indicate that the effect was due in part at least to the capacity of the spark

[1] W. Duddell, The Electrician, Vol. 61, p. 796, 1908. See also discussion, p. 134.

gap.   It is probable, however, that the resistance of the spark gap in Duddell's experiment plays a relatively more important part than in the phenomena of the Lepel arc.

Summary of Results.

For electrodes of the short arc oscillation generator copper was found to be most satisfactory material, and with plates not more than one quarter inch thick water cooling was found to be unnecessary.   The kind of paper used for separating the plates is immaterial, but much difficulty attends the use of mica.   Charring the paper and breaking a short circuit across the arc in oil is the most effective method of starting the arc when only a low voltage is available. Connecting arcs in parallel, each with its own choke coil and inductance, gives double power for the same applied voltage, and has other advantages over series connection.

The Lepel arc has a falling characteristic much steeper than those of the Duddell or Poulsen arcs and operates on smaller currents. The separation of the two characteristics obtained with and without oscillations is much greater for the Lepel arc than for the others. The characteristics for increasing arc-lengths lie one above the other at large currents, but at small currents the order is reversed. This is found to be due to marked maxima in the voltage across the arc which occur at smaller arc lengths as the current is increased.

The maxima in the voltage are due indirectly to the variation in the arc capacity, which is quite large on account of the small distance between the plates which form the electrodes.   The maxima in the voltage are caused by corresponding minima in the oscillations produced by the arc.   The capacity of the arc and of the condensers are in parallel, but the inductance in series with the condensers. partly neutralizes the effect of their capacity and makes the capacity of the arc more important.   Decreasing the inductance of the main circuit has the same effect on the voltage as increasing the capacity of the arc.

The minima in the oscillations and corresponding maxima in the voltage across the arc occur because for certain values of sparking, potential, arc current, capacity and inductance, the time of discharge is a minimum compared with the time of charge.   The minima in

the oscillations furnish conditions especially favorable for strong quenching of the oscillation and for impact excitation effects.

Experiments with Braun cathode ray tube show that the maximum instantaneous or sparking potential for the arc increases as the arc current is decreased. The sparking potential is especially high in the region of the minima in the oscillations.

The direct current through the arc required to maintain the voltage constant as the arc-length is varied is found to have a maximum where the oscillations are a minimum. This is chiefly a capacity effect also, but is similar to certain phenomena in spark gap excitation in which the variation in the resistance of the arc plays a more important part.

In the succeeding sections on the intensity and frequency of the oscillations, it will be shown that the minima in the oscillations occur at larger values of the direct current through the arc as the capacity in the shunt circuit is increased or the inductance diminished, and that the intensity of the oscillations themselves increase. It will also be shown that the frequency of the oscillations increases with the arc current and decreases with the arc length, and that the formula derived by the author for the frequency of the singing arc holds true also for the frequency of the oscillations produced by the new short arcs between metal electrodes.

Physical Laboratory,
    Cornell University,
        June 25, 1910.

EXPERIMENTS IN IMPACT EXCITATION.

BY GEORGE W. NASMYTH.

II. THE INTENSITY OF THE LEPEL ARC OSCILLATIONS AS A FUNCTION OF THE ARC CURRENT, CAPACITY AND INDUCTANCE.

IN the preceding sections on "The Characteristics of the Short Arc Metal Electrode Generators" the apparatus used and the results of the experiments on the relation between the voltage and current of the Lepel arc, consisting of two copper plates separated by a few sheets of paper, were described. It was found that the Lepel generator resembles the arc more closely than the spark gap, and has, in common with the Duddell and Poulsen arcs, a falling voltage-ampere characteristic. Certain curious maxima in the d.c. voltage across the arc were observed as the arc length was varied at constant current, and the cause of these was finally traced to the influence of the capacity of the arc itself. It was found that these maxima in the voltage are accompanied by corresponding minima in the intensity of the oscillations. In the following sections the effect of the variation of the arc current and the capacity of the condenser circuit upon the intensity of the oscillations will be investigated, and later the question of the frequency will be taken up. For the present the connections remain as shown in Fig. 1, the Lepel arc being connected to a 500-volt d.c. generator through a choke coil and variable resistance, with an oscillation circuit containing capacity, inductance and a thermoammeter, in shunt.

§ 11. *The Intensity of the Oscillations.*

Two striking phenomena meet the observer who sets out to investigate the intensity of the oscillations in the condenser circuit. First, the alternating current, as measured by an unshunted hot wire ammeter in the condenser circuit, is two or three times greater than the direct current measured by an ammeter in the main circuit.

Second, if the direct current in the main circuit is varied, the intensity of the oscillations passes through a well-defined minimum. The value of the direct current at which this minimum in the oscillations occurs depends on the constants of the circuit, increasing when the capacity is increased, and decreasing when the inductance is increased. The first characteristic indicates that these oscillations belong to the third class of "Wien" or "impact" oscillations; the second characteristic has interesting consequences in the theory of the relation of the intensity of the oscillations to the other factors of the circuit.

Inasmuch as the frequency of the oscillations varies with the current in the main circuit, the difficulties of calibration at varying frequencies made it impossible to use any kind of an ammeter with a shunt. Accordingly a special ammeter was constructed without the use of iron, in which the entire current passed through a small copper wire, 0.22 mm. in diameter, and about 9.5 cm. long. This ammeter was connected in the oscillation circuit, as shown by the dotted lines in Fig. 1 of the preceding article (see p. 76), and calibrated before and after each set of observations by comparison with the direct current ammeter in the main circuit.

The results of the experiments on the variation in the intensity of the oscillations with the arc current are given in Table II. and graphically in Fig. 11. The readings of the d.c. ammeter in the main circuit, called for brevity arc amperes, were varied by changing the resistance in the main circuit and the corresponding oscillation amperes as indicated by the hot wire ammeter in the condenser circuit were noted. The corresponding readings of the d.c. voltmeter connected to the two electrodes of the Lepel arc are also given.

Each curve in Fig. 11 corresponds to a definite capacity in the oscillation circuit, one Leyden jar representing a capacity of 0.00188 microfarad. The same inductance, 14,400 cm., was used for the nine curves, so that the frequency range was from 324,000 with 9 jars capacity, to nearly a million with one jar.

In each of the curves in Fig. 11 the oscillations pass through a minimum, well-defined in the curves for the larger capacities, less so at small capacities. These minima move over into the region of larger arc currents as the capacity is increased, following ap-

TABLE II.

Variation of Oscillation Intensity with Arc Current and Primary Capacity.   Arc Length 0.15 mm. $L_1 = 14{,}400$ cm.   Copper electrodes six inches diam.

Capacity, 1 Jar = 0.00188 mfd.

Arc Amperes D.C.	Oscillation Amperes.	Arc Volts D.C.
0.50	1.00	——
0.70	1.20	176
1.00	1.40	108
1.15	1.70	94
1.25	1.70	82
1.34	1.86	84
1.60	2.00	72*
1.70	1.81	64
1.75	1.65	63
1.85	1.65	62
1.90	1.63	62
2.00	1.65	61.5
2.20	1.86	60
2.40	2.03	57
2.50	2.06	57

Capacity, 2 Jars = 0.00376 mfd.

Arc Amperes D.C.	Oscillation Amperes.	Arc Volts D.C.
0.76	2.40	——
1.03	2.20	——
1.50	2.32	76
1.55 min.	2.28	71
1.60	2.32	70
1.70	2.43	69
1.80	2.49	68
1.90	2.64	66
2.00	2.76	64
2.50	2.34	63*
2.65	2.30	59*

Capacity, 3 Jars = 0.00564 mfd.

Arc Amperes D.C.	Oscillation Amperes.	Arc Volts D.C.
1.30	3.00	——
1.41 min.	2.80	91
1.45	2.90	88
2.20	3.03	64
2.30	3.05	63
2.40	3.10	62
2.50	3.21	62
2.60	3.28	61

Capacity, 4 Jars = 0.00752 mfd.

Arc Amperes D.C.	Oscillation Amperes.	Arc Volts D.C.
1.45	3.85	96
1.60	3.25	82
1.70	3.17	79
1.75 min.	3.12	76.5
1.80	3.14	75.5
2.00	3.25	70
2.05	3.30	69
2.25	3.41	67
2.50	3.48	62
2.71	3.52	56
2.75	3.53	56
2.90	3.54	54.7
3.01	3.69	53
3.20	3.90	52
3.30	3.86	52
3.36	3.90	52
3.42	3.95	52
3.50	3.80	51.5
3.63	3.59	48
3.72	3.69	49
3.80	3.55	47
4.00	3.70	46

Capacity, 5 Jars = 0.00940 mfd.

Arc Amperes D.C.	Oscillation Amperes.	Arc Volts D.C.
1.08	5.56	——
1.30	5.15	146
1.48	5.00	126
1.60	4.92	116
1.70	4.62	104
1.80	4.27	88
2.00	3.82	73
2.15	3.80	70
2.25	3.57	66*
2.50	3.77	65
2.75	3.86	61
3.00	3.90	53*
3.26	4.01	50
3.50	4.12	50

Capacity, 6 Jars = 0.01128 mfd.

Arc Amperes D.C.	Oscillation Amperes.	Arc Volts D.C.
1.45	6.23	——
1.52	6.05	138
1.63	6.05	128
2.00	5.70	97
2.23	5.00	82
2.47 min.	4.04	68
2.54	4.09	——
2.67	4.25	64
2.75	4.35	63
2.94	4.62	59
3.00	4.76	58
3.25	4.76	56
3.50	4.83	53
3.75	4.72	49
4.00	4.83	47

Capacity, 7 Jars = 0.01316 mfd.

Arc Amperes D.C.	Oscillation Amperes.	Arc Volts D.C.
2.35	6.00	
2.50	5.83	
2.55	5.60	
2.75	5.24	
2.83	5.20	
2.97 min.	4.92	
3.02	4.98	
3.20	5.14	

Capacity, 8 Jars = 0.01504 mfd.

Arc Amperes D.C.	Oscillation Amperes.	Arc Volts D.C.
2.35	6.28	
2.75	6.00	
3.02	5.60	
3.05	5.50	
3.19	5.28	
3.29 min.	5.10	
3.45	5.35	
3.50	5.24	

Capacity, 9 Jars = 0.0169 mfd.

Arc Amperes D.C.	Oscillation Amperes.	Arc Volts D.C.
2.82	6.1	72
3.18	5.9	—
3.43	5.6	—
3.51 min.	5.2	50
3.80	5.6	50

Points starred * are not plotted, to avoid confusion in the figure.

parently a straight line law. The descent to the minima is much steeper from the region of smaller arc currents, and in some of the curves a maximum followed by another minimum can be seen in the region of larger arc currents.

It was shown in § 7 that there are maxima in the voltage across the arc corresponding to minima in the oscillations as the capacity
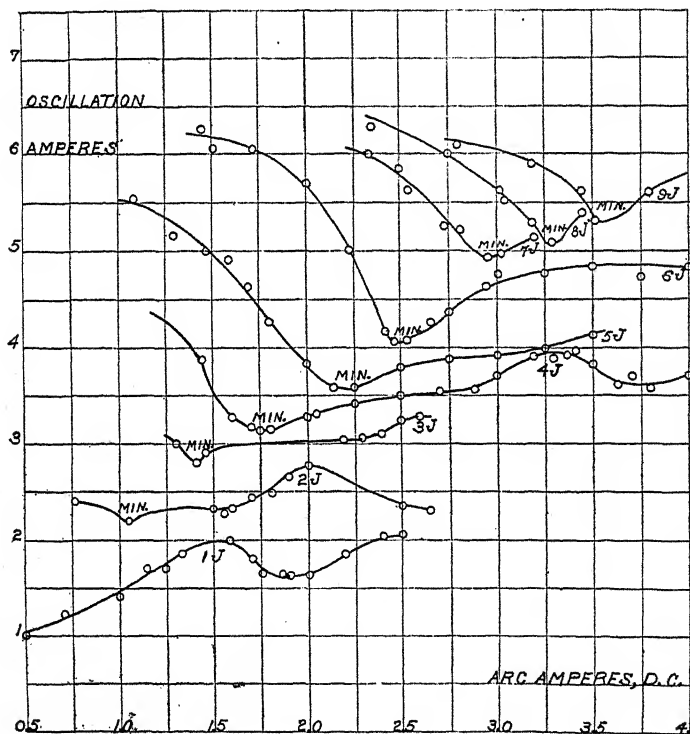
Fig. 11.

Variation in intensity of oscillations with arc current at constant capacities.

of the arc is varied at constant arc current. Moreover in § 5 it was shown that as the capacity of the arc was increased the maxima in the voltage moved into the region of larger arc currents.

This is evidently another phase of the phenomena shown by the shifting of the minima to larger arc currents as the capacity is increased, in Fig. 11, and furnishes additional evidence that the capacity of the arc itself must be considered as connected in parallel

with the capacity of the condenser-circuit as far as the effect upon the intensity of the oscillations is concerned.

The effect of the primary capacity upon the arc voltage is shown in Fig. 12, from the data of Table II. Besides the increase of voltage with added capacity there is also a change of the form and slope of the curves, the increase of voltage for small currents being greater than that for large currents. There is some indication of the maxima in the voltage corresponding to the minima in the
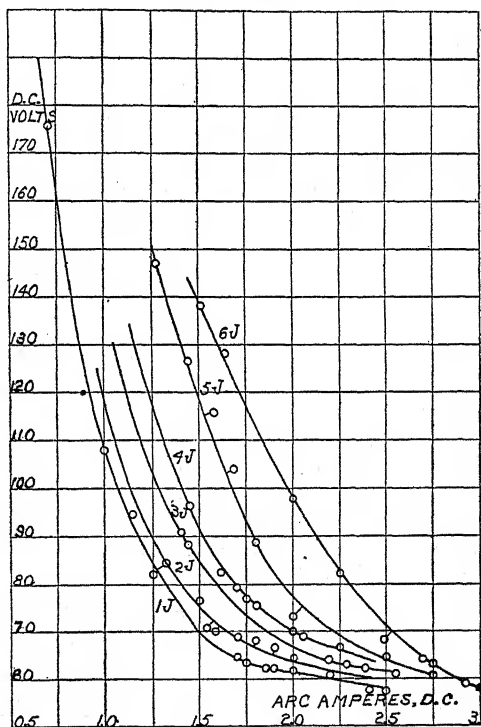
Fig. 12.

Increase of arc voltage with primary capacity.

oscillations of Fig. 11, but not enough points are available to bring out the humps of the curves clearly.

The influence of the minima in the oscillation intensity upon the frequency of the oscillations will be discussed in §§ 17–19. The bearing of the effect of capacity in increasing the oscillation intensity and changing the slope of the voltage-ampere characteristics upon the question of the frequency will appear in § 20.

In Table III. arc currents at which the minima in the oscillations in Fig. 11 occur, together with the intensity of the oscillations corresponding, are tabulated. Similar data are given for larger inductances with the addition of the d.c. voltmeter reading across the arc. In the data for 46,400 cm., the readings when a non-inductive resistance of one ohm was added to the condenser circuit are also given. The effect of resistance seems to be the same as that of capacity in increasing the arc voltage and the arc current at which the oscillation minima are found, but the change is too small to be conclusive.

TABLE III.

Shift of Oscillation Minima with Capacity and Inductance. Arc Length 0.15 mm

Primary Capacity Leyden Jars. Capacity, Microfarads.	3 0.00564	4 0.00752	5 0.00940	6 0.01128	7 0.01316	8 0.01504	9 0.0169
Primary Inductance $L_1 = $ 14,400 cm.							
Arc Current for Minimum Oscillations	1.33	1.84	2.12	2.34	3.02		
Minimum Oscillation Amperes	3.1	3.6	4.06	4.10	4.98		
Primary Inductance $L_1 = $ 46,400 cm.							
Arc Current for Minimum Oscillations	1.22	1.60	1.85	2.20	2.60	2.95	
Same, one ohm added	1.32	1.68	2.00				
Minimum Oscillation Amperes	3.08	3.23	3.42	3.51	3.90	4.08	
Same, one ohm added	3.04	3.27	3.51				
Arc Volts at Minimum Oscillations	64	56	46	41	41	—	
Same, one ohm added	76	64	64.2				
Primary Inductance $L_1 = $ 176,000 cm.							
Arc Current for Minimum Oscillations		1.45	1.68	1.98	2.25	2.65	2.90
Minimum Oscillation Amperes		—	1.49	1.77	1.99	2.46	2.95
Arc Volts at Minimum Oscillations		—	45	44	42.5	42	44
Primary Inductance $L_1 = $ 327,500 cm. Lower Minima.							
Arc Current for Minimum Oscillations		1.02	1.22	1.40	1.68	1.83	1.95
Minimum Oscillation Amperes		1.57	1.82	1.90	2.03	2.19	2.35
Arc Volts at Minimum Oscillations		76	66	58	53.5	49.5	47.5

In Fig. 13 the arc currents at which the minima in the oscillations occur are plotted as ordinates with the corresponding capacities as abscissas. The resulting curves are seen to be straight lines. As the inductance in the oscillation circuit is increased, the slope of these lines is seen to decrease, at first rapidly, then more slowly. The arc current for minimum oscillations is apparently some inverse function of the inductance, and directly proportional to the capacity.

At small inductances traces of a second minimum were observed at smaller currents, and at large inductances these secondary minima became more prominent than the others.   The lowest curve shows
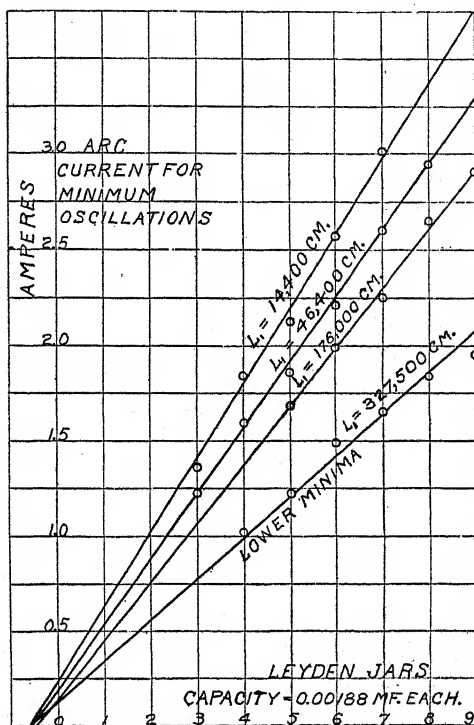
Fig. 13.

Shift of oscillation minima to larger arc currents with increasing capacity and decreasing inductance.

that this secondary minimum follows the same straight line law as the primary minima.

§ 12. *Influence of the Capacity of the Arc.*

The straight lines of Fig. 13 all give an x intercept corresponding to the capacity of the arc itself, considered as a condenser.   The capacity of the arc has been computed from the dimensions and distance apart of the plates to be about two thirds of one of the Leyden jars.   The mean value of the x intercepts in Fig. 13 by least squares was found to be 0.54 Leyden jars, equivalent to 0.01

microfarad. In the experiments the plates never overlapped completely, on account of the projection of the screws and connecting wires, and it was observed that when the overlapping was less complete, as in the experiments at 176,000 cm. of inductance, the x intercept corresponding to the capacity of the arc was smaller.

The capacity of the arc is added to the capacity of the condenser circuit as far as the intensity of the oscillations is concerned. But for the frequency of the oscillations the author's experiments, to be described later, show that the arc acts merely as a resistance and that its capacity effect does not enter directly. This is only to be expected, as the arc could hardly act as a condenser when its insulation is broken down and a current is flowing through it. An important corollary of these two observations is that the Lepel arc may be available as a source of very high frequency oscillations whose intensity shall not be too greatly decreased. In other words, with an arc whose capacity is large, it may be possible to produce electric waves shorter than one meter, having a mean intensity much greater than that of the short waves produced by a Hertz oscillator.

§ 13. *Theoretical Considerations.*

We can obtain some insight into the reason for the minima observed in the intensity of the oscillations and shown in Fig. 11 with the aid of a formula first derived by M. La Rosa[1] for the maximum value of the oscillatory current. If we write down the energy relations for the condenser circuit for the instant of sparking and for the instant when the condenser current is a maximum, we have

$$\frac{Li^2_{\text{max.}}}{2} = \frac{LI^2}{2} + \frac{Ce^2}{2}$$

where i is the condenser and I the generator current and e the sparking potential of the gap.

This equation states that the energy stored in the self induction ($\frac{1}{2}Li^2_{\text{max.}}$) at the instant when the current in the condenser is a maximum is equal to the energy contained in the self induction ($\frac{1}{2}Li^2$) at the instant of sparking increased by the energy given to

[1] La Rosa, Nuovo Cimento, Vol. 7, 5, 1904.

the condenser ($\frac{1}{2}Ce^2$) during the charging time T_1. Solved for the maximum condenser current, the equation becomes

$$i_{\mathrm{max.}} = \sqrt{(I^2 + Ce^2/L)}. \qquad (5)$$

The four assumptions under which the formula holds have been summarized by Vollmer as follows:

1. At the instant of sparking the condenser current $i = I$, the generator current.

2. The damping loss due to the resistance of the condenser circuit is negligible.

3. The energy produced in the condenser circuit on account of the falling characteristic of the arc during the discharge time T_2 is negligible. (In the Duddell oscillations this is the main source of energy.)

4. The energy $Ce_i^2/2$, stored in the condenser at the instant that the condenser current is a maximum is negligible. (e_b is the potential difference across the arc during the maximum discharge.)

In order that the formula shall hold for the short spark generators it is necessary to assume also:

5. The energy stored in the capacity of the spark gap itself is negligible.

Theory shows that the assumption 1 is permissible since for a sufficiently large choke coil in the main circuit the changes in the generator current are very small.

The errors introduced by assumptions 2 and 3 tend to neutralize each other, since the amounts of energy neglected in the two cases have opposite signs. Only the difference in these amounts of enegy will have an influence and this difference will usually be small.

Assumption 4 is unnecessary, as Vollmer has shown, if we use, instead of e^2, the difference of the squares of the sparking potential e and the potential difference across the arc during discharge, e_b.

The author's experiments have shown that the capacity of the arc cannot be neglected in the short spark generators, but as a first approximation we may treat the equation as it stands.

The experiments with the Braun tube described in § 8 have shown that the sparking potential decreases as the generator current I is

increased.   Barkhausen[1] has found the same effect in the Poulsen oscillations.   The sparking potential also increases with the arc length, as might be expected.   The data available are not sufficiently accurate or extended to determine the law of the variation of sparking potential with the arc current, but if we assume the simplest relation, that the sparking potential is inversely proportional to the generator current, we have

$$e^2 = k/I^2,$$

the proportionality factor k being a function of the arc length. For small variations in the arc length is it probable that k may be considered a constant, as the variation in k will be small in comparison with the total sparking potential even at very small lengths. Using this value of e, the approximate formula (5) becomes

$$i_{max.} = \sqrt{(I^2 + Ck/LI^2)}. \tag{6}$$

At large values of the generator current I, the first term under the radical will be of importance, and the second term small.   For small values of the generator current, the second term will be important, and the first term small.   Under the limiting conditions of very large or very small currents I, therefore, the condenser current maximum, $i_{max.}$ will be large, but at intermediate values the condenser current will have a minumim in its initial maximum value and in its corresponding effective value.   This is in agreement with the experiments of the author.

We can find the value of the current for which the oscillations will be a minimum by equating the differential $di_{max.}/dI$ to zero. The condition for a minimum is

$$I^4 = k\frac{C}{L}, \tag{7}$$

so that the critical current for minimum. oscillations should be a function of the capacity and inductance at constant arc length. This also is in agreement so far with the author's experimental results.   The experiments show that the critical current is directly proportional to C (Fig. 13) instead of to the fourth root, however.

[1]H. Barkhausen, Jahrbuch der Drahtlosen Telegraphie und Telephonie, Vol. 1, p. 251.

The difference may be due in part to the influence of capacity on the relations of the maximum and effective values of the condenser current, and in part to the influence of the frequency, which is a function of the capacity on the sparking potential. The indirect effect of an increase in the capacity in lowering the sparking potential (through the influence of the duration and intensity of the oscillatory current) would also modify the relation between the capacity and arc current for minimum oscillations.

The capacity of the arc is in parallel with the capacity of the condensers and the divided circuit may give rise to the two minima, observed in the oscillations. The effective capacity of the condenser branch of the divided circuit is reduced by the inductance in series, however, and with a large inductance in the condenser circuit the capacity of the arc becomes more important. This is probably the reason why the second minima become more important than the first at the large inductance of 327,500 cm. as shown in Fig. 13.

Impact Excitation.—The bearing of the minima in the value of the effective and initial maximum condenser current upon the problem of impact excitation is apparent. It is evident that the oscillations will be most quickly damped out when their initial value is least. The average intensity of the oscillations will be least also in this case. To secure the most favorable conditions for impact excitation, therefore, the arc current or arc length should be adjusted to give a minimum in the intensity of the oscillations. If the arc current is varied by means of a rheostat in the main circuit at constant arc length, capacity and inductance, the desired minimum can easily be found with the aid of a hot wire ammeter.

SUMMARY.

The intensity of the oscillations in the condenser circuit of the Lepel arc is in general two or three times greater than the direct current supplied to the arc by the generator.

The intensity of the oscillations increases with the capacity very rapidly at small arc currents, more slowly at larger currents. Inductance and resistance decrease the intensity of the oscillations.

As the arc current is increased at constant capacity the oscillations

fall rapidly to a minimum and then increase slowly, passing through a maximum to a secondary minimum.

As the capacity is increased or the inductance decreased, the minima and maxima in the oscillations shift into the region of larger arc currents. A linear relation exists between the arc current for minimum oscillations and the capacity. The range through which oscillations can be obtained shifts with the minima.

The explanation for the minima in the oscillations is found in the effects of the energy stored at the instant the gap breaks down. The energy stored in the inductance is large for large arc currents, while the energy stored in the capacity is large, on account of the increased sparking potential, at small arc currents, giving a minimum at intermediate arc currents.

The condition of minimum oscillations is most favorable for strong quenching and impact excitation.

The d.c. voltage across the arc increases with the capacity of the condenser circuit. The increase is most rapid at small arc currents, so that the form of the voltage-current characteristics is changed by the capacity in parallel.

The capacity of the arc is added, as if it formed a parallel circuit to the capacity of the condenser circuit, for the intensity of the oscillations, but not for the frequency.

STUDIES IN THERMO–LUMINESCENCE.

IV. The Distribution of Light in the Luminescence Spectrum of Sidot Blende.[1]

By C. A. Pierce.

Introductory.

THE characteristic decay curve of phosphorescent substances is well known. If $1 \div \sqrt{I}$ is plotted as ordinates with t as abscissæ, the decay curve appears to consist of two straight lines, of different slopes, which gradually merge into one another. One of the early explanations of the decay of phosphorescence was that by H. Becquerel, who assumed that phosphorescence light was due to molecular vibrations which were set up by the exciting light. It was shown by him that, if the damping forces were proportional to the square of the molecular speed, the decay curve, in the case of a substance which had but one band in its phosphorescence spectrum, would be of the form[2]

$$I = \frac{1}{(a + bt)^2}.$$

For substances which have more than one band, Becquerel proposed for the law of decay of the total light,

$$I = \Sigma \frac{1}{(a + bt)^2}.$$

Using $1 \div \sqrt{I}$ and t as coördinates, the first law gives a straight line curve, while the second law gives a more complicated curve.

Working with a single substance, Sidot blende, Nichols and Merritt showed[3] that the spectrum did not change its form during the first three or four seconds of decay. The measurements were made

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[1] This is a continuation of a preceding article which appeared in the Physical Review, Vol. XXX., No. 6, June, 1910, p. 663.

[2] Comptes Rendus, Vol. 113, p. 618, 1891.

[3] Phys. Rev., Vol. XXI., No. 4, Oct., 1905, p. 247.

upon the one broad bright band of the spectrum. If there were other bands in the spectrum, they decayed too rapidly to be measured.

Later the same authors studied[1] the decay of the same substance at different wave-lengths and found the decay in each case to follow the straight line law for the first ten seconds, although the slopes of the several lines were different, indicating a difference in the rapidity of decay.

Still later the same authors studied[2] the decay of the total band and found the curve to consist of two straight lines, of different slopes, which gradually merged into one another.

In an article[3] on thermo-luminescence, the author of the present article showed that Becquerel's law for the decay of light consisting of two bands represented the decay of the total band, as found by Nichols and Merritt, as closely as could be expected. By assuming that a change of temperature affected the decay of one of the hypothetical bands more than the other, the data on decay curves at different temperatures were accurately explained by the same law. In another article[4] by the present writer, the same law was used to explain the so-called hysteresis effect, which is due to the previous history of the phosphorescent substance.

Since the typical decay curve is always more complex than a single straight line, it is necessary to assume more than one band taking part in the decay in order to apply Becquerel's law. This led to the suggestion in the article above that, if there were always two bands, one might be due to some secondary effect. The assumption of more than one band is in contradiction to the data obtained by Nichols and Merritt, which indicated that the band did not change its shape with decay, but they did not study the band late enough in the decay to be certain that it did not change at the time the decay curve exhibited the typical bending. Their data upon decay curves corresponding to different wave-lengths offer no evidence for or against the existence of two bands, for while the straight line decay found at each wave-length seems to contra-

[1]Phys. Rev., Vol. XXII., No. 5, May, 1906, p. 279.
[2]Phys. Rev., Vol. XXIII., No. 1, July, 1906, p. 37.
[3]Phys. Rev., Vol. XXVI., No. 4, April, 1908, p. 312.
[4]Phys. Rev., Vol. XXVI., No. 6, June, 1908, p. 454.

ct this view, the different slopes of the straight lines are in agreement with it. The symmetrical smooth-sided band showed in the article preceding this would indicate that, if it were composed of two bands, they must be nearly of equal size.

More recently, in a theoretical article,[1] E. Merritt has considered the effect of different factors upon the form of the decay curve. It was shown by him that absorption of light by the phosphorescent material and irregularities in the distribution of the active component of the powder would explain part of the deviation of the typical decay curve from a straight line, but he decided that the two effects were not of sufficient moment to explain all of the deviation. Complex molecular grouping would account for the whole change. The author stated, however, that no data were available which definitely contradicted Becquerel's law, although several objections could be raised to it.

The present article and the one preceding it were undertaken to provide data with which to prove or disprove the two-band theory of phosphorescence decay. The work is based on the assumption that the typical decay curve is represented accurately by the summation of two straight-line decay curves. It has been suggested that the decay curve in its earliest parts is not in accordance with such an assumption, but recent data[2] by C. A. Zeller would seem to definitely prove the fallacy of these suggestions. The straight-line component parts of the typical decay curve differ widely in slope. Hence, on the assumption of two bands, most of the light before the bend in the curve is due to the band corresponding to the steeper straight line[3]; at the bend, the light is due more or less equally to both bands; and after the bend, the light is due mostly to the second band. With a method available for studying the light distribution before and after the bend, the two-band explanation would be proved or disproved depending on whether a change was or was not found in the distribution of light.

A method was developed in the first of these two articles with which the energy distribution of fluorescence light could be

[1]PHYS. REV., Vol. XXVII., No. 5, Nov., 1908, p. 367.
[2]PHYS. REV., Vol. XXXI., No. 4, Oct., 1910, p. 367.
[3]Fig. 3, p. 316, PHYS. REV., Vol. XXVI., No. 4, April, 1908.

studied by photographic means. This method was developed in such a manner as to avoid the assumption that a constant product of intensity of light at any wave-length times the length of exposure would give a constant density on the photographic negative, an assumption which was found not to hold even approximately.

The results of the first part of the work showed that the energy distribution of the fluorescence light of the substance used consisted of one prominent, symmetrical, smooth-sided band with a maximum at about $\mu = 0.55$. The band extended, approximately, from $\mu = 0.46$ to $\mu = 0.60$. Furthermore, the energy distribution immediately after excitation is the same as in the fluorescence light.

The present article is concerned with the light distribution before and after the bend in the decay curve and also with the effect of different temperatures and the effect of infra-radiation upon the fluorescence spectrum.

EXPERIMENTAL.

The method employed to photograph the decaying band of phosphorescence was based on the assumption that the conditions and phenomena could be reproduced indefinitely, a fact practically proven true for the substance under investigation. The phosphorescent powder, except in the case of Fig. 49, was emanations pulver, called by the writer Sidot blende, the same sample of powder having been used in the investigations previously published.

The apparatus was practically that used in the previous investigation, except electrically operated shutters were substituted to excite the powder, to expose the plate and to kill off the remaining phosphorescence with infra-radiations before repeating the excitation. With this apparatus the decay curve could be photographed between any two points time and time again until an impression had been made on the photographic plate. Since, at best, the necessary exposure was very long, varying from a few hours to a much longer time, no attempt was made to deduce the energy distribution, but the photographic spectra were compared with each other, the principle weight being attached to the position of the maximum of the band. The necessary attention was given to obtain negatives of about the same average density and of not too great density so

that no complications could result from widely different or complete chemical change of the films at any wave-length.   With these precautions, negatives were obtained which showed definite maxima and the results could be repeated as many times as desired.

The distribution of denseness on the photographic film was measured, at first, with a photometer, as in the previous article, but this method was soon abandoned because of the eye strain induced by comparing very faint fields of light.   The apparatus which was substituted consisted of a brightly illuminated slit placed before a very sensitive thermo-couple system.   The photographic
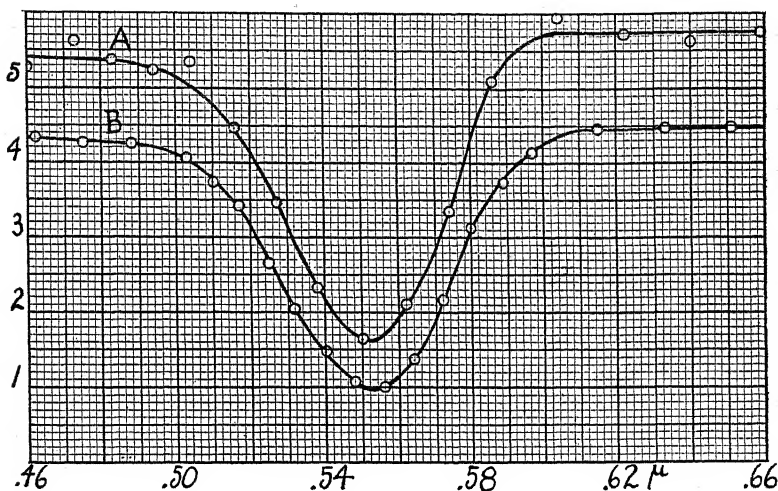
Fig. 47.

plate was pushed past the slit by means of a screw calibrated in wave-lengths.   Since the deflection of the galvanometer, which measured the current from the thermo-couple, depended upon the length of time that light was allowed to pass through the slit, a pendulum switch was made which would light and turn out the electric lamp, which illuminated the slit, at predetermined intervals. With this apparatus, results were obtained which were consistent with those obtained with the photometer, and the results could be repeated.

Fig. 47 shows the data obtained by measuring a film with the photometer, curve *A*, and with the thermo-couple, curve *B*.   In

this article all ordinates represent intensities of transmitted light.
These two curves are consistent as regards general shape and posi-
tion of the minima points of the curves.   Since intensities of trans-
mitted light are plotted, the minima of the curves correspond to
the maxima intensities of the spectra.   Inspection will show that
curve B is not the exact duplicate of curve A; the ratios of the
ordinates at long wave-lengths are not the same as at shorter wave-
lengths.   This lack of similarity did not exist in many of the com-
parisons and was, quite likely, due either to eye fatigue or else to
the gradual warming up of the plate under successive exposures to
the light which illuminated the slit.

Fig. 48 shows three curves corresponding to the decay of phos-
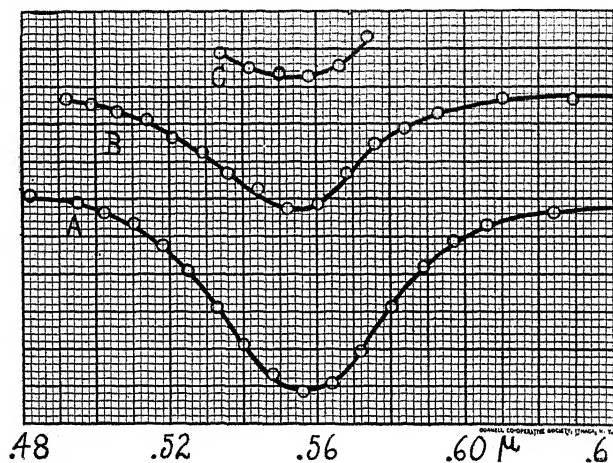
Fig. 48.

phorescence.   In curve A, the Sidot blende was excited for 9¾
sec. and the plate was exposed for 8½ sec. immediately after excita-
tion.   Then the powder was exposed for about one minute to infra-
red rays and the process repeated.   In curve B the powder was
excited for 21 sec., allowed to decay for 14 sec., then the plate was
exposed for 15 sec.   In curve B the x-axis was raised so that curve
B would not intersect with curve A.   In curve C, the excitation
was 5½ min., the decay 1½ min. and the plate was then exposed
for one minute.   The total time to produce the negative from which
curve C was made was 72 hours.   The negative was faint and the

film was somewhat fogged, but remeasurements on the film always gave approximately the same curve.   In fact, curve C is the average of several remeasurements.   The minima of these curves occur at wave-length $\mu = 0.555 \pm 0.003$.

The bend in the curve of decay of the phosphorescence of Sidot blende occurs between 10 and 20 sec. after the end of excitation and is more pronounced the longer the excitation.   Hence curve A, in Fig. 48, is due mostly to the light corresponding to the decay before the bend, while curve B corresponds to conditions near the bend, and curve C corresponds to conditions far beyond the bend. Or in terms of the two band theory, curve A corresponds mainly to band 1; curve 2, to bands 1 and 2; and curve 3 to band 2 almost entirely.   This set of curves shows no change in the maxima of the spectra, and no set was obtained which showed any appreciable change.

Because Balmain's paint, a powder on which data for a previous article were obtained, shows a more pronounced bend in the decay curve than Sidot blende, two curves, Fig. 49, were obtained showing
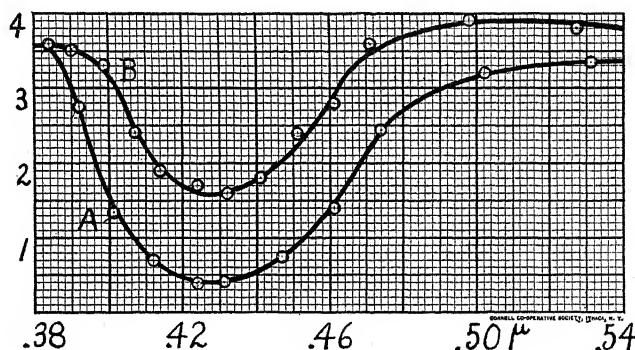
Fig. 49.

the spectrum before and after the bend.   For each curve, the excitation was 5 min.   For curve A, the plate was exposed 20 sec. immediately after excitation.   For curve B, the phosphorescence was allowed to decay 1½ min., then the plate was exposed for one minute.   No shift in the minima of the curves is evident, while a decided shift in the minima or a decided change in the shape of the curves would be expected if the two-band theory is correct.

The effect of infra-red radiations in suppressing phosphorescence and fluorescence is well known. Nichols and Merritt[1] found that all parts of the fluorescence spectrum were suppressed in the same ratio. As a check on the above work and on the present work, and because, on the basis of the two-band theory, all available data lead to an assumption that one band is suppressed more than the other, several runs were made in which the fluorescence spectrum of Sidot blende under the influence of infra-red rays was photographed. These rays were obtained, as in the whole work, from a 16 c.p. lamp held two inches from the powder. The visible portion of light was screened off by means of thin rubber.
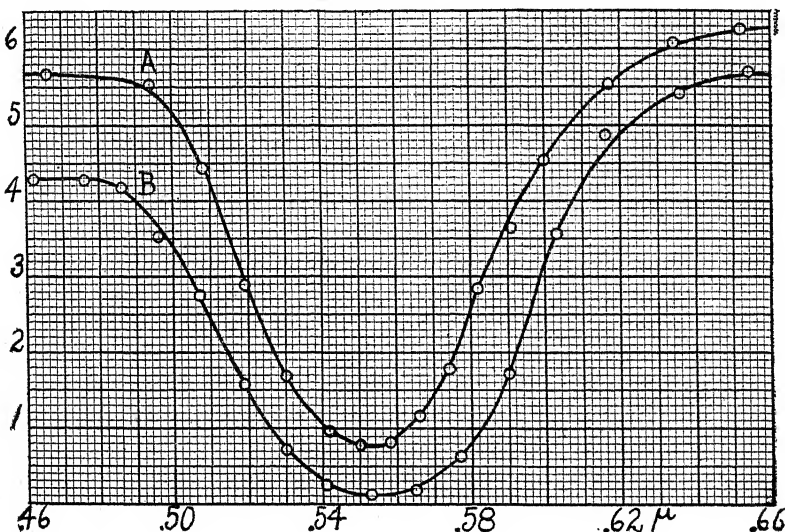
Fig. 50.

In Fig. 50 curve A corresponds to the fluorescence spectrum without infra-red excitation, and curve B, with infra-red. The exposure of the plate in the latter was twice as long as in the former case. The minimum is not changed appreciably from $\mu = 0.555$, and the curves correspond very well except at short wave-lengths.

In Fig. 51 curve A corresponds to the fluorescence spectrum without infra-red; curve B, with infra-red; and curve C corresponds to the phosphorescence spectrum just after excitation and is added

[1]Phys. Rev., Vol. XXV., No. 5, Nov., 1907, p. 362.

for ease of comparison. In each case the minimum of the curve occurs at $\mu = 0.555$ approximately.

The only effect of infra-red excitation on the fluorescence spectrum, according to all of the curves obtained, is to decrease the intensity of the band, but not to change its general shape. This result is in accordance with that found by Nichols and Merritt and is not in favor of the two-band theory of decay.

It is known that bands in the spectrum of fluorescent substances
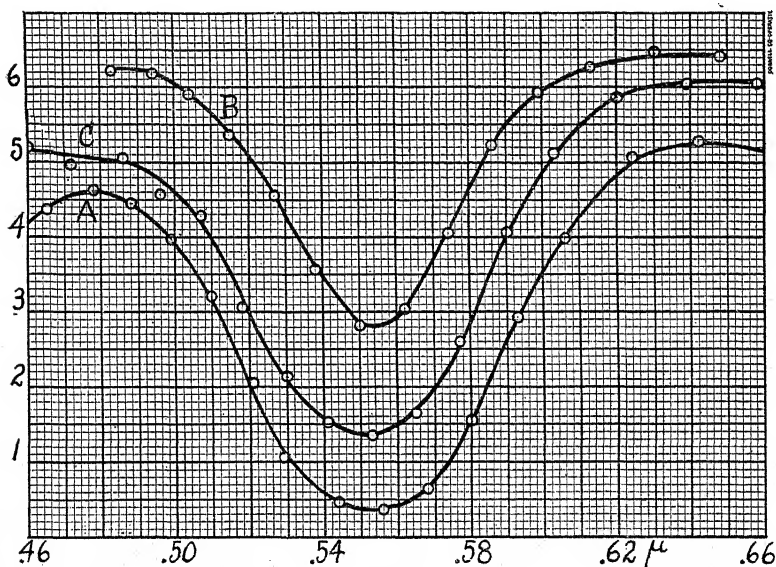
Fig. 51.

grow brighter or die out with changes in the temperature of the substance. This effect is due to the temperature change alone since many of the substances will show normal spectra when the temperature is brought back to normal. Some runs were made to see if any change could be detected in the fluorescence band of Sidot blende at different temperatures higher than normal. It is well known that the phosphorescence decay varies in rapidity with change of temperature and this could be easily explained by means of the two-band theory of decay.

Fig. 52 shows four curves corresponding to the fluorescence of Sidot blende at different temperatures. In each case the length of

excitation was varied so as to give, approximately, negatives of equal average density, hence the dimming of the band at higher temperatures is not evident, though it occurred, the powder practically ceasing to exhibit fluorescence when the temperature was raised sufficiently. When the temperature was lowered, the powder
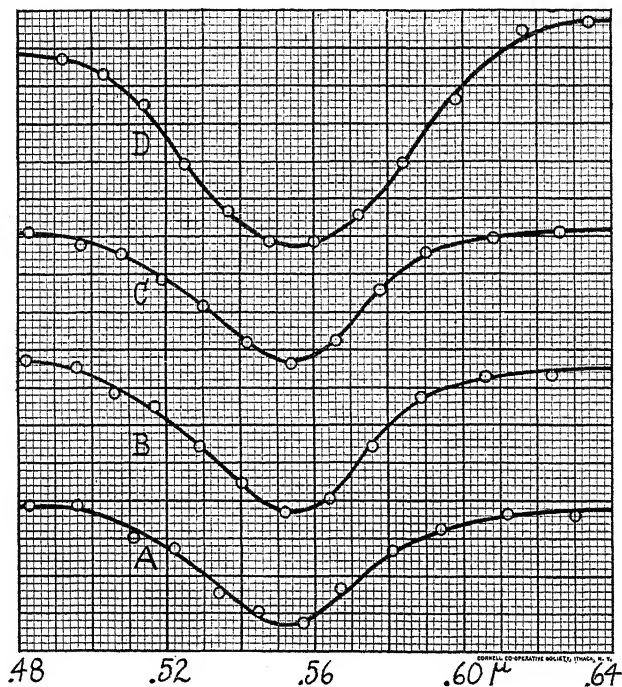
Fig. 52.

would again show the same spectrum as before if a certain critical temperature was not reached. The x-axis is changed for each curve so that the points may be entirely distinct from one another. Curves *A*, *B*, *C* and *D* were obtained at temperatures 22°, 67°, 88° and 120° respectively. No shifting of the minimum is evident.

DISCUSSION OF RESULTS AND CONCLUSIONS.

The accuracy of measurements made with a non-direct method, as the one above, is not great. Conclusions must be drawn from the indications of many runs, rather than from the too exact inter-

pretation of a single run.   A general survey of the curves shown will lead to the conclusion that the minima of all of the curves occur within the limits, $\lambda = 0.555\mu \pm 0.003$.   These limits were seldom exceeded and the variations were not consistent with each other. For some sets of curves, the limits could be contracted to $\pm 0.002\mu$ or possibly to $\pm 0.001\mu$.   Hence, if the maximum of the band changed under any of the conditions it must have been within the limits $\pm 0.003\mu$ and probably within even smaller limits.   From certain considerations, such as the bend in the decay curve of phosphorescence, and the smooth symmetrical shape of the whole band, one would expect, if the two-band theory is correct, a considerable change in the band under the conditions studied.   No changes were found that were not explained by the limitations of the method.

Hence the following conclusions are drawn from the work above:

1. That the fluorescence and phosphorescence bands of Sidot blende coincide with each other.

2. That no change in the position of the band of phosphorescence occurs with decay.

3. That no change in the position of the band of fluorescence occurs under the action of infra-red rays.

4. That no change in the position of the band of fluorescence occurs with a change in temperature of the powder.

5. That no evident change in the shape of the band was found under the above conditions.

6. That no change in the position or shape of the band of phosphorescence of Balmain's paint occurs with decay.

7. That the two-band theory of the decay of phosphorescence is not sustained.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY,
August 22, 1910.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE FIFTY-FOURTH MEETING.

THE Thanksgiving Meeting of the American Physical Society was held in the Ryerson Physical Laboratory of the University of Chicago on Saturday, November 26, 1910.

In the absence of the secretary, Dr. Jakob Kunz was elected secretary for the session.

The following papers were read:

Limitations Imposed by Slip and Inertia Terms upon Stokes' Law for the Motion of Spheres through Liquids.   H. D. ARNOLD.

Echoes in an Auditorium.   F. R. WATSON.

The Hall Effect and Some Allied Effects in Alloys.   ALPHEUS W. SMITH.

The Conduction of Electricity through Gases at various Temperatures. ROBERT F. EARHART.

The Production of Light by Canal Rays.   GORDON S. FULCHER.

On the Ionization of Gases by the Alpha Particles from Polonium.   T. S. TAYLOR.

On the Initial Velocity of Electrons emitted by Alkali Metals as a Function of the Wave-length of Light.   JAKOB KUNZ.

The Increase in the Positive Potentials assumed by Metals under the Influence of Ultra-violet Light.   R. A. MILLIKAN.

The Photo-electric Potentials of Aluminum as a Function of the Wave-Length of the Incident Light.   J. R. WRIGHT.

The Electrical Properties of Light-positive and Light-negative Selenium. F. C. BROWN.

A New Formula for the Vapor Tension of Water between 0° and 200° C. K. E. GUTHE and A. G. WORTHING.

A Null Method for Measuring Diamagnetic Susceptibility, and some Results obtained with it.   C. E. MENDENHALL and W. F. LENT.

A Comparison of the Influence of Planes of Transverse Section on the Magnetic Properties of Iron and of Nickel Bars.   E. H. WILLIAMS.   Introduced by A. P. Carman.

The Spectrum of the Spark under Pressure and an Application of the Results to the Spectrum of the Chromosphere.   HENRY G. GALE and WALTER S. ADAMS, Ryerson Physical Laboratory.

Results of Measurement on the Ultra-red Spectra of Sn, Pb, As, Bi, and Sb. H. M. RANDALL.

On the Emissive power of Wedge-shaped Apertures and their Use in Temperature Measurements.   C. E. MENDENHALL.

Valve-action of Al and Mg and their Alloys.   C. NUSBAUM.

Luminous Efficiency of Incandescent Lamp Filaments.   W. E. FORSYTHE.

The Effect of Temperature on the Di-electric Strength of Porcelain Insulators.   GEO. O. WEIMER..

The Kinetic Pressure-drop Correction in the Transpiration Method for Gas Viscosity.   (By title.)   WILLARD J. FISHER.

On the Free Vibrations of a Lecher System using a Blondlot Oscillator. F. C. BLAKE and Miss RUPPERSBERG.

On the Free Vibrations of a Lecher System using a Lecher Oscillator. F. C. BLAKE and CHARLES SHEARD.

The Distribution of Discharge between a Point and Plane under Varying Pressures.   ROBT. F. EARHART and F. W. POTE.

Elastic Properties of Bismuth Wire.   K. E. GUTHE and J. E. HARRIS.

Some Developments in the Construction and Use of the Optical Pyrometer. C. E. MENDENHALL.

The Theory of Light-positive and Light-negative Selenium.   F. C. BROWN.

Experimental Determination of the Temperature in a Fog Chamber immediately after Expansion.   E. K. CHAPMAN.

On the Absorption of Gamma Rays of Radium by Solids and Liquids. (By title.)   S. J. ALLEN.

Professor A. P. Carman took the chair during the absence of President Crew in the afternoon session.

The members of the American Physical Society were invited by the University Club to luncheon.   A vote of thanks to the University of Chicago for their hospitality proposed by Dr. Watson was unanimously carried. The meeting was closed at five o'clock.

DR. JAKOB KUNZ,
Secretary pro tempore.

NEW BOOKS.

A Text Book of Physics. By H. E. HURST and R. T. LATTEY. New York: D. Van Nostrand Company, 1910. Pp. ix + 638. Price, $3.00.

Physics. By C. R. MANN and G. R. TWISS. (Revised edition.) New York: Scott, Foresman and Company, 1910. Pp. x + 424.

An Introduction to Thermodynamics. By JOHN MILLS. Boston: Ginn and Company, 1910. Pp. viii + 136. Price, $2.00.

Gesammelte Abhandlungen. By FRIEDRICH KOHLRAUSCH. Leipzig: Johann Ambrosius Barth, 1910. Pp. xxxv + 1108. Price, M. 25.

Das Kristallisationsmikroskop. By O. LEHMAN. (Festschrift zur Feier des Dreiundfünfzigsten Geburtstages Friedrich II. von Baden.) Braunschweig: Friedrich Vieweg und Sohn, 1910. Pp. 112.

Grundzüge der Mathematisch-Physikalischen Akustik. By Dr. ALFRED KALÄHNE. Leipzig: B. G. Teubner, 1910. Pp. vii + 130. Price, M. 3.20.

Lehrbuch der Kristallphysik. By WOLDEMAR VOIGT. Leipzig: B. G. Teubner, 1910. Pp. xxiv + 964. Price, M. 30.

Prinzipien der Atomdynamik. By Dr. J. STARK. Leipzig: S. Hirzel, 1910. Pp. x + 124. Price, M. 3.20.

Annuaire Pour L'An 1911. Publie par le Bureau des Longitudes. Paris: Gauthier-Villars, 1910. Pp. vii + 639.

THE

PHYSICAL REVIEW.

MEASUREMENTS OF THE RATE OF DECAY OF GAS PHOSPHORESCENCE.

By C. C. Trowbridge.

THE present paper contains the results of a series of measurements of the rate of decay of the gaseous afterglow which is produced by the electrodeless ring discharge. It will be shown by the experiments to be described, that there is good reason for considering that this afterglow is a true phosphorescence of the gas similar to that which occurs in solids, such as the sulphides of zinc and calcium and similar substances, and therefore the phenomenon which usually has been referred to by writers as the afterglow, has been called gas phosphorescence in this paper.

The results of these experiments on the decay of the luminosity of a phosphorescent gas are presented in the following order:

1. Conditions under which the gas phosphorescence is formed.

2. A new photometric device for the measurement of the decay of phosphorescence.

3. The general law of decay of gas phosphorescence.

4. The relation of intensity of gas phosphorescence to gas pressure.

5. The change of the rate of decay with change of gas pressure.

1. Conditions under which the Gas Phosphorescence is Formed.

The formation of gas phosphorescence depends on many conditions. Not only can the electrical discharge which produces the phenomenon be varied in many ways but the gas itself is subject

to changes in constitution as well as variation in gas pressure. The phosphorescence is affected by changes in all of these conditions. In a previous paper[1] an account was given of a series of experiments which consisted in producing the phosphorescence at different gas pressures and then determining the time it remained visible. The exciting discharge and other conditions were systematically varied.

As these experiments were the first quantitative measurements made on gas phosphorescence a good deal was learned from them. A series of smooth curves was obtained showing the duration of the luminosity of the phosphorescence at various gas pressures. It was found that phosphorescence could be formed at a gas pressure as high as 2.4 millimeters and as low as about .02 millimeter. In the neighborhood of from .08 to 0.4 millimeter gas pressure, the glows were usually both bright and of long duration. The decay experiments to be described were made for the most part within the latter range of pressures.

The color of gas phosphorescence in either air or in nitrogen is greenish yellow, which agrees with the observations of H. F. Newall.[2]

The phosphorescence in air is probably chiefly due to the nitrogen in its composition, because it has been found that phosphorescence is more readily produced in nitrogen than in air. When it was difficult to produce a strong afterglow in air, a change of gas to nitrogen has always made it possible to obtain the desired results. Strong phosphorescence has been obtained in air, however, which at times has been of a very long duration.

Form of the Discharge.—At high gas pressures, or in the neighborhood of 0.5 mm., a gas is usually made phosphorescent only by continued discharges in the electrodeless vessel. This is particularly the case when low electrical intensities are acting on the gas, but when once the phosphorescence has been formed it is usually very bright, and excited by a short discharge. On the other hand, at low gas pressures the gas responds more readily to the discharge and will usually show phosphorescence with a very brief excitation and with even small electrical intensities. In the latter case, however, the phosphorescence is relatively faint.

[1] Trowbridge, PHYS. REV., Vol. XXIII., No. 4, October, 1906.
[2] Proc. Camb. Phil. Soc., Vol. IX., p. 295, 1897.

In the course of some experiments with other forms of tubes than the electrodeless type, it has been found that a good after-glow can be produced in a tube *with* the ordinary electrodes provided an oscillatory discharge of high frequency is produced in the tube. The phosphorescence thus formed has not been found yet to give very long duration but it is fairly bright and evidently of the same general nature as that formed by the electrodeless discharge. The experiments on the decay of gas phosphorescence have all been made with an electrodeless tube.

2. A New Photometric Device for the Measurement of the Decay of Phosphorescence.

In order to measure the intensity of the phosphorescence of the gas at short intervals of time after its formation by the electrodeless
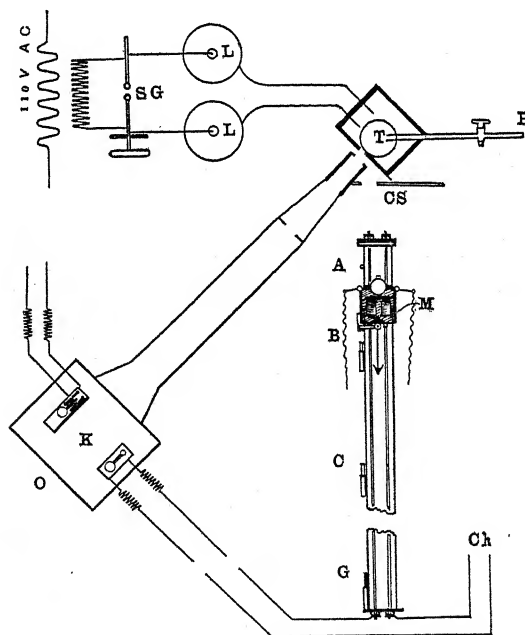
Fig. 1.

Diagram of apparatus. *I*, induction coil; *SG*, spark gap; *LL*, Leyden jars; *T*, bulb for gas phosphorescence; *P*, pump connection; *CS*, comparison screen placed against *T*; see Fig. 4; *M*, carriage with lamp which moves to stops *A*, *B*, *C*, etc.; *O*, observer; *K*, keys operating coil, carriage, and chronograph; *Ch*, chronograph.

discharge and thus determine the rate at which the luminosity diminished, a new photometric device was constructed, and with it the measurements described in this paper were made.

The method of determining the intensity of the phosphorescence consisted in a direct comparison of the luminosity of the phosphorescent gas contained in a glass bulb with a screen which was illuminated by an electric lamp of known candle power. A diagram of the apparatus is shown in Fig. 1, the novel part of which is the mechanism for moving the photometer carriage which carries the
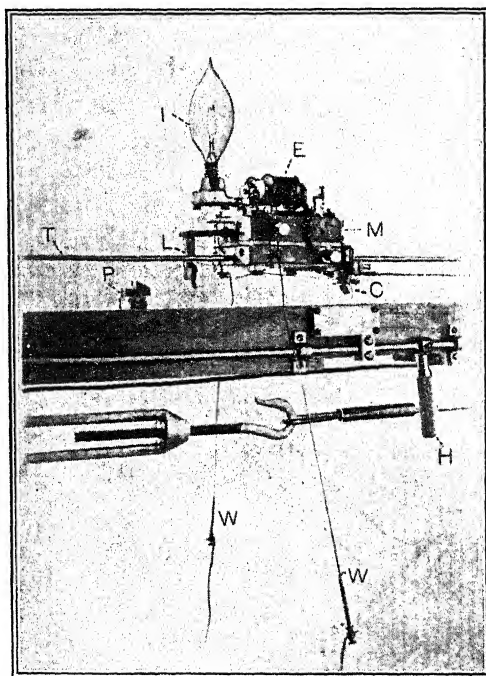
Fig. 2.

Photometer carriage. *M*, movable base which slides on track *T*; *L*, a lever which operates on projection *P* and locks the catch *C*; *E*, electromagnet for releasing the catch *C*, so that the carriage can move to the next stop; *H*, handle for adjusting with one movement all of the air check stops; *I*, incandescent lamp about 6 c.p.; *W*, *W*, wires leading to storage battery.

electric lamp so that a number of different photometric settings can be made in quick succession. This carriage is controlled elec-

trically in such a manner that by means of a contact key in the hands of the observer, a complete set of readings for a decay of the phosphorescent gas giving a number of intensities and corresponding times is recorded by use of the key alone. A long collimater tube is used for observing the comparison screen and the gas phosphorescence in order to make the latter appear as if it were a luminous plane surface. A photograph of the electrically controlled carriage of the photometer is shown in Fig. 2. The carriage M runs on a track T, made of two brass rods 3.5 meters long, which are under strong tension. It is so controlled that it can be moved rapidly by means of weights along the track to different stops at the will

of the observer. This is accomplished by a small electromagnet on the carriage which operates a catch. The latter holds the carriage at certain stops, A, B, C, etc., until released by the electro-magnet. (These stops are not shown in Fig. 2.) The connection between the observer's key and the electromagnet on the carriage is made by means of the brass photometer track. The comparison electric lamp can thus be moved by steps from A to a new position B, and then to C, etc., away from the screen which it illuminates, the latter being placed close to the tube containing the phosphorescent gas. The apparatus for moving the photometer carriage is shown by Fig. 3.

When a set of readings is to be taken the luminosity of the comparison screen is made less bright than the initial intensity of the gas phosphorescence by preliminary trials. The gas phosphorescence is then formed and when it fades to a luminosity equal to that of the screen, the key is pressed and the carriage moves from A to the next position B. The key
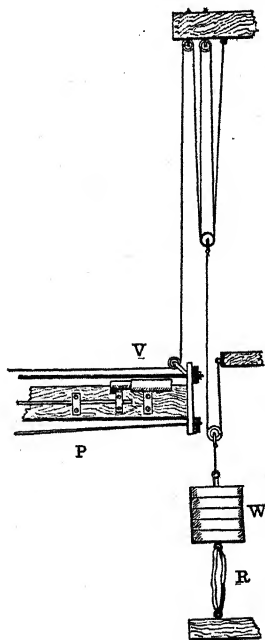
Fig. 3.

Device for moving the photometer carriage. P, photometer bar supporting the brass tracks; V, air check stop or arresting the movable carriage; W, weights with set of pulleys; R, rubber bands for producing decreasing tensions.

both registers the time of the observation on a chronograph and sets the standard lamp in a new position for a reading at a lower intensity of the phosphorescence. The readings can be readily taken. Seven different intensities of the phosphorescent gas can be recorded in about ten seconds, if it is necessary to make the observations rapidly.

Some difficulty was experienced in making the carriage move quickly enough in the portion of the photometer track corresponding to the first stage of the decay of the gas phosphorescence. The desired rapidity of movement was accomplished by attaching strong rubber bands directly to the weights as shown in Fig. 3. Since a system of pulleys is used in applying the force due to the weights a contraction of the elastic bands of a few centimeters is made to correspond to a movement of about a meter on the photometer track. This auxiliary traction force produces a rapid movement of the carriage at first, but its effect becomes gradually less as the carriage moves along the track which is exactly what is desired. A helical steel spring is now used in place of the rubber bands. The time when the exciting discharge ceases and the decay of the phosphorescence begins is automatically recorded on the chronograph. The arrangement of the coils for producing the electrodeless discharge is shown in Fig. 4.

Fig. 4.

Discharge bulb and helical coils for producing phosphorescence by the electrodeless discharge. *H C*, helical coils in parallel; *T*, tube constricted in the middle to remove influence of discharge; *CS*, comparison screen of green paper.

3. THE LAW OF DECAY OF GAS PHOSPHORESCENCE.

The first measurements of decay of the gas phosphorescence with the apparatus just described proved sufficiently accurate to demonstrate that the equation for the rate of decay in terms of the intensity and the time is the same as that for phosphorescent solids. The expression is as follows:

$$I = \frac{1}{(a + bt)^2}.$$

This formula was originally suggested as an empirical one by E. Becquerel and has been verified by a large series of experiments on phosphorescent solids by Nichols and Merritt[1] in their extensive studies in luminescence.

The observations contained in Table I. represent typical decays of gas phosphorescence, and shows the time that the phosphorescence remains brig⁻ t enough to take satisfactory readings of intensity. The value of the intensity I is an arbitrary one, and in most of the experiments was the light reflected from a paper screen illuminated by a 110-v. electric lamp of 6.4 candle power placed 69.3 centimeters distant from the screen, which was tinted green to match the color of the gas phosphorescence.

I, at position A, was given the value of 100 for convenience.

As stated in a previous paragraph, in order to make an observation of a decay all that is required is that the readings should be registered with the key. The apparatus was thus designed so that the attention of the observer could be as far as possible entirely concentrated on the photometric comparisons, which appears to be a very important requisite for accurate observations of this kind. Moreover in experiments on gas phosphorescence there are more conditions to be controlled, and therefore more that may divert the attention of the observer than in the case of experiments on phosphorescence in solids. The results of the measurements are better than might be expected for those based on rapid photometric observations and the smallness of the experimental error can be seen by an inspection of the observations in Table I. and also some of the curves given in the figures based on the observations of but one decay of the phosphorescence, namely, curves A and B, Fig. 5, and A to F, Fig. 6.

For convenience in plotting curves of intensity and time the stops on the photometer track were made to correspond with simple fractions of that intensity which corresponded to the first stop on the track.

The measurements of decay of gas phosphorescence were made

[1] PHYS. REV., Vol. XXII., No. 9, May, 1906, etc.

with a good deal of difficulty because they were often prevented on account of one of the two following conditions: (1) The low degree of the intensity of the phosphorescence. (2) The rapidity of the rate of decay of luminosity. Even under what appeared to be proper conditions for exciting the gas it was not always possible to obtain a strong phosphorescence.

TABLE I.

Observations of the Decay of Gas Phosphorescence (Nitrogen).

Intensity $(I = 100)$.	$\dfrac{1}{\sqrt{I}}$	No. 86. Excitation 4.2 Seconds.	0. 87. Ex ation 4.1 Seconds.	No . . Excitation 3.95 Seconds.	No. 89. Excitation 4.2 Seconds.
		Sec	Sec.	Sec.	Sec.
100	.100	3.47	3.77	3.70	3.30
50	.141	5.62	5.95	5.85	5.50
25	.200	8.72	8.95	8.85	8.85
15	.258	11.50	11.80	11.30	11.80
10	.316	14.74	15.10	14.60	14.67
6	.408	19.72	19.05	19.50	19.00
3.5	.534	25.5	22.9	25.65	24.95

Nos. 86–89, four successive decays, spark-gap of induction coil 8 mm. or equivalent to 30 volts per centimeter acting on the gas.

The conditions under which the observations of the two decays of the phosphorescence A and B were made, which are plotted in Fig. 5, were approximately the same. The curves are nearly alike, but show the following difference: when the reciprocal of the square root of the intensity, $I^{-\frac{1}{2}}$, is plotted with the corresponding times, one curve, A, is quite straight, while the other, B, is slightly bent away from the t axis. This curve B shows two rates of decay, both in accord with the expression $1/\sqrt{I} = a + bt$. This upward bend was found in many subsequent observations, and thus far has almost always been very slight. An exception was found in one series at a very low gas pressure (.045 mm.) in which the bend was very prominent. The curves in Fig. 5 were selected for illustration because although under apparently identical conditions in one, A, the rate of decay is constant while in the other, B, there are two rates of decay.

The small change in the rate of decay indicated by the bend in the line, B, and in other similar curves was at first attributed

to instrumental errors, but when most of the curves showed the slight bend and after the apparatus had been remeasured carefully it was necessary to conclude that the bend was due to a slight change in the rate of decay. The significance of this bend will be
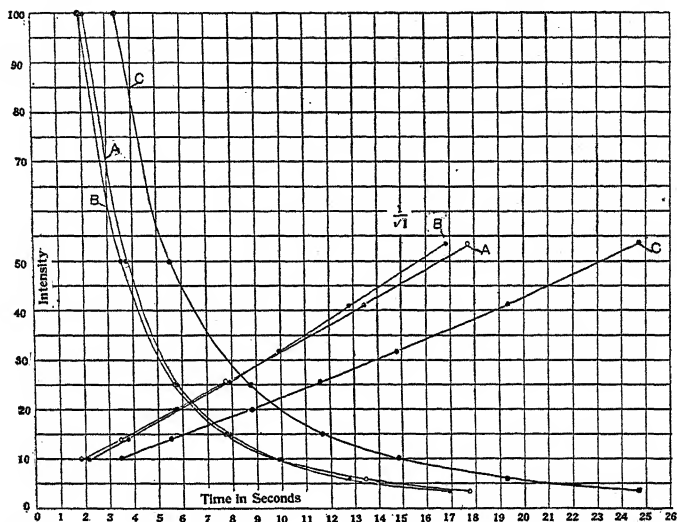
Fig. 5.

Decay of phosphorescence in air. *A* is a straight line, *B* shows a slight upward bend, when $I^{-\frac{1}{2}}$ is plotted, instead of *I*. *C* is slightly convex towards the axis of *t*.

discussed later, as certain of the experiments to be described show some effects which seem to indicate the cause of this upward bend in the curves.

The gas pressure in the case of the decays *A* and *B* shown by curves in Fig. 5 was 0.22 mm. and the electrical intensity acting on the gas which produced the phosphorescence was about 30 volts per centimeter.

The precise value of the electrical intensity is uncertain because it could only be determined by a test coil placed within the helical coil which was used to excite the phosphorescence.

Effect of Successive Discharges.—Fig. 6 contains six curves of the decay of phosphorescence in atmospheric air which were made under the same conditions and in succession. They are plotted in terms of the reciprocal of the square root of the intensities of phosphores-

cence and the corresponding times, giving thus the straight line
form of the curve,

$$\frac{I}{\sqrt{I}} = a + bt.$$

If the curves were plotted with reference to the same axes they
would overlap, hence in order to show each curve by itself they
are separated in the figure by displacing the ordinates, $I^{-\frac{1}{2}}$, a
different amount for each decay curve.

A comparison of the curves shows that the lines are nearly parallel,
indicating that the rate of decay is the same in all.   Furthermore
while the lines are not perfectly
straight throughout their length
certainly they show no tendency
to bend towards the axis of t,
as do the corresponding curves
in the case of the decay of phos-
phorescence in solids, as illus-
trated by the decay of phos-
phorescent zinc sulphide shown
in Fig. 7, but on the contrary
all the curves show a slight bend
convex to the axis of t.   The
curves also do not show any
systematic variation in the point
where they intercept the axis
of y, and since from the y in-
tercept the initial intensity of
phosphorescence is obtained the
intensities   are   approximately
the same.   It is evident that in
this case very little change in the
nature of the gas or in the gas
pressure occurred during the time occupied by the measurements of
the six decays. The small experimental error of the separate obser-
vations is demonstrated since the points fall very close to the straight
line in all cases.   It is shown from these curves (A–F, Fig. 6) that

Fig. 6.

Six decays of phosphorescence in air
showing that the rate of decay b (the
slant) is approximately the same after
successive excitations.   In curves B to F
the ordinates $I^{-\frac{1}{2}}$ are displaced ($y+D$, y
$+2D$, etc.) in order to separate the curves.

if conditions are kept constant, successive curves if few in number will not vary appreciably in the value of the slant (b) or in the y intercept (a), hence these constants in the expression $1/\sqrt{I} = a + bt$ will remain the same. This fact has an important bearing on a series of experiments described later on in this paper relating to the measurement of the variation of initial phosphorescence with the change of gas pressure. It is however evident from other experiments that a large number of discharges change the nature of the gas and reduce the gas pressure, hence the evidence of this series of observations holds only for a small number of consecutive discharges in the electrodeless vessel.

COMPARISON OF THE DECAY OF PHOSPHORESCENCE IN A GAS AND IN A SOLID.

A comparison between the decay of gas phosphorescence and the decay of the phosphorescence of a solid such as sidot blende (phosphorescent zinc sulphide) is shown in Fig. 7. The luminosities in the two examples of phosphorescence used in the comparison are of the same order of magnitude. A is a decay of gas phosphorescence, B a typical decay of phosphorescence of zinc sulphide. The observations for both were made on the phosphorescent photometer by the writer and the observations are plotted in Fig. 7 on the same scale, but the intensities are not exactly comparable because it was necessary to place a green tinted screen in front of the comparison lamp when making the measurements of the decay of phosphorescence of zinc sulphide. The initial intensities of the gas and solid phosphorescence are very nearly the same as shown by the converging curves, the value of I being somewhat less in the case of B owing to the use of the colored screen.

The curves serve to illustrate the character of the two types of phosphorescence. In the phosphorescence of the gas the rate of decay is nearly the same throughout, there being a slight upward tendency of the line at the end, while in the phosphorescence of the zinc sulphide the rate of decay changes, being slowest for long-time phosphorescence. This change in the rate of decay in solids as shown by Nichols and Merritt[1] may be due to irregular distribu-

tion of the active material and absorption effects, or it can be accounted for by assuming the presence in the solid of two phosphorescent substances which decay at different rates; a solution which has been suggested by other writers.   Whatever may be the true explanation, in a gas the conditions are perhaps relatively more
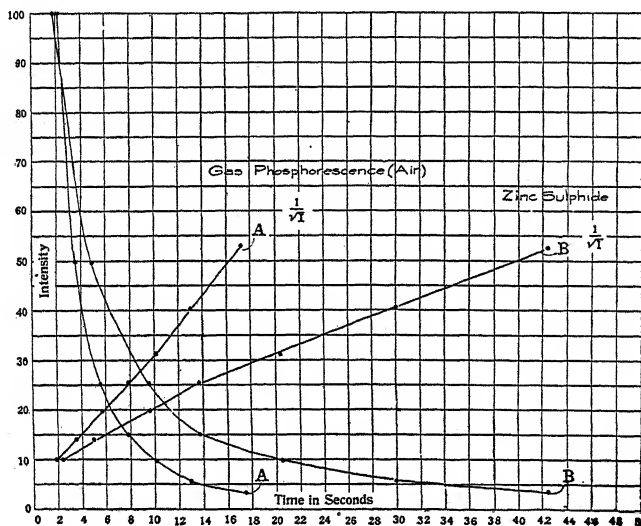
Fig. 7.

Decay of phosphorescence in air, *A*, and phosphorescence of zinc sulphide, *B*. The comparison of the intensities is approximate only because it was necessary to tint the standard light slightly with a green screen in the case of *B*.

simple resulting in a constant rate of diminution of luminosity. Thus in the formula $1/\sqrt{I} = a + bt$, b remains nearly constant in most cases of gas phosphorescence, while it changes in the case of the phosphorescence of solids.   The rate of decay in the two curves in Fig. 7 is quite similar but the fact must be kept in mind that the rate of decay in both types of phosphorescence may vary considerably, and depends in both cases on the condition of the phosphorescing matter, and on the method of excitation.

4. THE RELATION OF INTENSITY OF GAS PHOSPHORESCENCE TO GAS PRESSURE.

The intensity of luminosity of gas phosphorescence depends on various factors of the exciting discharge and on the condition of

the gas itself.   In gas phosphorescence the change of gas pressure brings in a new type of variation in the condition of the luminescent substance which is not found in solids, and it is therefore of special interest.   The effect on the intensity of the phosphorescence produced by a variation of gas pressure has been studied experimentally and the facts determined are presented below.

In the course of the decay experiments it was found that the intensity of luminosity of the phosphorescing gas depended in a marked manner on the pressure of the gas, and that the intensity
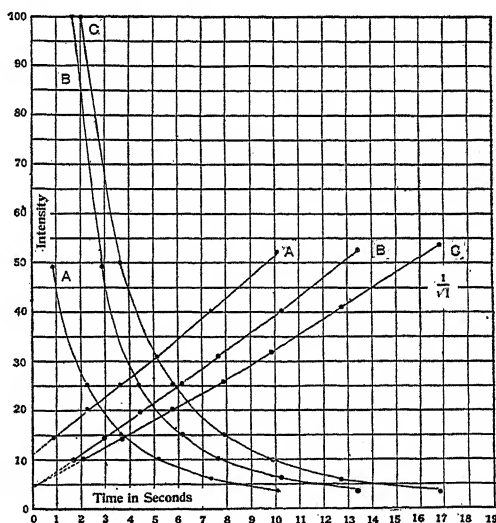
Fig. 8.

Decay of gas phosphorescence at different gas pressures.   Curves *B* and *C* are at approximately the same pressure but have different rates of decay.   Note the marked decrease in initial intensity in the case of curve *A*, which is at lower pressure.

was very much less when the gas pressure was low than when it was high.

That this is the case is demonstrated by an inspection of decay curves *A*, *B* and *C* in Fig. 8.   In *A* the phosphorescence formed at a gas pressure of 0.139 millimeter give an initial intensity of 81 (an arbitrary value depending on the scale of the figure), and *B* formed at 0.227 millimeter which gives an initial intensity of 560, on the same intensity scale.   A further inspection shows that

the straight line form of curves A and B are nearly parallel indicating that the intensity of B was greater throughout than A, both having about the same rate of the decay. Decay C was at a gas pressure of 0.222 millimeter but slightly differing from that of B, yet a marked difference in the rate of decay is shown, although the initial intensities of the two were approximately equal.

Thus it is evident that the rate of decay may vary considerably while the gas pressure remains constant. The cause of this variation in the rate of decay has not yet been determined.

In order to study systematically the effect of change of gas pressure on the phosphorescence, a series of glows were formed as far as possible under exactly like conditions; and the decay curves determined. The gas pressure was then varied a small amount and another series formed. In this way the gas pressure was changed by seven steps from 0.44 millimeter to 0.08 millimeter.

TABLE II.

Relation of Initial Intensity of Phosphorescence (I_0) to Gas Pressure (p).

Set.	Gas Pressure (p) mm.	a	Initial Intensity, I_0.	$\sqrt[3]{I_0}$	$\dfrac{\sqrt[3]{I_0}}{p}$
A[1]	.440	.007	20,400	27.08	61.4
B	.323	.0148	4,567	16.59	51.3
C	.257	.0213	2,200	13.01	50.6
D	.192	.0333	902	9.66	50.3
E	.144	.0524	364	7.14	49.6
F	.106	.0760	173	5.58	52.3
G	.080	.126	63.3	3.99	49.8

The results are shown in Table II. The value of a, from which the initial intensity of the phosphorescence was computed, is given in the third column and was determined for each pressure as follows:

From four to six decays were measured at each pressure.

These decays were plotted on a large sheet of cross-section paper, and the y intercepts (a) averaged. A mean curve was also drawn from the averaged values of t for like intensities, and another value of the y intercept found. The mean of these two values was used for a, given in Table II. The errors arising from this graphical

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[1]The value of I_0 in series A was unreliable owing to the fact that for small values of a the error in determining a by the graphical method became relatively large.

method of determining the value of a were thus minimized. The fourth column in Table II. contains the initial intensity ($1/a^2$) designated by I_0, and the fifth the cube root of I_0. The seventh contains the cube root of I_0 divided by the gas pressure, which is seen to be a constant. The variation in this constant is small except in the measurements at the highest gas pressure 0.44 millimeter, but the reason for the discrepancy at this pressure is evident enough, since the y intercept was so close to the axis of x as to make a small error in determining the y intercept produce a large error in the value of a and I_0. It is however also to be considered that the gas pressure 0.44 millimeter is very near the pressure value where it has been found, in work previously done, that the intensity is very variable, and where even the phosphorescence is sometimes not formed, hence uncertainty at this highest pressure was to be expected. There was no way to determine the initial intensity at this pressure and its value must be regarded as uncertain, rather than one that is not consistent with the law shown by the values found at other pressures. It is seen that the initial intensity varies with the cube of the gas pressure, or,

$$I_0 = Cp^3.$$

In this formula the constant is, of course, dependent on the value assigned to I and on the various conditions under which the phosphorescence is formed.

In these experiments the duration of excitation was made approximately four seconds, or as near to this interval as a hand key could be timed to the sound of a relay. A variation in the intensity of the phosphorescence occurs with a change of time of excitation. Since however in four seconds the intensity had nearly reached a maximum the errors arising from the fact that the duration of the electrodeless discharge was not exactly equal were probably small. The electric intensity acting on the gas was about 30 volts per centimeter, according to some experiments made with a test coil, in series with a small variable spark gap, the voltage being obtained from Paschen's[1] tables. The sparking distance across the terminals of the secondary of the induction coil was eight millimeters.

[1] Wied. Ann., 37, 1889.
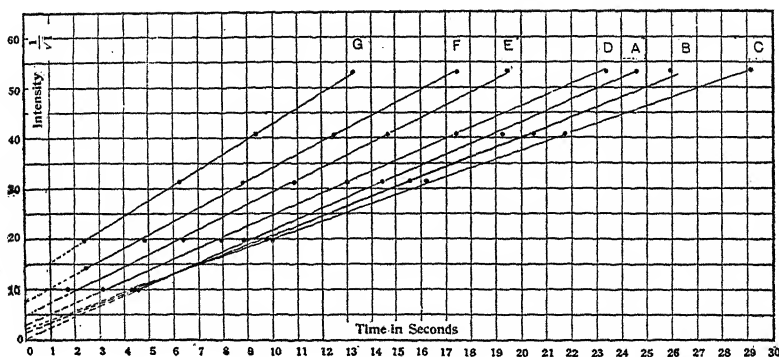
Fig. 9.

The decay of phosphorescence in nitrogen gas at gas pressures from 0.44 mm. to 0.08 mm. The approximate values of a are indicated by the y intercept.

A, 0.440 mm. gas pressure; *B*, 0.323; *C*, 0.257; *D*, 0.192; *E*, 0.144; *F*, 0.106; *G*, 0.080.

The gas used was atmospheric nitrogen, containing the usual impurities which accompany it, when it is obtained from air by the oxidation of phosphorus, probably including some oxygen.

In these experiments it was thought not advisable to attempt to use very pure gas until something could be learned of the general laws of gas phosphorescence. The gas was not freed from mercury vapor and when pure gas is used this should be done.

Indeed the mercury vapor may play a very important rôle in the behavior of the gas phosphorescence and the gas in these experiments should be considered a mixture of nitrogen and mercury vapor. On the other hand experiments by H. F. Newall,[1] P. Lewis[2] and the writer[3] seem to agree that the phosphorescent glow in nitrogen is stronger than in some other gases, and has a yellowish color which is quite characteristic.

Fig. 9 contains the mean curves of the decays at each pressure given in Table II., the straight line form of the curves being alone used. The change of initial intensity with the change in gas pressure is shown by the y intercepts of the various curves.

In Fig. 10 the initial intensities and the gas pressures are plotted

[1]Newall, l. c.
[2]Lewis, Astrophys. Jour., XII., July, 1900.
[3]Trowbridge, l. c.
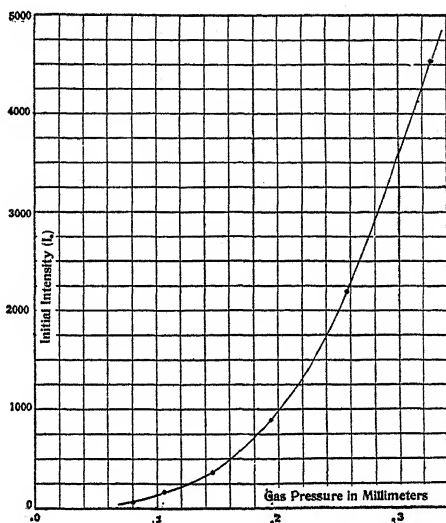
Fig. 10.

Initial intensities of gas phosphorescence I_0 (I at zero time) and corresponding gas pressures. I is taken at 100, and is the intensity of the comparison screen illumined by a 6.4-c.p. lamp placed at a distance of 69.3 cm.
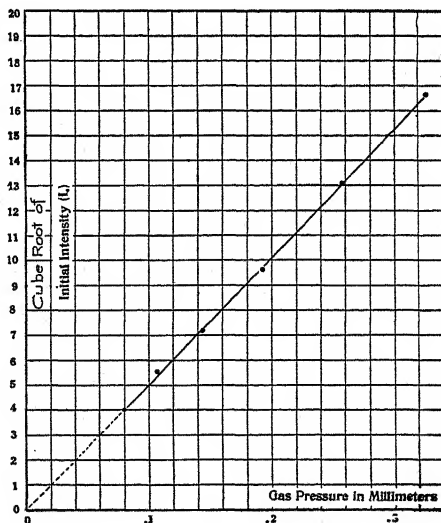
Fig. 11.

The cube root of the initial intensity of gas phosphorescence and corresponding gas pressures. The curve appears to pass through the origin.

showing an even curve. In Fig. 11 the cube root of the initial intensities and the corresponding pressures are plotted showing the relation $I_0 = Cp^3$ to be true. It is noteworthy that the straight line obtained passes directly through the origin. The value of the cube root of the intensity for gas pressure at 0.44 millimeter is omitted as being unreliable as already explained.

5. Change in the Rate of Decay with Change of Gas Pressure.

General Considerations.—There are several causes which bring about a change in the rate of decay of the phosphorescence of a gas. Among these is that of continued sparking of the gas. Another is one which produces a gradual change in the gas when it is left in the bulb and pump with no discharges for a considerable time, a matter of days, and without the admission of new gas. The explanations of these effects are difficult to determine, and they are not known; the rate of decay appears to depend on the susceptibility of the gas to the passage of the exciting discharge. There is evidently also a hysteresis effect which makes the behavior of the phosphorescence depend on the previous history of the gas, although the electrodeless discharge itself appears to suffer no alteration. There are certainly changes in the gas which last for hours or longer after a series of discharges have passed through the gas, which affect both the subsequent intensity of the phosphorescence as well as its rate of decay.

Change of Gas Pressure.—The effect of a change of gas pressure alone on the rate of decay is clearly illustrated by Fig. 9, where the decays at seven different pressures are seen to vary greatly. The computed values of b in the formula $1/\sqrt{I} = a + bt$, or the slants of these curves are given in Table III. In Fig. 12 a curve is drawn with the values of b as ordinates and the gas pressures as abscissas. As the pressure is reduced by steps from 0.44 millimeter the rate of decay diminishes until it reaches a minimum value of b of about .0175 at a gas puressre of 0.28 millimeter after which b increases down to the lowest pressure used in this particular series of observations, or .08 millimeter. It is interesting to note that this point of maximum duration is near the gas pressure at which

the electrodeless discharge is most easily started, and although slowest in rate of decay it is not as bright as at higher pressures as shown by the results given in a previous paragraph.

Omitting the highest pressure value of b (p .44 mm.) the curve was found by trial to be not much different from a parabola in form. It is necessary to state that in all these curves, especially where the rate of decay is concerned, there are probably conditions which have slightly affected the form of the curves, due perhaps to a sort of hysteresis, and which, in the present stage of the investigation, it is not possible to control. This is illustrated by Fig. 8 where the curves show a marked difference in rate of decay at the same gas pressure, with apparently quite similar exciting conditions of the discharge.

In the observations on which the curves in Figs. 9–12 are based particular care was taken to keep all conditions the same except the systematic change of gas pressure, and the fact that the indi-
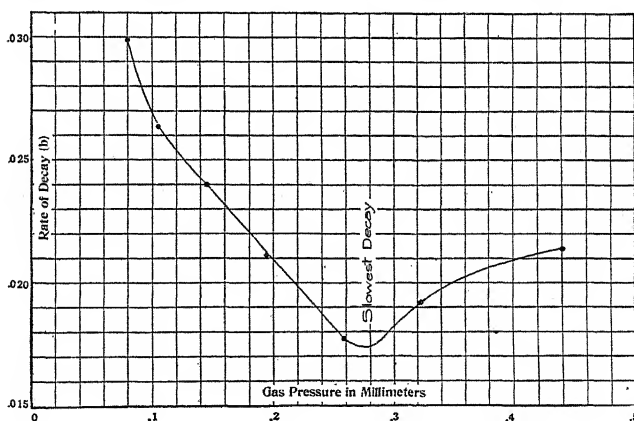
Fig. 12.

The change of rate of decay of phosphorescence with change of gas pressure. The decay is slowest at about .28 mm. gas pressure.

vidual decay curves at each of the various pressures had approximately the same value of b make it appear that the results are fairly free from error. It was, however, noticed that both in the values of the initial intensity determined and in the value of b there was a very slight progressive change which was due to a

corresponding change in the gas pressure, since the gas pressure is reduced a small amount by every discharge in the electrodeless vessel, but this very small progressive change was taken care of as far as possible by averaging both the gas pressures and the values of the slant in each set.

TABLE III.

Relation of the Rate of Decay (b) to Gas Pressure.

Set.	Gas Pressure (p).	Computed Value of b.	Set.	Gas Pressure (p).	Computed Value of b.
A	.44	.0214[1]	E	.144	.0240
B	.323	.0191	F	.106	.0263
C	.251	.0177	G	.080	.0299
D	.192	.0211			

Hysteresis Effects.—In the case of solids which phosphoresce it has been shown by Nichols and Merritt that there is an effect produced on the solid when it is made phosphorescent which does not disappear when the phosphorescence ceases and which affects the subsequent phosphorescence in a marked manner. The solid may, however, be made to return to its original condition by allowing the infra-red rays from some source such as an arc light to fall on the solid. In the case of gas phosphorescence there appears to be a similar effect on the gas produced by the excitation.

The nature of this effect on the gas produced by the stimulating discharge has not been discovered. A number of experiments have been made to determine if the heating effect of the discharge on the glow affects the conclusions drawn from the curves in this paper, and it is apparently evident that it does not do so. It is sometimes necessary to pass a discharge again and again through a gas before it will phosphoresce regularly and brightly, hence all the measurements of the decay made and here described should be considered as dealing with a gas which is in the susceptible condition where like discharges will produce approximately equal degrees of phosphorescence, and it is partly on this account that hysteresis effects do not need to be considered as affecting the re-

[1]The value of b in all cases except in set A was computed from the formula $1/\sqrt{I} = a + bt$. In set A the value of a being unreliable b was determined graphically.

sults obtained. If it was possible to obtain decays of gas phosphorescence free from the after effects of previous discharges, a series of curves could be obtained which would no doubt be slightly different than those presented in this paper. On the other hand, it is extremely doubtful if these would show different general laws than those determined.

Summary of the Experimental Results.

In conclusion the results of the foregoing experiments on luminescent gases may be summarized as follows:

1. *Form of the Decay Curve.*—The decay of the phosphorescence of air and nitrogen is expressed by the relation

$$I = \frac{1}{(a + bt)^2}.$$

The curves obtained by plotting the values of $I^{-\frac{1}{2}}$ as ordinates and the corresponding values of t as abscissas is approximately a straight line for a considerable range of t. A small variation in b (the slant) has been observed however in many curves making the curve slightly convex towards the axis of t. In other curves the rate of decay seems to change at one point only, making two rates of the decay, the more rapid decay being for long-time phosphorescence. The effect is exactly opposite to that observed in phosphorescent solids.

This change in the rate in the decay curves of gases may be due to a change in the pressure of the gas due to its heating by the discharge. That this is at least one cause is indicated by a series of experiments made for the purpose of inquiring into this upward bending in the decay curves as t increases. Except in one series of decays at low gas pressures this change of b (the slant) has been so slight in most cases as to be readily overlooked. The relation may be considered therefore as approximately a linear one.

2. *Relation of Intensity to Gas Pressure.*—The phosphorescence of a gas when it is formed at various pressures increases very rapidly in intensity with increase of gas pressure. This is apparently true up to a point near the gas pressure, where the exciting discharge ceases to produce the glow. The relation between the *initial* intensity of the phosphorescence and the gas pressure is

$$I_0 = Cp^3.$$

In this formula I_0 is the initial intensity of the phosphorescence, or the intensity at zero time determined from the y intercepts of the straight line form of the decay curves, the only assumption being that a very decided change in the rate of decay does not occur for short-time phosphorescence.

3. *Change of Rate of Decay with Change of Gas Pressure.*—When the phosphorescence is formed at various pressures the rate of decay decreases as the pressure is reduced, and reaches a minimum where the glow is most persistent. From thence it increases to the lowest pressures. There is thus a pressure, which appears to be near where the electrodeless discharge passes most readily, at which the phosphorescence has the longest decay. A portion of the curve made by plotting the values of b as ordinates and the corresponding values p as abscissas closely resembles a parabola in form.

4. *Intensity and Duration of Excitation.*—In general the intensity of gas phosphorescence becomes greater with an increase of either the intensity or the duration of the exciting electrical discharge. This matter is being studied at present.

The results of this investigation make it apparent that the afterglow formed by the electrodeless ring discharge is a phenomenon which is of the same nature as the phosphorescence which occurs in solids.

The two types of phosphorescence have the same law of decay and exhibit many other characteristics that are common to both. That the rate of decay changes in some cases in gases to a slightly more rapid decay for long-time phosphorescence is no indication whatever of a different phenomenon than phosphorescence in solids. Indeed it has been shown by Nichols and Merritt[1] in their discussion of the general theory of the subject that according to the dissociation hypothesis suggested by Wiedemann and Schmidt and extended by the first mentioned authors that under certain circumstances and with simplified conditions the decay of phosphorescence should be more rapid as t increases.

A more rapid decay for long time-phosphorescence is what has been found in gases; although the effect is very slight. It is im-

[1]Nichols and Merritt, Phys. Rev., Vol. XXVII., No. 5, November, 1908, p. 389.

portant to take into consideration in this connection that phosphorescence in gases in comparison with that in solids is free from certain conditions which may tend to alter the form of the decay curve in solids such as absorption, slow diffusion and irregular distribution of the phosphorescent material. In the curves shown as examples, drawn from theoretical data, by Nichols and Merritt illustrating the special case referred to in the previous paragraph, it was necessary to neglect the effect of these three conditions.

A discussion of gas phosphorescence from the theoretical standpoint must be deferred until more facts are learned concerning the phenomenon, although sufficient experimental data have been gathered to show that the afterglow in a gas is a true phosphorescence, the study of which should prove of value in solving some of the problems of luminescence.

PHŒNIX PHYSICAL LABORATORY,
COLUMBIA UNIVERSITY, August, 1910.

EXPERIMENTS IN IMPACT EXCITATION.

By George W. Nasmyth.

III. The Frequency of the Lepel Oscillations.

§ 14. *Importance of the Variation in Frequency.*

THE frequency of the oscillations produced by the short-arc gen-
erators is of considerable practical importance on account of the
effect of variations in the frequency upon the sharpness of tuning
which can be obtained. The frequency of the oscillations produced
by the singing arc is not determined by the inductance and capacity
of the shunt circuit alone. With the Duddell arc the variations in
the pitch caused by a change in the arc current or arc length are
very prominent. In the Poulsen arc the variations in frequency
due to changes in the arc are so marked that the advantages of
sharp tuning which undamped oscillations ought to give cannot be
obtained in practice. On account of the general similarity of the
characteristics of the Lepel arc to those of the Poulsen arc, it might
be inferred that the frequency of the oscillations produced by the
Lepel arc would vary in the same way as the frequency of the singing
arc. In view of the importance of the question, however, it was
thought desirable that the problem be made the subject of an experi-
mental investigation.

§ 15. *The Frequency of the Duddell and Poulsen Oscillations.*

From theoretical considerations the author has derived[1] a formula
for the frequency of the singing arc agreeing closely with all the
experimental data which have been published on the Duddel and
Poulsen arcs. In its general form this formula is

$$n = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{(R + dV/dA)^2}{4L^2}}, \tag{1}$$

[1] Nasmyth, "The Frequency of the Singing Arc," Phys. Rev., *27*, No. 2, p. 117,
August, 1908.

where n is the frequency, L is the inductance and C the capacity and R the resistance of the oscillatory circuit, and dV/dA is the slope of the volts-amperes characteristic curve. If we take the value for the slope as given by the author's experiments at high frequencies,

$$\frac{dV}{dA} = -\frac{c + ld}{A},$$ (2)

where l is the arc length, A the arc current, and c and d are constants depending upon the electrodes and the atmosphere in which the arc is formed, the formula for the frequency becomes

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{\{R - (c + ld)/A\}^2}{4L^2}}.$$ (3)

If further, we neglect c and R, which are usually small in comparison with the term ld, we get the approximate formula,

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{l^2 d^2}{4L^2 A^2}}.$$ (4)

This formula shows that if the inductance is increased while the product LC is kept constant, the frequency will increase because the second term under the radical decreases. This effect was first observed experimentally by A. Banti.[1] If the arc current is increased the second term under the radical will decrease and the frequency should increase. Moreover, if the square of the frequency is plotted against the reciprocal of the square of the arc current a straight line with a slope $= -l^2 d^2/4L^2$ should result, and the author has shown that this is the case both in his own experiments and in those of L. W. Austin.[2] Austin's data plotted in this way shows that the formula holds not only for the fundamental but for the harmonics which are present in the singing arc oscillations as well. The author has shown that the frequency decreases with increasing arc length and that if the square of the frequency is plotted against the square of the arc length a straight line results. Finally, the formula (3) shows that if the resistance of the oscillation circuit is

[1] A. Banti, Ellettricista, *12*, p. 1, January, 1903. See also L'Elettricita, March 8, 1908, p. 145.

[2] W. Austin, Bulletin, Bureau of Standards, *3*, p. 325, May, 1907.

increased the second term under the radical will approach zero, and the frequency should increase. This prediction has been verified by K. Vollmer,[1] who found that a resistance of 3.3 ohms added to the oscillatory circuit raised the frequency from 350,000 to 359,000 when the arc length was 0.3 mm. and the arc current 1.3 amperes, and similar, though smaller, increases of frequency on the addition of resistance at larger currents and arc lengths. It may be considered, therefore, that the law of the variation of the frequency of the singing arc is fairly well known, and may serve as a guide in the investigation of the frequency of the short-arc, metal-electrode generators.

§ 16. *The Frequency of the Short Arc Oscillations.*

The object of the first experiments on the frequency of the Lepel arc oscillations made by the author was to determine the dependence of the frequency on the arc current. A loosely coupled secondary with variable air condensers and a hot wire ammeter was used to determine the frequency, the connections being the same as in Fig. 1 in the section on the characteristics of short arcs.

Five Leyden jars having a total capacity of 0.0094 mfd. were used in the primary. Each of the three variable air condensers in the secondary had a maximum capacity of 0.00055 mfd., and differences of capacity of 0.000003 mfd. could be determined accurately by means of circular scales mounted on each of the air condensers and calibrated with considerable care by comparison with standard forms of capacity. The resistance of the primary circuit as measured with a Wheatstone bridge, was less than 0.10 ohm, and since stranded wires were used throughout, it is probable that the high frequency resistance did not greatly exceed this value.

The inductance of the primary circuit consisted of a solenoid of 15 turns, 20.55 cm. in length, and 12.85 cm. mean diameter. The stranded conductor was 0.65 cm. in diameter, and consisted of 36 strands each 0.6 mm. in diameter. The coefficient of self induction of this solenoid, as computed by Russell's formula,

$$L = (\pi dn)^2 l[1 - 0.424(d/l) + 0.125(d/l)^2 - 0.0156(d/l)^4],$$

[1] K. Vollmer, Jahrbuch der Drahtlosen Telegraphie und Telephonie, *3*, p. 143 (table 11), December, 1909.

where L is the inductance, d the diameter and l the length, in cm., and n is the number of turns per cm., was 14,100 cm. The value given by comparison with an Ayrton Perry standard at a frequency of 1,000 cycles was 14,400 cm. The small difference is probably due chiefly to the connecting wires at the end of the solenoid. Since these wires were used in the actual connections, the larger value given by direct measurement is probably the more correct. All the remaining connections in the primary were made so as to give a minimum of inductance, twisted pairs of wires being used wherever possible, and it is probable that the self inductance of the entire primary circuit was within one or two per cent. of 14,400 cm.

The inductance in the secondary circuit was a solenoid of 75 turns, 27.38 cm. long, and 8.87 cm. mean diameter. The computed coefficient of self induction was 140,200, and the measured value slightly greater than this, giving a mean of about 141,000 cm.

The primary and secondary circuits were coupled by arranging these two solenoids coaxially, with a distance of 7.2 cm. between their adjacent ends. In response to an inquiry concerning the best formula to use for the coefficient of mutual induction of the two solenoids, Dr. E. B. Rosa, of the Bureau of Standards, very kindly furnished a complete calculation of the coefficient by two formulas. According to an adaptation of the general formula given by Gray the mutual inductance was found to be 1,089.8 cm., while the formula of quadratures by Rayleigh gave 1,089.3 cm. The coefficient of mutual induction may be considered to be very closely $M = 1{,}090$ cm. therefore. Accordingly, the coupling coefficient is $k = M/\sqrt{L_1 L_2}$ $= 1090/\sqrt{14{,}400 \times 141{,}000} = 0.0242$ or 2.42 per cent.

§ 17. *Variation with Arc Current.*

With the arc current maintained at a constant value, it was found that resonance could be obtained at a number of different frequencies by varying the capacity in the loosely coupled secondary circuit. It very soon became evident that the frequencies at which resonance could be obtained varied with the arc current. The results of the first experiments on the frequency are given later, in Fig. 17. As some of the resonance frequencies were close together, considerable difficulty was experienced in trying to follow the varia-

tions of a single one as the current was changed, and it was finally decided to make a complete exploration of the field. The results of the more exhaustive experiments at arc lengths of 0.10 and 0.15 mm. are given in Tables IV. and V., and graphically in Fig. 14.

TABLE IV.

Change of Frequency with Arc Current. (Fig. 14.)

Capacity. in M.F.	Fre-quency.	Curve 1. Funda-mental.		Curve 2a.		Curve 2.		Curve 2b.		Curve 3a.		Curve 3b.	
		D.C.	H.W.	D.C.	H.W.	D.C.	H.W.	D.C.	W.	D.C.	H.W.	D.C.	H.W.
.000545	565,000			2.81	.39	1.85	.39	1.42	.33	1.10	.28	.80	.25
.000697	500,000			2.10	.31	1.54	.44	1.22	.44			Curve 3.	
.000822	460,000			1.75	.36	1.40	.34	1.10	.33	.78	.41	.60	.41
.001091	400,000			1.75	.31	1.20	.40	.96	.41				
.001365	357,000			1.44	.30	1.06	.29	.87	.27				
.001635	327,000	2.59	.53	1.19	.23							Curve 1a.	
.001880	305,000	2.09	.29			.98	.14	.82	.12			2.58	.3
.002153	284,000	1.78	.22			.90	.12						
.002425	268,000	1.67	.15			.86	.10					Curve 1a.	
.00270	254,000	1.47	.15			.80	.07					1.85	.08
00297	242,000	1.30	.07										
.00325	232,000	1.23	.07										

D.C. indicates current through the arc, measured by a d.c. ammeter.

H.W. indicates intensity of oscillations, measured by a hot wire ammeter.

Arc. length, 0.10 millimeters, two sheets W. S. & B. Paragon paper.

Computed frequency from formula $n = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{L_1 C_1}} = 430{,}000.$

Primary capacity 0.0094 mfd. Inductance 14,400 cm.   (October 29, 1909).

It is evident that the frequency of the oscillations produced by the Lepel arc increases with the arc current, as in the singing arc. From the figure, the curves seem to occur in groups of three, and from the tables it is seen that the middle curves in each group, marked respectively 1, 2 and 3, represent the largest amounts of energy for a given current, the companion curves above and below representing comparatively weak oscillations. Curve 1 is apparently the fundamental, and at large currents of three amperes or more, this is the only resonance frequency that appears. It ap-

TABLE V.

Change of Frequency with Arc Current. (*Fig. 14.*)

Capacity in M.F.	Frequency	Curve 1. Funda-mental.		Curve 2a.		Curve 2.		Curve 2b.		Curve 3a.		Curve 3.	
		D.C.	H.W.	D.C.	H.W.	D.C.	H.W.	D.C.	H.W.	D.C.	H.W.	D.C.	H.W.
.000545	565,000	(Fig. 16)		2.70	.44	1.85	.33	1.42	.34	1.15	.33	.95	.33
.000697	500,000			2.36	.33	1.57	.42	1.26	.44	.85	.41	.70	.41
.000822	460,000			2.10	.44	1.45	.47	1.11	.48	.80	.50	.60	.44
.001091	400,000			1.76	.46	1.23	.41	.96	.44	.70	.42	Curve 3b.	
.001242	375,000			1.16	.42	1.18	.42					500,000	
.001365	357,000	3.47	.51	1.52	.38	1.06	.31					.60	.42
.001503	340,000	3.03	.50	1.43	.29								
.001635	327,000	2.58	.47	1.35	.23			.78	.17				
.001880	305,000	2.08	.30			1.05	.14			Curve 1a.		Curve 1b.	
.002153	284,000					.95	.14			2.43	.12	1.46	.21
.002425	268,000	1.76	.09			.89	.13			2.16	.13	1.34	.14
.00297	242,000	1.46	.07							1.85	.09		
.00352	223,000									1.63	.08		

D.C is current through the arc, measured by a d.c. ammeter.

H.W. is intensity of oscillations, measured by hot wire ammeter.

Arc length 0.15 millimeter, three sheets W. S. & B. Paragon paper.

Computed frequency from formula $n = \dfrac{1}{2\pi} \sqrt{\dfrac{1}{L_1 C_1}} = 430{,}000$.

Primary capacity = 0.0094 mfd. Inductance 14,400 cm. (November 11, 1909.)

proaches asymptotically the frequency given by the Thomson formula $n = I/2\pi\sqrt{L_1 C_1}$ as the arc current is increased. The curve 2 is evidently the first harmonic; at 2 amperes, for example, when curve 1 indicates a frequency of 300,000 for the fundamental, the frequency for curve 2 is very closely twice as great, 600,000. The range is not quite large enough to make sure that curve 3 is the next harmonic, but this is probably the case. At one ampere, for example, curve 2 indicates a frequency of about 300,000 and curve 3 about 600,000.

To avoid the necessity for the computation of a large number of frequencies corresponding to a varying capacity in the secondary, the capacity was held constant at the different values and the arc current varied by means of the resistance in the main circuit until

resonance was obtained. At higher frequencies than about 565,000 the oscillations in the secondary became so weak, on account of the small capacity, that accurate settings could not be obtained. The observations for the curves at 0.10 mm. were made before the
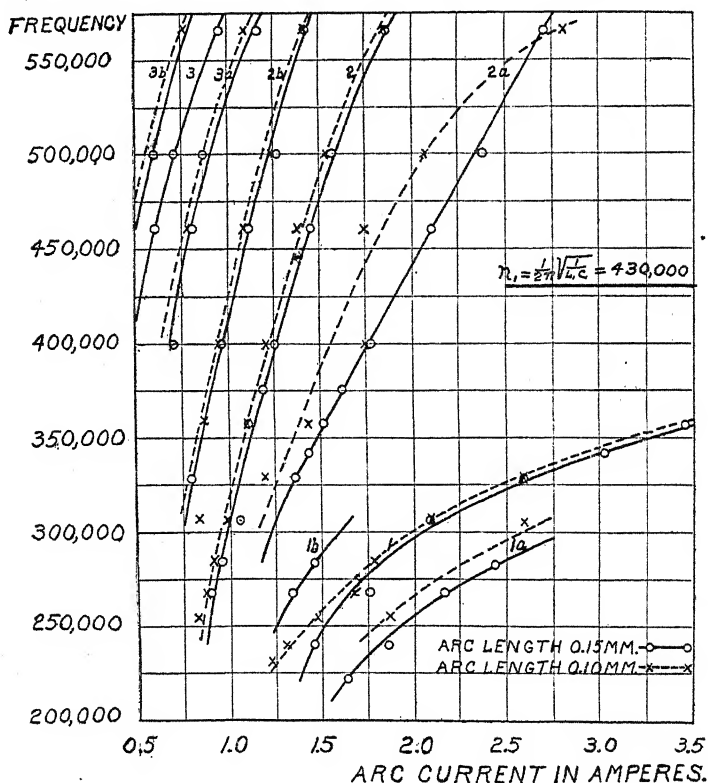
Fig. 14.

Change of frequency with arc current.

experiments at the longer arc length were undertaken, and the curves 1*b* and 3, which had been overlooked in the first observations were discovered in the more accurate experiments at 0.15 mm.

Influence of the Oscillatory Current.—The interpretation of the apparent discrepancies in the two sets of curves is interesting. The reason for the flatness of the curve 2*a* at 0.15 mm. in the region from about 1.5 to 2.5 amperes is clear from Fig. 7 in the section on the characteristics. Fig. 7 shows that the region of maximum voltage

at 0.15 mm. is between 1.5 and 2.5 amperes, and Fig. 8 shows that the region of minimum oscillations corresponds with the region of maximum voltage.   The currents plotted in Fig. 14 were measured by a d.c. ammeter in the main circuit, whereas the actual current which determines the resistance and characteristics of the arc includes not only this current but the oscillatory current as well. The effective current traversing the arc is accordingly less, relatively in the region where the full line curve 2a is flat, and if total currents through the arc were plotted instead of the d.c. current, the flat part of the curve would bend over into its proper place in the region of smaller currents.   The apparent shift of the curve 1a, and of most of the points in curve 1, in the same region, is explained in the same way.   At 0.10 mm. the region of high voltages and low oscillation currents moves to higher currents as shown in Fig. 7, and the effect of this is apparent in the falling off of the dotted curve 2a at 2.81 amperes d.c.   No readings at higher currents than 2.59 amperes were obtained at 0.10 mm. and the dotted curve has been drawn as if it lay above the full line curve all the way out, but the two curves probably cross at larger currents, on account of the influence of the minimum in the oscillatory current.   At small currents no irregularities are observed, as no maxima of voltages or minima of oscillations occur in this region.

If the effective currents through the arc were plotted instead of the readings of the d.c. ammeter, the curves at low currents would be affected more than the others, since the oscillatory currents are large even when the d.c. ammeter readings are small.   For the same values of effective current, the frequencies indicated by all the curves would be less, but the frequencies at small d.c. readings would be reduced more than the others, and the curves would be steeper.

Influence of Mutual Inductance.—Although the coupling between the primary and the secondary resonance circuit was very loose, the coefficient being only 0.0242 or 2.42 per cent., it was thought possible that the two companion curves above and below the curves 1, 2 and 3 might be accounted for as the two coupled circuit waves which appear even in impact excitation.   It is difficult to see how indications of the two coupled waves could be obtained without the

aid of a loosely coupled tertiary circuit, and moreover the largest effect of the loose coupling employed turns out to be far too small to account for the phenomena. At resonance the frequencies[1] of the two waves of the coupled circuit should be

$$T = \sqrt{T_0^2 \pm \theta^2}; \quad (\theta^2 = 4\pi^2 M \sqrt{C_1 C_2})$$

where T_0 is the period of each circuit before coupling, M is the coefficient of mutual induction between the two circuits, and C_1 and C_2 are the capacities in the primary and secondary circuits respectively. Using the actual capacity, 0.0094 mfd. in the primary and the theoretical capacity from the Thomson formula, 0.00092 mfd. in the secondary, we get for the two frequencies 425,000 and 435,000 respectively. This gives a difference of only 1.16 per cent. instead of the difference of more than 10 per cent. required to account for the observations. Using the actual values of the secondary capacity, for a frequency of 350,000 the coupled circuit frequencies would be 347,800 and 352,200, giving a still smaller difference of less than three fourths of one per cent. The phenomenon seems to be therefore a characteristic of the generator itself rather than of the coupling. The presence of the large number of harmonics at which resonance can be obtained with the Lepel arc marks another point of resemblance between it and the singing arc. Austin[2] found three frequencies at which resonance could be obtained, all of which increased with the arc current. In the author's experiments on the frequency of the singing arc formed between a copper and a carbon electrode in illuminating gas six such frequencies were found by means of a loosely coupled resonance circuit, with indications of others at capacities too small to measure accurately.

§ 18. *Variation with Arc Length.*

The full line and dotted line curves for arc lengths of 0.15 and 0.10 mm. given in Fig. 14 show that the frequency decreases with increasing arc length, as in the case of the singing arc and in agreement with the formula (3) or (4). On attempting to follow the changes in the curve 1 as the arc length was increased apparent

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[1] See "The Principles of Electric Wave Telegraphy," J. A. Fleming.
[2] Austin, *loc. cit.* See also PHYS. REV., 30, p. 134, August, 1908.

inconsistencies were again met with, the curves falling below each other for a time, then rising to higher frequencies as the arc length was increased beyond a critical value and finally falling again.   The attempt to trace the course of the whole curve I was finally abandoned, and the variation of the frequency of the fundamental with

TABLE VI.

Fig. 15.   Variation of Frequency with Arc Length.

Sheets Paper.	2	3	4	6	8	10	12	14
Arc length, mm.	0.10	0.15	0.20	0.30	0.40	0.50	0.60	0.70
Arc length squared	0.01	0.023	0.04	0.09	0.16	0.25	0.36	0.49
Fundamental at 2.0 amperes. C_1=0.0094 mfd.								
Frequency $\div 10^3$	303	299	315	305	295	288	275	—
(Frequency)$^2 \div 10^{10}$	9.18	8.94	9.92	9.40	8.70	8.29	7.56	—
Harmonic at 1.0 ampere. C_1=0.00564 mfd.								
Frequency $\div 10^3$	1,095	—	999	885	910	850	715	662
(Frequency)$^2 \div 10^{10}$	119.9	—	99.8	78.4	82.8	72.3	51.1	43.8
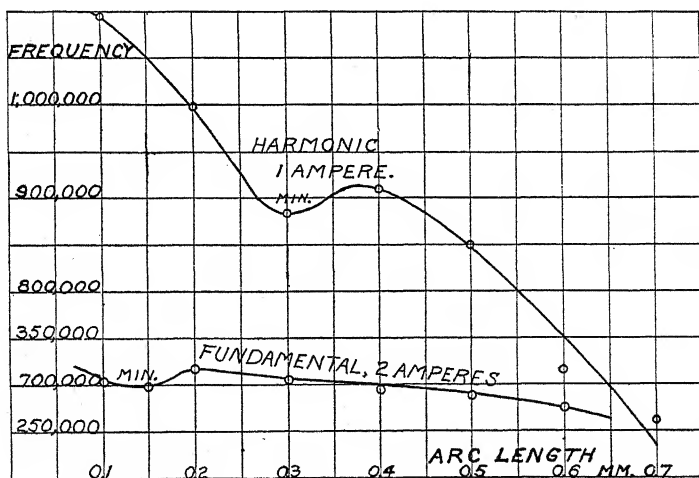
Fig. 15.
Variation of Frequency with Arc Length.

arc length at a constant current of 2 amperes was investigated. The results obtained are given in Table VI. and graphically in Fig. 15.   With a primary capacity of 0.00564 mfd. and the same primary

inductance of 14,400 cm., the variation of frequency of one of the harmonics at 1.0 ampere was also observed and the corresponding curve, which is of the same general form, has been plotted. The values of the squares of the arc lengths and frequencies are also given for use later in § 19.

Influence of the Oscillatory Current.—In both the curves of Fig. 15 the frequency decreases with increasing arc length except for regions of strongly marked minima in each case. On referring to
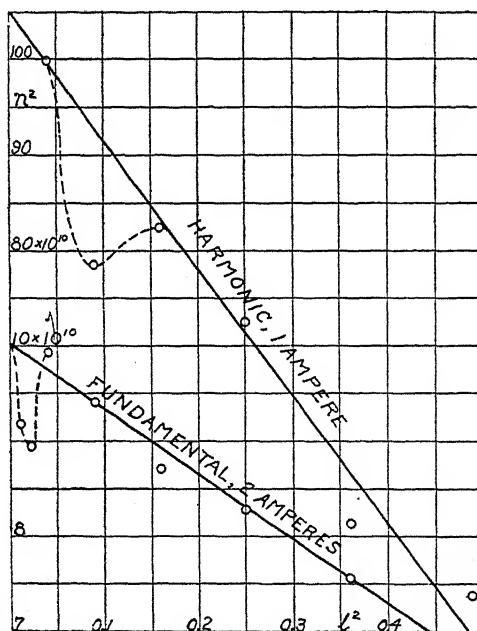
Fig. 16.

Squares of Frequency and Arc Length.

Physical Equation of Straight Lines, $n^2 = -\dfrac{d^2}{16\pi^2 L^2 A^2} \cdot l^2 + \dfrac{1}{4\pi^2 LC}.$

Fig. 7 again, it is found that these minima correspond to regions of voltage maxima and oscillatory current minima, so that the reason for the apparent discrepancies are traced as before, to a decrease in the effective current through the arc. Thus, the minimum in the oscillations and in the frequency for the 2 amperes d.c. curve both come at 0.15 mm. For the harmonic a smaller capacity and a

smaller current were used than for the curves in Fig. 7, and the minimum should be shifted into the region of longer arc lengths or smaller capacities, as it is.   If this allowance is made, we may say that the minima in the oscillations and the frequency both come at about 0.3 mm.   A second minimum, less pronounced, seems to be indicated at about 0.6 mm. in the curve for the frequency, and it is possible that the corresponding voltage maximum lies just outside the range of Fig. 7.

§ 19. *Proof of the Formula for the Frequency.*

It is apparent from the above curves that the frequency of the Lepel arc agrees qualitatively with the formula which has been found to hold true for the singing arc.   With the exception of the

TABLE VII.

Figs. 17 and 18.   D.C. Arc.   Frequency and Arc Current.

Curve 1.

Current$=I$.	$1/I^2$.	Frequency $=n$.	n^2	
3.47	0.083	357,000	12.74×10^{10}	Copper electrodes.
3.03	0.109	340,000	11.56	Arc length $= 0.15$ mm.
2.58	0.150	327,000	10.69	$L_1 = 14.400$ cm.
2.08	0.230	305,000	9.30	$C_1 = 0.0094$ mfd.
1.76	0.320	268,000	7.18	$N_0 = 430,000$.
1.46	0.460	242,000	5.86	
		Curve 2.		
1.18	0.720	322,000	10.36×10^{10}	
1.29	0.600	332,000	11.02	
1.38	0.530	335,000	11.22	
1.43	0.490	337,000	11.36	
1.48	0.456	342,000	11.70	
1.52	0.430	347,000	12.04	
1.55	0.417	352,000	12.39	Copper$+$, brass $-$, electrodes
1.595	0.393	357,000	12.76	Arc length $= 0.35$ mm.
1.66	0.362	362,000	13.10	$L_1 = 14,400$ cm.
1.70	0.346	369,000	13.60	$C_1 = 0.00564$ mfd.
1.79	0.312	366,000	13.40	$M = 1089$ cm.
1.83	0.299	370,000	13.70	
1.87	0.286	379,000	14.30	
1.19	0.705	329,000	10.82	
1.08	0.858	315,000	9.91	
.98	1.040	298,000	8.90	
.875	1.310	275,000	7.58	

Curve 3.

2.34	0.183	711,000	50.7×10^{10}	
2.19	0.209	694,000	48.1	
2.06	0.235	678,000	46.0	Copper$+$, brass$-$, electrodes.
1.93	0.269	668,000	44.6	Arc length 0.15 mm.
1.79	0.312	650,000	42.3	$L_1 = 14{,}400$ cm.
1.61	0.389	625,000	39.1	$\qquad \begin{cases} 0.00376 \\ \end{cases}$
1.40	0.510	571,000	32.6	$C_1 = \begin{cases} \\ 0.00564 \end{cases}$
1.30	0.590	534,000	28.5	

Fig. 17.

Frequency and arc current.

irregularities due to the oscillation minima the curves are of the same form as the corresponding singing arc curves. A more rigorous test of the formula is obtained by plotting the square of the frequency against the square of the arc length, as in Fig. 16. Both these curves prove to be straight lines outside the regions of the above mentioned irregularities due to a decrease in the intensity of the oscillatory current through the arc. If the effective current through the arc A, as measured by a hot wire ammeter, had been maintained constant instead of the direct current I, the agreement

of the curves with the straight line requirement of the formula would have been as satisfactory as in the case of the Poulsen arc.

Additional data for the variation of frequency with the arc current are given in Table VII., and the curves corresponding are plotted in Fig. 17. The data were obtained with the same connections and coupling as for the curves in Fig. 14. The effect of the minima in the oscillations is plainly visible in curve 2, and in the corresponding straight line in Fig. 18. The data for curve 2 are

Fig. 18.

Squares of frequency and current reciprocals.

Physical equation of straight lines, $n^2 = -\dfrac{l^2 d^2}{16\pi^2 L^2} \cdot \dfrac{1}{A^2} + \dfrac{1}{4\pi^2 LC}$.
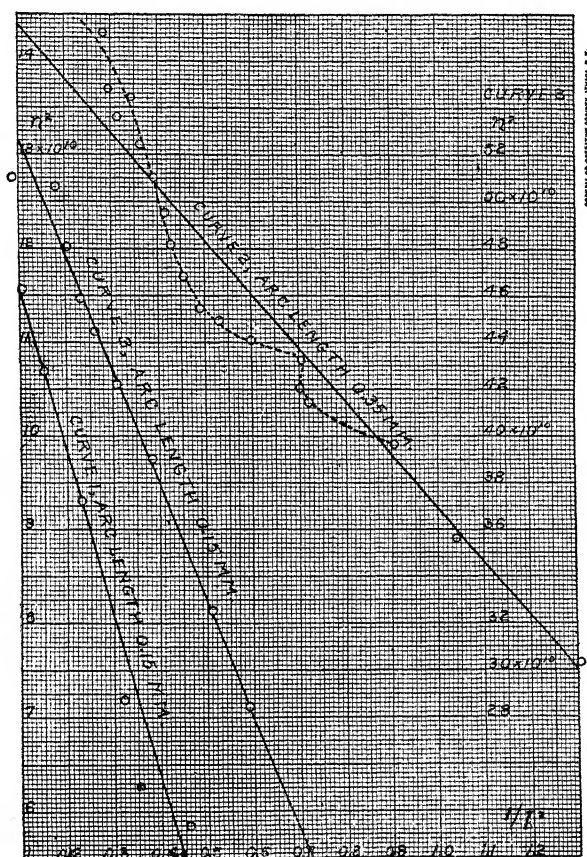
given in Table VII. in the same order in which they were obtained, and it is apparent that the points taken near the end of the rim form a much smoother curve than the earlier ones. It is possible that in this case there was some irregularity in the arc which caused the minima in the oscillations, in addition to the causes which have been traced in the preceding sections on the intensity of the oscillations.

With the exception of the discrepancy caused by using the arc current as measured by a d.c. ammeter instead of the effective current in curve 2, the agreement with the straight line requirement of the formula is again well within the limits of errors of observation.

§ 20. *Related Experiments on Poulsen Oscillations.*

The experiments of the author have shown that the Lepel arc is closely related to the singing arc, especially in its frequency characteristics. Since it appears that the formula derived by the author for the singing arc holds also for the short arc between metal electrodes, experiments on the singing arc which afford additional tests for this formula are of interest to investigators working with the new impact generators.

Some excellent experimental data recently obtained by K. Vollmer[1] working under the direction of Professor Max Wien at Danzig, afford material for a severe quantitative test of the formula as applied to Poulsen oscillations. Vollmer used flat discs of copper and carbon as his electrodes, and formed the arc in an atmosphere of hydrogen. His experiments cover a wide range of frequencies, from 156,000 to 1,000,000 per second corresponding to wave-lengths of from 1,915 to 300 meters. The inductances used range from 33,300 cm. to 1,358,000 cm. and the capacities from 0.00076 mfd. to 0.00585 mfd.

Vollmer measured his arc lengths directly, and did not take account of the variations in the length as indicated by the changes in voltage. Nevertheless the two curves[2] which he plotted to verify the author's formula for the frequency give a good straight line between n^2 and l^2 as formula (4) requires when l is large enough so

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[1] Vollmer, Jahrbuch der Drahtlosen Telegraphie und Telegraphonie, *3*, pp. 117-173.
[2] Vollmer, *loc. cit.*, p. 146.

VARIATION OF FREQUENCY WITH ARC LENGTH; POULSEN ARC.

TABLE 2V.

$n_0 = 156,600$; $L_1 = 1,358,000$ cm.; $C_1 = 0.76 \times 10^{-3}$ mf.; $\lambda_0 = 1915$ meters.
(I_g = generator or arc current as measured by a d.c. ammeter; I_c = oscillatory current.)
$I_g = 2.7$ amp. $I_c = 2.0$ amp.

l	l^2	n^2	
1.2 mm.	1.44	2.433×10^{10}	$m = 11 \times 10^6$
1.8	3.24	2.4313	$\sqrt{m} = 3310$
2.4	5.76	2.4298	$d = 154$
2.7	7.29	2.4267	
3.3	10.89	2.4250	$d\sqrt{C_1} = 4.25$
3.6	12.96	2.4212	

TABLE 3V.

$n_0 = 156,600$; $L_1 = 532,000$ cm.; $C_1 = 1.94 \times 10^{-3}$ mf; $I_g = 2.7$ amperes; $I_c = 2.01$ amp.

l	l^2	n^2	
0.3 mm.	0.09	2.4414×10^{10}	
0.6	0.36	2.4388	
0.9	0.81	2.4363	$m = 27 \times 10^6$
1.2	1.44	2.4348	$\sqrt{m} = 5200$
1.8	3.24	2.4280	$d = 94$
2.1	4.41	2.4267	
2.4	5.76	2.4230	$d\sqrt{C_1} = 4.14$
2.7	7.29	2.4200	
3.0	9.00	2.4154	
3.3	10.89	2.4154	

TABLE 4V.

$n_0 = 156,600$; $L_1 = 176,500$ cm.; $C_1 = 5.85 \times 10^{-3}$ mf; $I_g = 2.7$ amp.; $I_c = 2.1$ amp.

l	l^2	n^2	
0.3 mm.	0.09	2.434×10^{10}	$m = 92 \times 10^6$
0.6	0.36	2.421	$\sqrt{m} = 9,600$ (9,600)
0.9	0.81	2.419	$d = 57.5$
1.2	1.44	2.416	$d\sqrt{C_1} = 4.40$
1.5	2.25	2.396	
1.8	3.24	2.3815	
2.1	4.41	2.374	
2.4	5.76	2.357	
3.0	9.00	2.343	
3.6	12.96	2.326	
3.9	15.21	2.282	

TABLE 5V.

$n_0 = 375,000$; $L_1 = 237,000$ *cm.*; $C_1 = 0.76 \times 10^{-3}$ *mf.*; $\lambda_0 = 800$ *meters*; $I_g = 2.7$ *amp.*;
$I_c^r = 2.15$ *amp.*

l	l^2	n^2	
1.1	1.21	13.684×10^{10}	$m = 5 \times 10^8$
1.2	1.44	13.65	$\sqrt{m} = 2.24 \times 10^4$
1.35	1.82	13.624	$d = 180$
1.5	2.25	13.583	$d\sqrt{C_1} = 4.96$
1.8	3.24	13.55	

TABLE 6V.

$n_0 = 375,000$; $L_1 = 92,900$ *cm.*; $C_1 = 1.94 \times 10^{-3}$ *mf.*; $I_g = 2.7$ *amp.*; $I_c = 2.15$ *amp.*

l	l^2	n^2	
0.3 mm.	0.09	13.86×10^{10}	
0.45	0.203	13.80	$m = 10 \times 10^8$
0.6	0.36	13.73	$\sqrt{m} = 3.16 \times 10^4$
0.9	0.81	13.68	$d = 100$
1.2	1.44	13.63	
1.5	2.25	13.51	$d\sqrt{C_1} = 4.40$
1.8	3.24	13.46	
2.1	4.41	13.35	
2.4	5.78	13.13	

TABLE 7V.

$n_0 = 375,000$; $L_1 = 30,800$ *cm.*; $C_1 = 5.85 \times 10^{-3}$ *mf.*; $I_g = 2.7$ *amp.*; $I_c = 2.25$ *amp.*

l	l^2	n^2	
0.15 mm.	0.0225	13.8×10^{10}	
0.3	0.09	13.4	$m = 33 \times 10^8$
0.6	0.36	13.1	$\sqrt{m} = 5.75 \times 10^4$
0.75	0.5625	12.8	$d = 60$
0.9	0.81	12.6	
1.2	1.44	12.32	$d\sqrt{C_1} = 4.59$
1.5	2.25	12.1	
1.8	3.24	11.7	
2.1	4.41	11.3	
2.4	5.76	11.0	

that the constant c may be neglected. The Thomson frequencies
chosen for the two tests were 156,600 with a capacity of 0.00585
mfd. and an inductance of 176,500 cm.; and 375,000, with a capacity
of 0.00076 mfd. and an inductance of 237,000 cm. Although the

TABLE 8V.

$n_0 = 1,000,000$; $L_1 = 33,300$ cm.; $C_1 = 0.76 \times 10^{-3}$ mf.; $\lambda_0 = 300$ meters; $I_g = 2.7$ amp.; $I_c = 1.7$ amp.

l	l^2	n^2	
1.2 mm.	1.44	92.2×10^{10}	$m = 248 \times 10^8$
1.4	1.96	91.0	$\sqrt{m} = 15.7 \times 10^4$
1.5	2.25	89.7	$d = 177$
1.7	2.89	88.7	$d\sqrt{C_1} = 4.89$

TABLE 9V.

$n_0 = 1,000,000$; $I_1 = 13,060$ cm.; $C_1 = 1.94 \times 10^{-3}$ mf.; $I_g = 2.7$ amp.; $I_c = 2.1$ amp.

l	l^2	n^2	
0.3 mm.	0.09	91.4×10^{10}	$m = 436 \times 10^8$
0.45	0.2025	90.2	$\sqrt{m} = 20.9 \times 10^4$
0.9	0.81	86.4	$d = 93$
1.2	1.44	84.2	
1.5	2.25	81.2	$d\sqrt{C_1} = 4.10$
1.7	2.89	78.8	

Recapitulation.

No.	L_1	C_1	I_c	$\sqrt{m}$	d	$d\sqrt{C_1}$
156,600	1,358,000	0.76×10^{-3}	2.0	0.33×10^4	154	4.25
156,600	532,000	1.94	2.01	0.52	94	4.14
156,600	176,500	5.85	2.1	0.96	57.5	4.40
375,000	237,000	0.76	2.15	2.24×10^4	180	4.96
375,000	92,900	1.94	2.15	3.16	100	4.40
375,000	30,800	5.75	2.25	5.75	60	4.59
1,000,000	33,300	0.76	1.7	15.7×10^4	177	4.89
1,000,000	13,060	1.94	2.1	20.9	93	4.10

$I_g = 2.70$ amperes.   Mean 4.47

curves given are straight lines, as required by the formula, the value of the constant d as found from the slope is greater for the large capacity and low frequency curve than for the other. The author found that the product of the constant d and the square root of capacity gives a constant for these cases. Testing Vollmer's other remaining data in the same way, it was found that the product of $d\sqrt{C_1}$ gave a constant within the limits of the errors of observation, with a mean value of 4.47, in all his experiments. Vollmer's data

are not in convenient form for testing the formula, as he gives wavelengths instead of frequencies. The data are therefore given in the form for direct verification of the results in tables 2V. to 9V., corresponding to Vollmer's table numbers, 2 to 9.

It was predicted in the derivation of the formula[1] that the constant d would change with the Thomson frequency on account of the skin effect of resistance in the arc vapor. It would also change with the capacity in parallel with the arc on account of the effect of capacity on the arc voltage-current characteristic. It appears from the above results, however, that if we use for d the value $d = k/\sqrt{C_1}$, where $k = 4.47 \pm 0.4$, the formula may be extended from a single Thomson frequency (constant value of L_1 and C_1) for which it was at first derived, to include all of Vollmer's experiments from 156,600 to 1,000,000 frequency range and with all values of capacity and inductance used in the practice of radiotelegraphy and radio-telephony.

Using the above value of d, the approximate formula (4) becomes

$$n = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{l^2 k^2}{4L^2 C A^2}}, \qquad (5)$$

where n is the frequency, L and C the inductance and capacity, A the arc current, l the arc length, and k is a constant for given arc electrodes and atmosphere. The agreement of the generalized formula (5) with the results of experiments may be shown graphically if we plot $n^2 L^2 C$ against l^2. The equation of the straight line in the form $y = mx + b$ is

$$n^2 L^2 C = -\frac{k^2}{16\pi^2 A^2} l^2 + \frac{L}{4\pi}.$$

For a constant current A all the lines should have the same slope, $- k^2/16\pi^2 A^2$. The results from Vollmer's data are given in convenient form in Table X., and the curves at constant slope are plotted in Fig. 17.

The agreement is seen to be well within the limits of the errors of observation in Vollmer's work.

Vollmer measured his arc currents with a d.c. ammeter, whereas

[1] Phys. Rev., 27, p. 127, formula 13.

the author's experiments on the variation of the frequency of the Lepel arc with current and arc length have demonstrated that the effective current through the arc, including the oscillations, should be used as the arc current A in the formula. The oscillatory current in general increases with the capacity, so that the correction on this account which should be made in Vollmer's data is in the right direction. Whether the Thomson frequency and the oscillatory current corrections together are sufficient to account for the constancy of the product $d\sqrt{C_1}$ must be determined by further experiments in which the arc current is measured by a hot wire ammeter or some equivalent instrument. The relation is at any rate interesting, and enables us to extend the formula from a single frequency as determined by the capacity and inductance, to the general case.

Vollmer's data show also, as mentioned above, that the frequency increases with the resistance in the oscillation circuit. This is in agreement with the prediction of formula (3) and contradicts all

TABLE X.

(*Figure* 19.)   *Frequency and Arc Length; Poulsen Arc.   Values of* n^2L^2C.

j^2	Table 2V.	Table 3V.	Table 4V.	Table 5V.	Table 6V.	Table 7V.	Table 8V.	Table 9V.
.0225						.766		
.090		13.403	4.437		2.176	.744		.303
.203					2.167			.299
.360		13.389	4.413		2.156	.727		
.5625						.7104		
.81		13.375	4.410		2.148	.699		.286
1.21				5.841				
1.44	34.11	13.367	4.404	5.829	2.140	.684	.777	.279
1.82				5.817				
1.96							.767	
2.25			4.368	5.801	2.121	.672	.756	.269
2.89							.748	.261
3.24	34.08	13.330	4.341	5.786	2.113	.6494		
4.41		13.323	4.328		2.096	.627		
5.76	34.06	13.302	4.297		2.061	.611		
7.29	34.01	13.286						
9.00		13.261	4.271					
10.89	33.99	13.261						
12.96	33.94		4.240					
15.21			4.160					
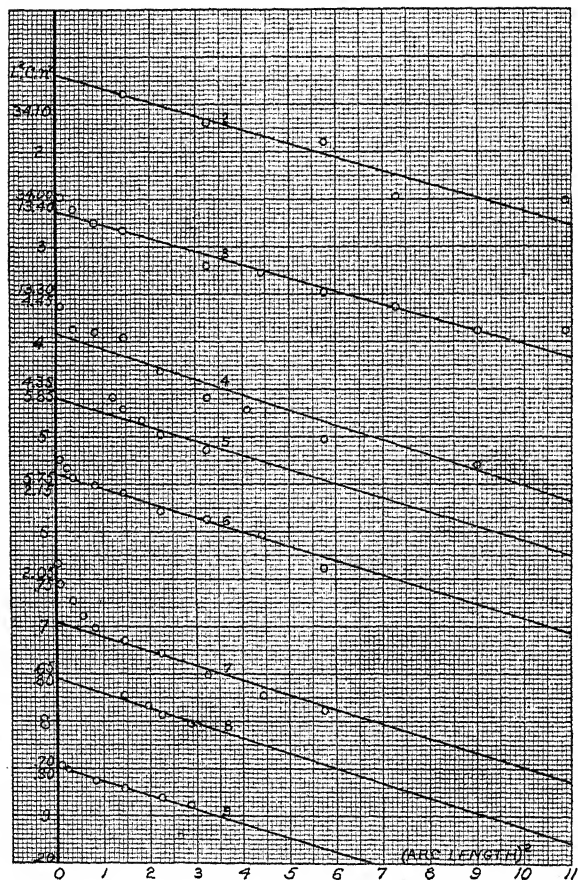
Fig. 19.

Frequency arc length curves with constant slope.

previous assumptions of investigators, based for the most part on reasoning by analogy from the effect of friction in mechanical pendulums and other systems.   There is reason to suspect that an increase of frequency with resistance will be found also in the ordinary spark gap excitation as well as in the short spark generators.

§ 21. *Impact Excitation and Forced Undamped Oscillations.*

The resonance curve for the antenna or secondary of the two closely coupled circuits is of interest on account of its bearing upon
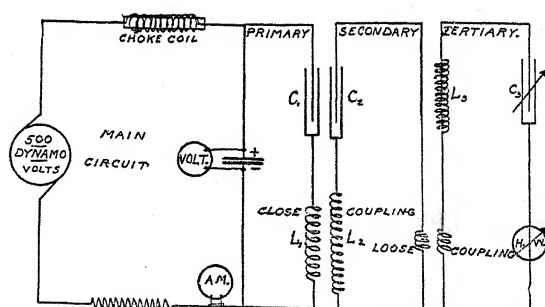
Fig. 20.

Diagram of connections for resonance curves.
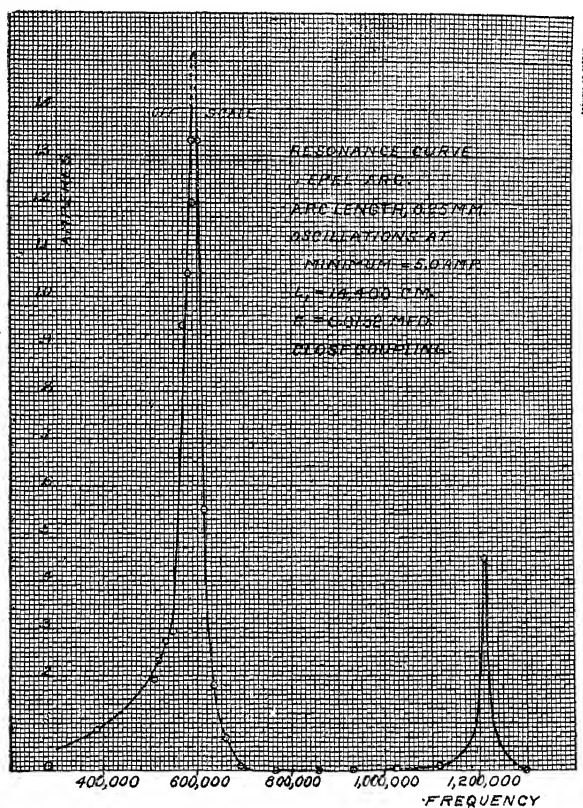
Fig. 21.

Resonance curve for Lepel arc.

the question of impact excitation. It may be determined by means of a tertiary circuit in addition to the connections used thus far in the experiments and shown in Fig. 1, loosely coupled to the

TABLE XI.

Resonance Curve for Lepel Arc.

Frequency.	Hot Wire Ammeter Reading.	
385,000	.26	Primary capacity, 7 ja s = 0.0132 mfd.
385,000	.40	Primary inductance, 14,400 cm.
399,000	.40	Arc length, 0.15 mm. Volts, 80.
415,000	.46	Arc current, 2.15 amperes d.c.
420,000	.48	Oscillations at minimum, 4.6 amperes.
440,000	.46	Secondary capacity, 2 jars in series = 0.00094 mfd.
450,000	.43	Secondary inductance, about 180,000 cm.
470,000	.48	
482,000	.52	
492,000	.625	
502,000	.725	Secondary and primary closely coupled. Secondary
506,000	.83	solenoid of 141,000 cm. inductance placed inside
519,000	.94	primary solenoid of 14,100 cm. inductance with
520,000	1.166	lower ends flush. See § 16.
525,000	1.37	
Maximum, off scale.		
567,000	1.37	Tertiary capacity variable.
579,000	.92	Tertiary inductance 160,000.
594,000	.79	Tertiary and secondary loosely coupled.
596,000	.68	
615,000	.40	
660,000	.26	For the resonance curve of Figure 21 the primary and
702,000	.185	secondary capacities and inductance were the same,
765,000	.15	but the arc length was 0.25 mm., and the arc current
980,000	.28	3.03 amperes (volts 66) giving a minimum of 5.0
1,060,000	.51	amperes in the oscillations.
Maximum.		
First harmonic.		
1,000,000	.29	
1,150,000	.20	
1,220,000	.10	

secondary. The complete diagram of connections for determining the form of the wave in the antenna or secondary is given in Fig. 20. The primary and secondary circuits are maintained constant, and

the readings of the hot wire ammeter are read as the capacity in the loosely coupled tertiary circuit is varied.

With the arc current arranged so as to give a minimum in the intensity of the oscillations at an arc length of 0.25 mm. the resonance curve shown in Fig. 21 was obtained.

The primary and secondary circuits were not tuned to the same frequency, the lowering of the frequency of the oscillations by the arc itself increasing the so-called "untuning." The arc current was 3.03 amperes d.c., and the voltage 66. The remaining constants are given in connection with Table XI., which gives the data for a similar resonance curve (not plotted) at an arc length of 0.15 mm. and with the oscillations again at a minimum.

Both the resonance curves show but a single wave, the smaller maximum being evidently the harmonic of double frequency. So far they are in agreement with the theory of impact excitation, the steep curves and sharp peak being especially characteristic. On the other hand, the frequency of the oscillations should remain constant, as long as the secondary circuit is not changed, whereas the frequency of the oscillations changes with the constants of the primary circuit, according to the above figure and table. Further experiments gave the following increase of frequency of the oscillations in the secondary circuit with increasing primary arc current.

TABLE XII.

Arc Current Amperes, D.C.	Arc Volts, D.C.	Oscillation Amperes.	Resonance Frequency.
1.30	90	3.25	500,000
1.50	88	3.40	507,000
1.75	84	3.45	519,000
2.00	90	4.27	528,000
2.00	80	3.68	545,000
2.00	70	3.68	552,000
3.00	66	4.84	563,000

The conclusion is that in these experiments at least, the Lepel arc did not give free impact excitation, but *forced oscillations* which the form of the resonance curve shows to be almost *undamped*

at arc currents which give a minimum in the intensity of the oscillations.

<div align="center">SUMMARY.</div>

The Lepel generator resembles the older Duddell and Poulsen arc generators in having a number of harmonics, besides the fundamental, to which a loosely coupled resonance circuit may be tuned.

The following laws hold for both fundamental and harmonics, as in the case of the older generators:

1. The frequency of the oscillations increases with increasing arc current at constant arc length.

2. The frequency of the oscillations increases with decreasing arc length at constant arc current.

The variation in the frequency is least with a large primary inductance.

It is predicted, though not yet experimentally verified, that a small increase in the resistance of the oscillatory circuit will increase the frequency of the Lepel arc oscillations as in the case of the Duddell and Poulsen arcs.

The formula derived by the author in 1908 for the frequency of the singing arc, which includes all the known facts and is in agreement with the results of all the experiments on the frequency of the Duddell and Poulsen arc oscillations, holds also for the Lepel arc oscillations. The formula is

$$n = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{\{R - (c + ld)/A\}^2}{4L^2}},$$

where n is the frequency, L, C and R the inductance, capacity and resistance of the condensor circuit, l the arc length and A the arc current, and c and d are constants depending upon the electrodes and gas in which the arc is formed. Usually R and c are small compared with the other terms, so that the formula may be written in an approximate form,

$$n = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{l^2 d^2}{4L^2 A^2}}.$$

Recent extensive experiments on the Poulsen arc have shown that if the value of $d = k/\sqrt{C}$ where d is the arc constant and C

the primary capacity, k has the same value for a given arc for all the values of inductance, capacity and Thomson frequencies used in practice.   In its more general form the formula thus becomes

$$n = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{l^2 k^2}{4L^2 A^2 C}}.$$

The effect of capacity on the oscillation intensity and the form of the voltage current characteristics makes it very probable that the more generalized formula holds for the Lepel arc also, but sufficient data for a rigorous test are not yet available.

With a closely coupled secondary circuit and the arc current arranged to give minimum oscillations, a very sharp resonance curve with only a single wave is obtained by means of a loosely coupled tertiary circuit, as required by the theory of impact excitation. The maximum shifts with change of arc current, however, leading to the conclusion that the sharp resonance curve of the Lepel arc is due, not to free oscillations, but to *forced undamped* oscillations.

The author wishes to express his appreciation of the sympathetic aid and many valuable suggestions which he has received in the course of his researches from Dean Ernest Merritt, of the Graduate School of Cornell University.

THE HALL EFFECT AND SOME ALLIED EFFECTS IN ALLOYS.

By Alpheus W. Smith.

FROM a study of the data on the electrical conductivity of alloys together with the data from their thermal analysis, Guertler[1] has shown that the curve representing the relation between the electrical conductivity and the concentration of one of the components of a binary alloy will have a singular point, where the components unite to form a compound. It thus becomes possible from the behavior of the electrical conductivity to infer the presence of compounds in the alloy. In a subsequent paper by the same author,[2] it has been pointed out that similar conclusions may be drawn from the curves showing the relation between the temperature coefficient of the electrical resistance of the alloy and the concentration of one of its components. An examination of the thermoelectric properties of certain alloys by Becquerel[3] and others has indicated that the curve connecting the thermoelectric heights and the concentration of one of the components of the alloy shows a singular point, where a compound is formed. While the present investigation has been in progress a paper by Haken[4] has appeared in which a careful study has been made of the relation between the constitution, the thermoelectric heights and the electrical resistance of the following series of alloys: tellurium-antimony, tellurium-bismuth, tellurium-lead, antimony-silver and copper-phosphorus. He finds that wherever compounds appear in these systems, the curves connecting the thermoelectromotive with the concentration of one of the components have well-defined singular points. Similar conclusions are arrived at from the study of the electrical conductivity.

[1] Zeitschr. f. anorg. Chem., 51, p. 397, 1906.
[2] Zeitschr. f. anorg. Chem., 54, pp. 58–88, 1907.
[3] Ann. de Chem. et Phys. (4), 8, p. 408, 1866.
[4] Ann. der Phys. (4), 32, p. 291, 1910.

It seemed to the author that where the metals form compounds the Hall effect and the Nernst effect might be expected to behave in a way similar to that in which the thermoelectric height and the electrical conductivity had been found to behave. Such a result seemed the more to be expected from the fact, as Beattie[1] has pointed out, that there seems to be a sort of proportionality between the Hall effect and the thermoelectric heights.

Since in such an investigation so much depends on the purity of the metals from which the alloys are made, on the way in which the alloys have been allowed to cool and on their treatment after solidification, it seemed almost impossible to connect with certainty the data on the Hall effect and on the Nernst effect with the data which other observers had obtained on the thermoelectric heights, the electrical conductivity and the temperature coefficient of the resistance. Furthermore, the data on these quantities is for the most part meager. For the three series of alloys used in this investigation, viz: bismuth-antimony, antimony-cadmium and antimony-zinc, the author has been able only to find data on the thermoelectric heights of the three series and on the electrical conductivity of the bismuth-antimony series. In order, therefore, to make the points aimed at in this investigation, it seemed necessary to study besides the Hall effect and the Nernst effect, the thermoelectric heights, the electrical resistance and the temperature coefficient of the resistance. In the bismuth-antimony alloys the change of resistance in a magnetic field has also been studied.

Although the data obtained in the investigation cannot lay claim to a high degree of accuracy, the conclusions drawn from the data would not be changed by much greater errors than those which are present. Besides the errors of observation it is necessary to remember that there are errors due to slight impurities in the alloys, to small changes in the conditions under which they solidify and to slight differences in treatment after solidification. These sources of error may be of considerable consequence and since it is not possible to eliminate them, a high degree of accuracy is scarcely to be realized. The failure of different observers to obtain consistent values for the Hall effect and the Nernst effect in metals

[1]Roy. Soc. Edinb. Proc. (20), p. 481, and (21), p. 146.

which are the purest to be obtained, shows clearly the importance of the sources of error to which reference has just been made.

PREPARATION OF ALLOYS.

The bismuth, antimony, cadmium and zinc for the alloys were obtained from C. A. F. Kahlbaum. Ten or fifteen different alloys of each of the following series were prepared: bismuth-antimony, antimony-cadmium and antimony-zinc. The alloys were prepared synthetically and chemical analyses were not made after the preparation. After weighing, the metals were placed in closed hard glass tubes which contained such a small amount of air that the loss of either metal from oxidation would be small. These glass tubes were heated in an electric furnace until both of the metals were fused. When the metals had been sufficiently stirred to insure uniform mixing, they were allowed to cool as quickly as possible to room temperature. The alloys were then fused as quickly as possible and poured into a talc mould which was at room temperature. The mould was made up of two parts which when clamped together formed a rectangular receptacle about 3 cm. long, 1.5 cm. wide and 0.3 cm. deep. Care was taken that the molten alloys should be in contact with the air for as short a time as possible, so that unequal oxidation of the two components might not appreciably change the composition.

After the alloys had been cast in the form of plates the bismuth-antimony plates were ground on a rotating disc with carborundum powder for an abrasive, until they were about 0.1 cm. in thickness. Many of the antimony-cadmium and the antimony-zinc plates were so brittle that they could not be ground and were used nearly as they came from the moulds. They were about 0.3 cm. in thickness. The thickness of the plates was measured in a number of places and the mean of these measurements used in the subsequent calculations.

RESISTANCE.

It was desirable to measure the resistance and the temperature coefficient of the resistance in the specimens in which the Hall effect and the Nernst effect were to be determined. Since these specimens were in the form of rectangular plates of low resistance,

their specific resistances could not be determined easily. It was, however, sufficient for the purposes of the present investigation to know approximately the relative specific resistances of the different series of alloys. A simple method of getting the relative specific resistances of the different plates was by comparing the fall of potential over a given distance in one plate with the fall of potential over the same distance in the other plates, when the current density in all of the plates was the same. For this purpose lead wires were soldered to the ends of the plate (Fig. I). By means of these lead wires the plate was connected through a known resistance R to a battery
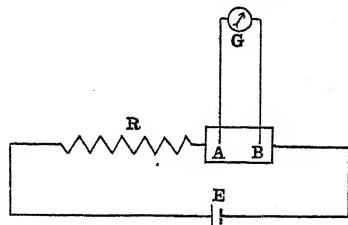
Fig. 1.

E of known electromotive force. At A and at B, points on the axis of the plate, was soldered a small copper wire which lead to the d'Arsonval galvanometer G. With the plate immersed in a mixture of ice and water, the deflection corresponding to a given current in the plate was noted. After reversing the current in the plate the deflection was again noted. The mean of four such observations was used in calculating the relative specific resistances of the plates. From the distance between A and B and the thickness and width of each plate, the deflection corresponding to the same current density in each plate was calculated, when the distance between A and B was 1.0 cm. These deflections were proportional to the specific resistances of the plates.

In order to obtain the temperature coefficient of the resistance, with a suitable current in the plate, the deflection of the galvanometer was noted, when the plate was immersed in a mixture of ice and water. The plate was then introduced into a steam bath at atmospheric pressure and when temperature equilibrium had been established, the deflection of the galvanometer was again noted. The temperature coefficient of the resistance of the plate is then given by the equation,

$$\alpha = \frac{d_{100} - d_0}{100 d_0},$$

where d_{100} is the deflection when the plate is in the steam-bath and

d_0 the deflection when the plate is in ice and water.   In the case
of the bismuth-antimony alloys, an oil-bath at room temperature
was used instead of the steam bath.   The temperature of the oil-
bath was determined by a thermometer graduated to 0.1° C.   The
temperature coefficient of these alloys is, therefore, the average
temperature coefficient between 0° and 22° C., while that of the
other alloys is the average temperature coefficient between 0° and
100° C.   The deflections of the galvanometer were nearly pro-
portional to the applied electromotive forces, so that no serious
error was introduced from this source.
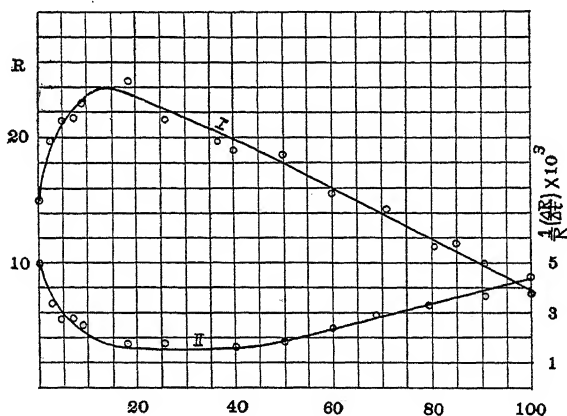
Fig. 2.

Bismuth-antimony alloys.   Curve I., electrical resistance.   Curve II., temperature
coefficient of resistance.

In Fig. 2 the results of the observations on the bismuth-antimony
alloys have been plotted.   Curve I. shows the relation between
the specific resistance at 0° C. and the percentage by weight of
antimony.   The ordinates are proportional to the specific resist-
ances and the abscissæ give the percentages by weight of antimony.
Curve II. shows the dependence of the average temperature coeffi-
cient of the resistance between 0° and 22° C. on the composition.
The ordinates represent the temperature coefficient of the resistance
and the abscissæ have the same meaning as they have in Curve I.
It will be observed that whereas the addition of a small amount of
antimony causes an increase in the resistance it causes a decrease

in the temperature coefficient of the resistance. When about 10 per cent. of antimony is present in the alloy, the resistance reaches a maximum but the temperature coefficient has a minimum. Further increase in the percentage of antimony causes a decrease in the resistance but an increase in the temperature coefficient. The general form of Curve I. is the same as that given by Haken,[1] when resistances instead of electrical conductivities are plotted for ordinates in his curve. The discrepancies may be attributed largely to impurities and to the different conditions under which the alloys solidified.
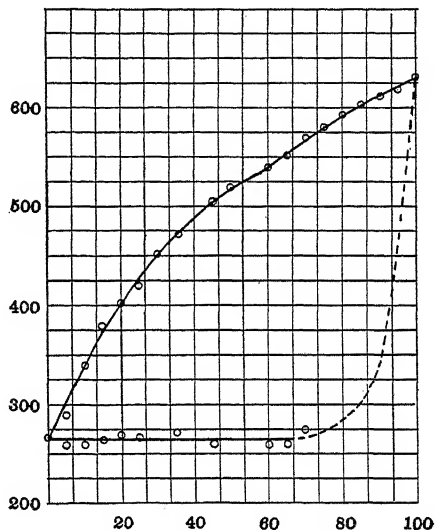
Fig. 3.

Fusion curve for bismuth-antimony alloys.

The fusion curve for bismuth-antimony alloys, as obtained by Hüttner and Tammann[2] is reproduced in Fig. 3. This curve shows that bismuth and antimony form a continuous series of mixed crystals without the formation of any compounds. It appears according to Guertler[3] that it is only where the metals form compounds, that either the curve relating resistance and percentage of one component or the curve relating temperature coefficient of the resistance and percentage of one component shows a singular point. Singular points were not, therefore, to be expected in the curves in Fig. 2. On the other hand, there are in these curves the well defined maximum and minimum which are characteristic of this class of alloys.

Fig. 4 and Fig. 5 give the results for the antimony-cadmium and antimony-zinc alloys, respectively. Curve I. in each figure shows

[1]Ibid., p. 327.

[2]Zeitschr. f. anorg. Chem., 44, p. 131, 1905.

[3]Ibid., p. 403.

the dependence of the resistance on the composition and Curve II. the dependence of the temperature coefficient of the resistance on the composition. In each of the curves marked I. the ordinates are proportional to the specific resistances at 0° C. and in the curves marked II. the ordinates give the average temperature coefficient between 0° and 100° C. The abscissæ in Fig. 4 are percentages
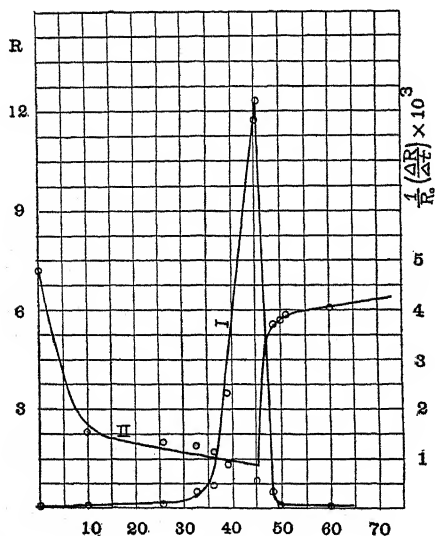
Fig. 4.

Antimony-cadmium alloys. Curve I., electrical resistance. Curve II., temperature coefficient of resistance.
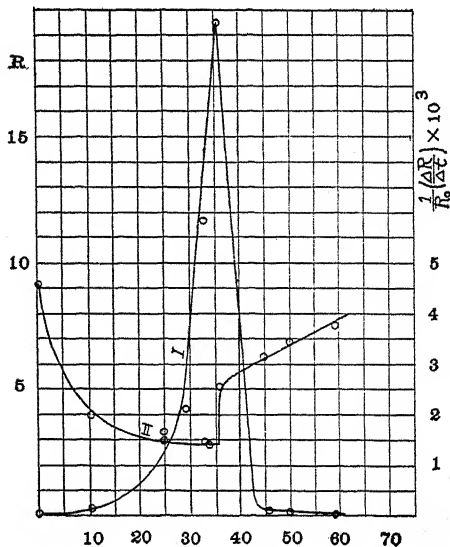
Fig. 5.

Antimony-zinc alloys. Curve I., electrical resistance. Curve II., temperature coefficient of resistance.

by weight of cadmium and in Fig. 5, percentages by weight of zinc. It will be observed from these curves that with an increasing percentage of cadmium or zinc, the resistance of the alloy increases, at first slowly and then very rapidly, until the metals are present in the ratio of their atomic weights, where the resistance of the alloy is enormously higher than that of either component entering into it. A further increase in the percentage of cadmium or zinc causes the resistance to drop suddenly to a value which is of the order of magnitude of the resistance of the cadmium or zinc.

The addition of either cadmium or zinc to antimony causes the temperature coefficient of the resistance to decrease. This decrease

is at first very rapid, until the alloy contains about 10 per cent. of either cadmium or zinc.   With further increase in the concentration of either cadmium or zinc the temperature coefficient of the resistance continues to decrease, more and more slowly, until equal chemical equivalents of the metals are present.   When this concentration has been reached for either series of alloys, a further increase of cadmium in one case or of zinc in the other case makes the temperature coefficient jump to a value which is somewhat less
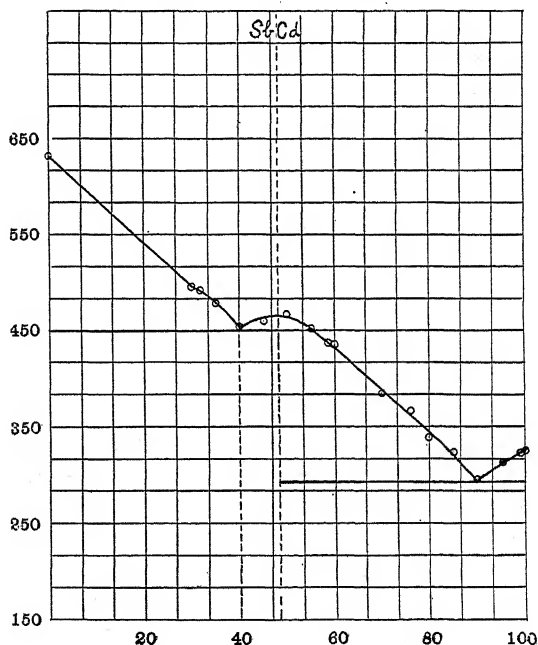
Fig. 6.

Fusion curve for antimony-cadmium alloys.

than that for a pure metal.   Still further increase in the concentration of either cadmium or zinc causes the temperature coefficient of the resistance to increase slowly and it seems to approach the value of that coefficient in the pure metal.

In Fig. 6 and Fig. 7 are given fusion curves for antimony-cadmium and antimony-zinc alloys respectively, the former by Treitschke,[1] the latter by Monkemeyer.[2]   From Fig. 6 it is seen that when the

[1]Zeitschr. f. anorg. Chem., 50, p. 217, 1906.
[2]Zeitschr. f. anorg. Chem., 43, p. 182, 1905.

antimony and cadmium are present in the ratio of their atomic weights, they form the compound, SbCd. Similarly, the melting curve in Fig. 7 shows that when the antimony and zinc are present in the ratio of their atomic weights, the compound SbZn is formed. An examination of Fig. 4 in connection with Fig. 6 brings out the fact that where the antimony and cadmium form the compound SbCd, the curve relating resistance to composition of the alloy and the curve relating the temperature coefficient of the resistance with the composition each have a singular point. Analogous results are arrived at for the antimony-zinc alloys by an examination of Fig. 5 in connection with Fig. 7. These results confirm the relation pointed out by Guertler[1] between resistance, temperature coefficient of resistance, and composition of the alloy.

Fig. 7.

Fusion curve for antimony-zinc alloys.

THERMOELECTROMOTIVE FORCES.

For the approximate determination of the thermoelectromotive forces the plates were placed in a rectangular brass box which was separated into two compartments by a partition of hard fiber. The plates were thrust through the partition so that one end of the plate was in one compartment, the other end in the other compartment. One compartment was filled with a mixture of ice and water, the other with water at room temperature. The temperatures of the ends of the plate were determined by means of copper-german-silver thermal couples which were soldered to the ends of the plates. The copper wires of these couples could by means of suitable

[1] Ibid., p. 412.

switches, be connected with a d'Arsonval galvanometer on which the thermoelectromotive force between the ends of the plate could be observed. To prevent error due to drift of temperature, the temperatures of the ends of the plates were determined immediately before and after the observation of the desired electromotive force. The mean of the respective temperatures thus obtained has been used in the calculation of the thermoelectric heights of the plates.
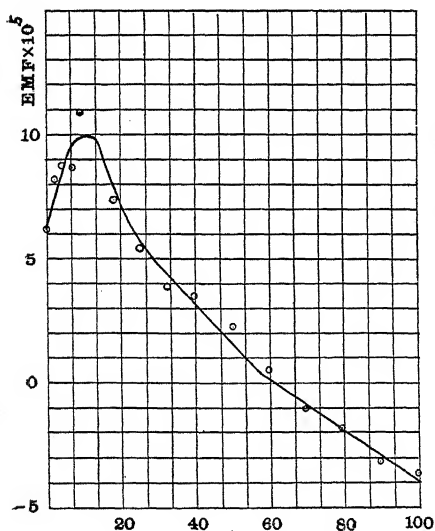
Fig. 8.

Thermoelectric heights of bismuth-antimony alloys.
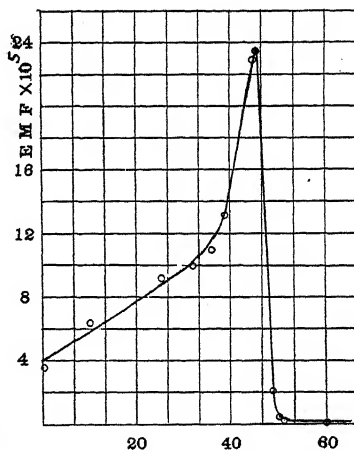
Fig. 9.

Thermoelectric heights of antimony-cadmium alloys.

The results of these observations for the bismuth-antimony alloys are to be found in Fig. 8; those for the antimony-cadmium alloys in Fig. 9; and those for the antimony-zinc alloys in Fig. 10. In each of these curves the ordinates are the average thermoelectric height between 0° and 20° C. The abscissæ in Fig. 8 are weight per cent. antimony; those in Fig. 9 weight per cent. cadmium; and those in Fig. 10 weight per cent. zinc. The character of the curve for the antimony-bismuth alloys is the same as that given recently by Haken[1] and that given earlier by Becquerel.[2] It shows

[1]Ibid., p. 327.
[2]Ibid., pp. 291–336.

a well defined maxmum when the alloy contains about 10 per cent. of antimony.   The curve, however, shows no singular point and a singular point was scarcely to be expected, since there is no compound formed in the bismuth-antimony series.   The analogous curves for the antimony-cadmium and for the antimony-zinc alloys agree in form with the curves given by Becquerel for these alloys. There are some discrepancies in the absolute values of the thermo- electric heights.   These discrep- ancies which may be attributed mainly to slight impurities in the alloys, are of no consequence in the present investigation, since the character of the curve was all that was desired.   The addition of either cadmium or zinc to antimony causes a de- cided increase in the thermo- electric heights.   This increase becomes more and more rapid with increasing concentration, until equal chemical equiva- lents of the metals are present. A further increase in the con- centration of either cadmium or zinc causes the thermoelec- tric heights to sink very rapidly to a value which is of the order of magnitude of the thermo-

Fig. 10.

Thermoelectric heights of antimony-zinc alloys.

electric height of the antimony.   The largest value of the ther- moelectric height in the antimony-cadmium series is about seven times that for antimony and the largest value in the antimony-zinc series is about five times that for antimony.

A study of Fig. 9, in connection with Fig. 6 and Fig. 10 in con- nection with Fig. 7, shows that the singular points in the curves relating the thermoelectric heights with the composition occur where the metals form compounds.   It is also seen from these

curves that the alloy which has the highest resistance and the
least temperature coefficient of the resistance, has the greatest
thermoelectric height, and that the singular points in each of the
curves appear for the same percentages of metals in the alloys.

The Hall Effect.

The arrangement of the apparatus for the study of the Hall
effect and the method of procedure were essentially the same as
that used by the author[1] in an earlier paper. The plates, the
dimensions of which have already been given, were arranged in
the ordinary way for the observation of the Hall electromotive
forces. The copper lead wires for connecting the plate to the
battery were soldered directly to the plate. On either edge of the
plate midway between its ends were soldered the copper lead wires
which lead to the galvanometer on which the Hall electromotive
forces were to be observed. The plates were mounted on rec-
tangular pieces of hard fiber and held in position by covering the
plate and fiber with paraffin. Suitable arrangements were made
for holding the plates in position between the poles of the magnet,
so that the lines of magnetic force would be perpendicular to the
plane of the plate. Observations were made at room temperature.

Experiment has shown that for a given metal or alloy, the Hall
electromotive force E is given by the equation,

$$E = R\frac{Hi}{d} \; ;$$

where H is the magnetic field in absolute units; i, the current in
absolute units; d, the thickness of the plate in centimeters; and
R, a constant which is a function of the temperature and the
strength of the magnetic field. Since the lead wires which were
soldered to the edges of the plate for getting the Hall electromotive
forces were not of the same material as the plate itself, there is
superimposed on the Hall electromotive force a thermoelectromotive
force which is caused by the inequality of temperature between
the edges of the plate, when it is in the magnetic field. It has
been found that this difference of temperature which was discovered
by Ettingshausen, is given by the equation,

[1] Phys. Rev., 30, p. 1, 1910.

$$\Delta T = P \frac{Hi}{d};$$

where ΔT is the difference in temperature between the edges of the plate for a given magnetic field; H, the magnetic field in absolute units; i, the current in the plate in absolute units; P, a constant for a particular metal, at a particular temperature and field strength. If E denotes the Hall electromotive force, E' the observed electromotive force, and θ the thermoelectric height of the plate with respect to copper, then $E = E' \pm \theta \Delta T$ and $R' = R \pm P\theta$, where R' is the value of the Hall constant calculated from the observed electromotive force without correction. According to Zahn,[1] $P = 2 \times 10^{-6}$ for antimony and $P = 5 \times 10^{-5}$ for bismuth. Approximate determination of P in the antimony-cadmium and in the antimony-zinc series showed that it would not have a value greater than the value in pure antimony. The error due to this source would not, therefore, exceed about three per cent. for the most unfavorable cases and has been neglected in this work. This source of error has also been neglected in the bismuth-antimony alloys, although it amounted in some cases to about seven per cent. Correction for this error would not have changed the character of the curve and little importance is being attached to the absolute value of the effect.
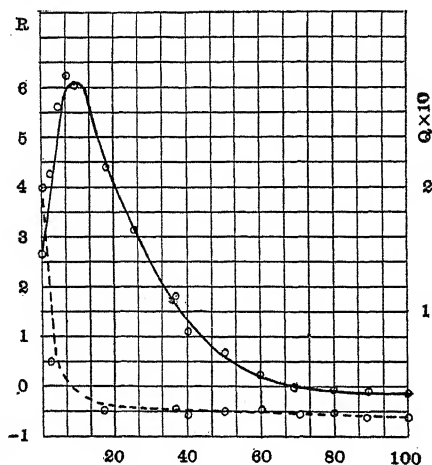
Fig. 11.

Bismuth-antimony alloys. Solid curve shows Hall effect. Dotted curve shows Nernst effect.

The values of R found for the bismuth-antimony alloys have been plotted in Fig. 11. The continuous curve in that figure shows the relation between R and the composition of the alloy. The ordinates

[1] Ann. der Phys., 14, p. 886, 1904.

are the values of R at room temperature, when the magnetic field is
10,400 absolute units and all of the quantities on which R depends
are measured in the units indicated above. The abscissæ are the
percentages by weight of antimony. It will be observed that the
addition of antimony to bismuth increases the value of R, until the
alloy contains about 10 per cent. of antimony when the Hall effect
is largest. A further increase in the percentage of antimony causes
a rapid decrease in R which becomes nearly zero in an alloy con-
taining about 70 per cent. of antimony. In alloys which contain
more than about 70 per cent. of antimony the direction of the
effect is the direction of the effect in antimony which is opposite
to that in bismuth. It is to be observed that the character of the
curve obtained for the Hall effect is very like that obtained for
resistance and that for thermoelectric heights. In each case the
curve has a maximum when the alloy contains about 10 per cent.
of antimony and an increase in the concentration of the antimony
above about 10 per cent. causes a continuous decrease in the resist-
ance, in the thermoelectric heights and in the Hall effect.

In the continuous curve in Fig. 12 the Hall constants for the
antimony-cadmium alloys have been plotted against the percentages
by weight of cadmium, the former as ordinates, the latter as
abscissæ. The strength of the magnetic field was 10,400 c.g.s. units.
The first addition of cadmium to the antimony causes only a small
increase in the Hall effect. With higher concentrations of cadmium
the Hall effect increases more and more rapidly, until its value
is about one hundred and fifty times the value in pure antimony.
This very large value is found where the metals are present in the
ratio of their atomic weights. When there is present in the alloy,
a small amount of cadmium more than the equivalent of the anti-
mony, the Hall effect drops suddenly to a value which is of the
order of magnitude of the value in antimony. With further increase
in the concentration of the cadmium the value of R continues to
decrease and seems to approach the value of R in cadmium. By
referring again to the fusion curves for the antimony-cadmium
alloys, it will be seen that the singular point in the curve relating
R and the percentage of cadmium, occurs where the antimony and
cadmium form the compound $SbCd$. The behavior of the Hall

effect in this respect is analogous to the behavior of the electrical conductivity, the temperature coefficient of the resistance and the thermoelectric heights.

The analogous observations on the antimony-zinc alloys are given in the continuous curve in Fig. 13. Here the abscissæ are the
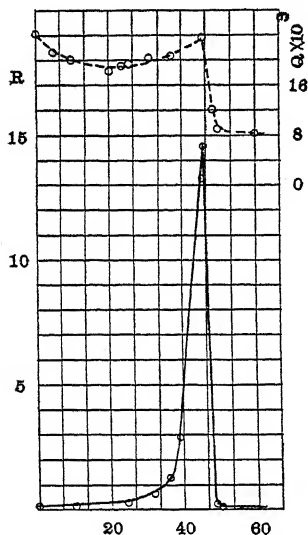
Fig. 12.

Antimony-cadmium alloys. Solid curve shows Hall effect. Dotted curve shows Nernst effect.
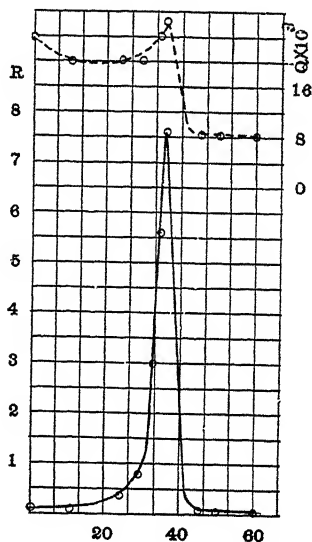
Fig. 13.

Antimony-zinc alloys. Solid curve shows Hall effect. Dotted curve shows Nernst effect.

percentages by weight of zinc. The temperature and strength of magnetic field were the same as in the case of the antimony-cadmium series. The first addition of zinc to the antimony seems to have a somewhat greater effect on the magnitude of the Hall effect than does the addition of cadmium. The largest value of the Hall effect in the antimony-zinc alloys is only about one half of the largest value in the antimony-cadmium series. Here again it is noticed that when the antimony and zinc are present in the ratio of their atomic weights the Hall effect has its largest value and the curve shows a singular point. The largest value of the effect in these alloys is about seventy times the value in pure antimony. As soon as there is present in the alloy somewhat more than 35 per cent.

of zinc, the value of the Hall effect sinks to a small fraction of its largest value, and with further increase in the concentration of the zinc the value of R continues to decrease, apparently approaching the value of R in pure zinc.   By reference to Fig. 7 it will be seen that the singular point in this curve which has just been discussed, occurs where the antimony and zinc form the compound SbZn.

NERNST EFFECT.

v. Ettingshausen and Nernst[1] have shown that when a metal plate through which heat is flowing, is brought into a magnetic field so that the lines of magnetic force are perpendicular to the plane of the plate there is set up a difference of potential between the edges of the plate which are parallel to the direction of the flow of heat.   Later experiments by Nernst[2] and others have shown that

$$E = QH\beta\frac{\delta T}{\delta x},$$

where E is the electromotive force between the edges of the plate in c.g.s. units; H, the magnetic field in c.g.s. units; β, the width of the plate in centimeters; $\delta T/\delta x$, the temperature gradient in the plate; Q, a constant which is a function of the temperature of the plate, and of the magnetic field and depends on the material of which the plate is made.
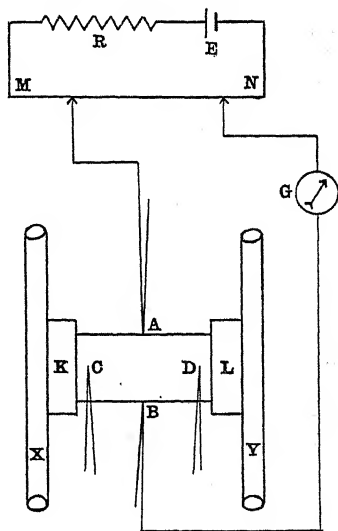
Fig. 14.

To study this effect the plates were soldered to heavy narrow strips of copper K and L (Fig. 14), which had been previously brazed to the copper tubes X and Y, respectively.   The tubes and plates were mounted on a suitable frame of hard fiber which afforded the necessary protection from strains and also a means of fastening the plates in the proper position between the poles of the magnet.

[1]Wied. Ann., 29, p. 343, 1886.
[2]Wied. Ann., 31, p. 760, 1887.

Steam at atmospheric pressure flowed through one of the tubes and through the other there was a continuous flow of water at room temperature. For the purpose of determining the temperature gradient in the plates, a copper-german-silver thermal couple was soldered to the plate at C and another at D. These points were so chosen that the couples, at a known distance from each other, were on the line midway between top and bottom of the plate. At A and B which lay on the vertical line midway between the edges of the plate was also soldered a copper-german-silver thermal couple. The copper lead wires lead to the d'Arsonval galvanometer on which the Nernst effect was to be observed. In series with the galvanometer was a slide-wire potentiometer MN by means of which any thermoelectromotive force in the circuit due to an inequality of temperature between A and B could be compensated.

The plate was placed between the poles of the magnet with its plane perpendicular to the lines of force and was well packed with cotton wool to prevent loss of heat by radiation. When temperature equilibrium had been realized, the thermoelectromotive forces in the circuit were compensated on the potentiometer and when the magnetic field had been established, the deflection of the galvanometer produced by the difference of potential between A and B was noted. Four such observations were made. The direction of the magnetic field for two of them was opposite to that for the other two.

If the lead wires are not of the same material as the plate which is being examined, the effect discovered by Leduc will always be superposed on the Nernst effect. Leduc[1] found that when a plate traversed by a current of heat is brought into a magnetic field in the way previously described, there occurs a rotation of the isothermal, so that A and B which were originally on the same isothermal are no longer at the same temperature. Since in these experiments the lead wires were of copper, this effect would give an uncompensated thermoelectromotive force in the circuit. It has been found that the value of ΔT, the change of temperature between A and B for a given magnetic field H, is given by the equation,

[1]Compt. Rend., 104, p. 1783, 1887.

$$\Delta T = SH\beta \frac{\delta T}{\delta x};$$

where β is the width of the plate and $\frac{\delta T}{\delta X}$, the temperature gradient in the plate. If θ is the thermoelectric height of the plate with respect to copper; E', the observed electromotive force; and E, the electromotive force from the Nernst effect, after correction for the Leduc effect; Q', the constant in the equation for the Nernst effect calculated from the observed electromotive force, and Q the same constant after correction for the Leduc effect, the following equations hold:

$$E = E' \doteq \Delta T\theta$$

and

$$Q = Q' \mp S\theta.$$

This source of error for most of the plates is quite large and has been the chief cause of uncertainty in the determination of the Nernst effect. It has been seen that the thermoelectric heights of some of the plates are quite large. In these plates any uncertainty in S introduces large errors in Q. By means of the thermal couples at A and B approximate values of S were obtained for the alloys studied in this paper. The value of S found for antimony was 2.0×10^{-6}, when distances are measured in centimeters; temperatures on the Centigrade scale and magnetic field in absolute units. This value is in fair agreement with that given by Zahn and other observers. With the addition of either cadmium or zinc to the antimony the value of S decreased, so that for an alloy containing 30 per cent. of zinc and 70 per cent. of antimony, or for an alloy containing 40 per cent. of cadmium and 60 per cent. of antimony, the value of S was about one tenth of its value in pure antimony. For alloys containing more than 30 per cent. of zinc and for alloys containing more than 40 per cent. of cadmium the value of S was so small that it was impossible to determine it with any accuracy. For bismuth S was found to be 2.0×10^{-6}. For alloys which contained less than 60 per cent. of antimony S was sometimes less and sometimes greater than in bismuth. The direction of the effect in bismuth and in the alloys which contained less than about 60 per cent. of antimony was opposite to that in

antimony. In alloys which contained more than about 60 per cent. of antimony the effect had the same direction as it has in antimony. For an alloy containing about 60 per cent. of antimony the value of S was nearly zero. The values of S obtained were only approximate and have not been tabulated in this paper, for they were desired only as corrections in the Nernst effect.

In three of the bismuth-antimony alloys, viz., Bi 91 per cent., Sb 9 per cent.; Bi 82 per cent., Sb 18 per cent., Bi 75 per cent., Sb 25 per cent.; it was found that the electromotive force from the Leduc effect was larger than that from the Nernst effect. Since the two electromotive forces have opposite signs, there was an apparent reversal of the Nernst effect for certain magnetic fields. The Leduc effect decreased rather rapidly with increasing magnetic fields, so that for the higher magnetic fields the electromotive force due to the Nernst effect exceeded that due to the Leduc effect and there was no longer an apparent reversal of the effect. The strength of the magnetic field above which there was no apparent reversal of the Nernst effect was considerably less for the plate containing 9 per cent. of antimony than for the plate containing 25 per cent. of antimony.

The results for the bismuth-antimony alloys have been plotted in Fig. 11. The dotted curve shows the relation between Q and the percentage by weight of antimony. The ordinates on the right hand side of the figure are for this curve. It will be observed that the direction of the effect is the same in bismuth and in antimony and in any of their alloys. The addition of antimony to bismuth causes a rapid decrease in the magnitude of the effect, so that the effect in an alloy containing 91 per cent. of bismuth and 9 per cent. of antimony is only a little larger than the effect in pure antimony. A further increase in the percentage of antimony causes the Nernst effect to decrease slowly and its value approaches ultimately the value of the effect in antimony.

In Fig. 12 and Fig. 13 the dotted curves give the values of Q in the antimony-cadmium and antimony-zinc alloys, respectively. In Fig. 12 the abscissæ are percentages by weight of cadmium and in Fig. 13 they are percentages by weight of zinc. Q has been plotted for ordinates in each figure. The addition of either cadmium or

zinc to antimony causes a decrease in the magnitude of the Nernst effect. With further increase in the concentration of either cadmium or zinc Q seems to pass through a minimum and then to increase to a value which is about the value of the effect in pure antimony. When the metals are present in the ratio of their atomic weights, where it has been seen that the compounds SbCd and SbZn are formed, each of the curves has a singular point which is very similar to the singular points in the curves which have been previously discussed. The increase in the concentration of the cadmium above that necessary to form the compound SbCd or that of the zinc above that necessary to form the compound SbZn, causes Q to sink rapidly to a value which is of the order of magnitude of its value in either cadmium or zinc. There is considerable uncertainty in the value of Q for the plates which contained equal chemical equivalents of the metals, because the thermoelectric heights of the plates is so large that any uncertainty in S which is small, makes a large error in Q. Concerning the break in the curves at the points indicated there can be no doubt.

Change of Resistance in a Magnetic Field.

To obtain the change of resistance in a magnetic field, the plate with the connections as indicated in Fig. 1 was placed between the poles of a large electromagnet in such a way that the lines of force were perpendicular to the plane of the plate. When temperature equilibrium had been established, with a suitable current in the plate and the magnetic field zero, the deflection of the galvanometer was noted. Everything else remaining unchanged, the magnetic field was established and the deflection of the galvanometer again noted. These observations which were made at room temperature were repeated several times. The direction of the magnetic field was reversed between successive observations. The change of resistance was calculated from the equation,

$$\frac{\Delta R}{R_0} = \frac{d_H - d_0}{d_0},$$

where d_H is the deflection with the plate in a magnetic field of strength H, and d_0 the deflection with the magnetic field zero.

Observations were made on the plates containing less than 50 per cent. of antimony for four different magnetic fields, and on those containing more than 50 per cent. of antimony for three different magnetic fields.
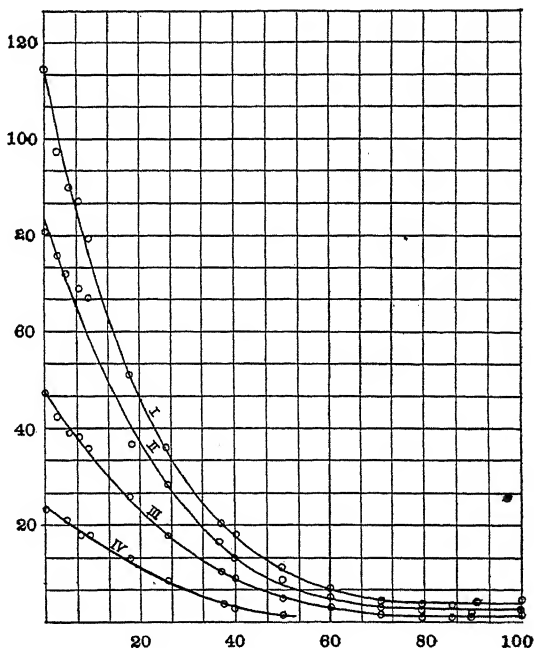
Fig. 15.

Bismuth-antimony alloys.   Change of resistance in four different magnetic fields.

The results of these observations have been plotted in Fig. 15, where the ordinates are the percentages of change of resistance and the abscissæ are the percentages by weight of antimony.   The curve marked I. is for 21,700 c.g.s. units; II., for 17,700 c.g.s. units; III., for 10,400 c.g.s. units and IV., for 5,500 c.g.s. units.   The addition of antimony to bismuth causes the change of resistance in the magnetic field to decrease and this decrease is considerably larger than is to be expected from the additive law, so that an alloy containing about 70 per cent. of antimony shows a change of resistance which is very little larger than the change of resistance in pure antimony. After the alloy contains about 70 per cent. of antimony a further

increase in the percentage of antimony seems to have little influence
on the change of resistance in the magnetic field.

In Fig. 16 curves are plotted showing the relation between the
change of resistance and the strength of the magnetic field, for
bismuth and four of these alloys.   The ordinates in these curves
are percentages of change of resistance and the abscissæ the strength
of the magnetic field in kilogausses per square centimeter.   Curve
I. is for bismuth; II., for Bi 91 per cent., Sb 9 per cent.; III., for
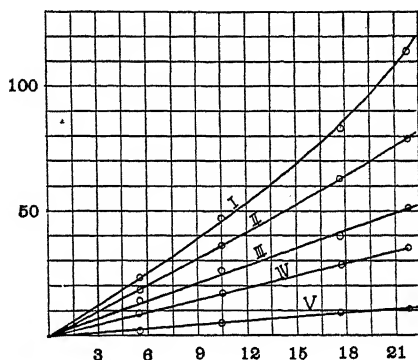
Fig. 16.

Bismuth-antimony alloys.   Relation between change of resistance and strength of
the magnetic field.

Bi 82 per cent., Sb 18 per cent.; IV., for Bi 75 per cent., Sb 25 per
cent.; and V., for Bi 50 per cent., Sb 50 per cent.   It will be seen
that in bismuth the change of resistance increases somewhat more
rapidly than the magnetic field.   In an alloy containing 91 per cent.
of bismuth and 9 per cent. of antimony the resistance still increases
somewhat more rapidly than the magnetic field but the deviation
from proportionality between change of resistance and strength
of magnetic field is less marked than in the case of bismuth.   In
an alloy containing 18 per cent. or more than 18 per cent. of anti-
mony the change of resistance seems to be nearly proportional to
the strength of the magnetic field.

SUMMARY.

The electrical resistance, the temperature coefficient of the resist-
ance, the thermoelectric heights, the Hall effect and the Nernst

effect have been studied in three series of alloys, viz., bismuth-antimony, antimony-cadmium, antimony-zinc. For each of these series of alloys curves have been plotted showing the relation, (*a*) between the amount of one component and the electrical resistance, (*b*) between the amount of one component and the temperature coefficient of the resistance, (*c*) between the amount of one component and the thermoelectric heights, (*d*) between the amount of one component and the Hall constant, (*e*) between the amount of one component and the Nernst effect. From these curves the following conclusions may be drawn:

1. In the case of the bismuth-antimony alloys in which no compound of bismuth and antimony is formed, it is found that none of the curves have singular points but that each of them except one shows a well-defined maximum or minimum for an alloy which contains about 90 per cent. of bismuth and 10 per cent. of antimony.

2. In the antimony-cadmium series of alloys in which the compound SbCd is formed, when the metals are present in the ratio of their atomic weights, each of the curves has a well-defined singular point corresponding to the compound SbCd.

3. Similarly, in the antimony-zinc series of alloys in which the compound SbZn is formed when the metals are present in the ratio of their atomic weights, each of the curves has a well-defined singular point corresponding to the compound SbZn.

The change in the resistance of the bismuth-antimony alloys when in a magnetic field has been studied and it has been found that the change of the resistance is not an additive property of these alloys.

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ON THE SECONDARY β RADIATION FROM SOLIDS AND LIQUIDS.

By S. J. Allen.

THIS paper is a continuation of one on the same subject published in the Physical Review for December, 1909, and deals chiefly with the effect of changing the angle of incidence of the primary beam of rays. In that paper only one angle of incidence, about 70°, was used, and it seemed desirable to test what effect different angles of incidence would have on the relative order of the secondary radiations from the various substances. As stated before McClelland had shown that the angle of incidence had a large effect upon the relative magnitude of the secondary radiations of light atoms as compared to heavy ones.

In my former paper the secondary radiation from a large number of solids, aqueous solutions of salts, and pure liquids, was given, and the results therein expressed seemed to show in my estimation many cases which were difficult to explain on the then existing theories of secondary radiations. In order to explain these anomalous results I advanced the possibility that the secondary radiation of compounds might in some cases be largely affected by the chemical arrangement of the molecules.

A preliminary investigation of the absorbing power of these liquids for the primacy β rays was also made, and the results obtained showed that in many cases the relative absorbing power followed the same order as the relative secondary radiation. In some cases this was not so, especially in those liquids which were highly volatile. Both in the case of secondary radiation and in that of absorption, the presence of heavy atoms could be easily traced by their increased effect.

Since this paper was published several papers on the same subject have appeared, which discuss at more or less length my results. Borodowsky in the April number of the Phil. Mag., 1910, gives the

results of his investigation into the relative absorbing power of certain compounds, both solid and liquid, and concludes that the effects obtained in every way coincide with the present theory of secondary β rays. Knowing the constitution of a compound and the relative absorption of its constituents, the relative absorption of the mixture can be calculated. He also comes to the conclusion that the chemical arrangement of the molecules has no apparent effect on the absorption, which is only affected by the internal arrangement of the atoms. He draws attention to the fact that in a number of cases his results do not agree with mine, notably in that of ethelene chloride, and ascribes the difference to the imperfection of my methods.

Mr. Borodowsky's method is the same in principle as my own, he expressing the absorbing powers of the compounds in relative terms of aluminium, whilst I made use of water. His method of procedure in covering the liquid with a vapor proof cover is certainly an advantage in many cases. In performing my experiments I was fully aware of the possibility of an error due to the column of vapor above the liquid and clearly mention this fact in my paper, though I did not think it was sufficient to account for the greatly anomalous results in many cases. The one substance which showed the most anomalous result was ethelene chloride, which gave an absorption much greater than one would expect. Borodowsky however found this substance quite normal.

In the May number of the REVIEW Mr. Bragg replied quite sharply to my criticism of his theory of secondary γ rays, and also criticises the conclusions which I draw from my results on the secondary β rays. Several of his students repeated my experiments on ethelene chloride and ethelene bromide, and found that these substances were quite normal and that ethelene chloride gave a very much less radiation than the bromide, quite the contrary to my results.

About the time of the appearance of this paper I had occasion to make some density tests on these two liquids and found very much to my surprise that the densities obtained did not agree with the values in Landolt and Börnstein's tables. As far as I could make out the substance which I thought was ethelene chloride

was ethelene bromide, and the ethelene bromide, ethyl bromide. It seems that the bottles in which these liquids were furnished to me must have been incorrectly marked. This is the first opportunity I have had of making this correction to my last paper. With this correction in view the results obtained are quite what one would expect for ethelene and ethyl bromide, and there is no need in these cases to consider any effect due to chemical grouping of the molecules.

In a number of important papers H. W. Schmidt[1] has made valuable contributions to our knowledge of secondary radiations, and also developed theories with regard to the relation between the secondary and primary β rays for both simple substances and complex compounds. In the case of simple elements the equations arrived at are as follows:

$$\frac{\mu}{D} = \sqrt{\frac{\alpha}{D}\left(\frac{\alpha}{D} + \frac{2\beta}{D}\right)},$$

an

$$p = \frac{1}{\beta}[\alpha + \beta - \sqrt{\alpha(\alpha + 2\beta)}],$$

where α represents the "absorption coefficient," β the "reflexion coefficient," D the density of the absorbing substance. $1/\mu$ is equivalent to the much used "penetrating power" of the rays, and p to the maximum "secondary radiation." α and β are connected in a simple manner with the density D, and the atomic weight A of the substance. From a large number of experiments on both solids and liquids Schmidt has deduced from empirical considerations the following:

$$\alpha = c_1 \frac{D}{\sqrt[3]{A}}; \quad \beta = c_2 A \cdot D,$$

where c_1 and c_2 are two universal constants, dependent only on the nature of the rays under consideration. If one knows μ/D and p for any substance, or u/D for any two substances, then c_1 and c_2 can be calculated. For example Pb has a value for μ/D of 7.9 and for Al 4.7. The values of c_1 and c_2 are therefore 7.66 and 0.113 respectively.

[1]Part. I., Phys. Zeit., 10, 929–948, 1909; Part II., Phys. Zeit., Jan. 11, 1910, 262–273.

Now since $D = NA$ (N = number of atoms in a cm.^{3}), then we have

$$\alpha = c_1 N \sqrt[3]{A^2},$$

$$\beta = c_2 N A^2.$$

These simple equations can be applied to the case of compounds in the following manner:

Let the compound be represented by $X_a, Y_b, Z_c, \cdots$. Then,

$$\alpha = c_1 N (a\sqrt[3]{A_x^2} + b\sqrt[3]{A_y^2} + c\sqrt[3]{A_z^2} + \cdots),$$

$$\beta = c_2 N (aA_x^2 + bA_y^2 + cA_z^2 + \cdots),$$

where A_x, A_y and A_z are the atomic weights of the elements entering into the compound, N the number of molecules in a cm.^{3} of the compound.

Put $N = D/M$, where M is the molecular weight of the compound. We have

$$\frac{\alpha}{D} = \frac{c_1}{M}(a\sqrt[3]{A_x^2} + b\sqrt[3]{A_y^2} + c\sqrt[3]{A_z^2} + \cdots),$$

$$\frac{\beta}{D} = \frac{c_2}{M}(aA_x^2 + bA_y^2 + cA_z^2 + \cdots).$$

Substituting these values of α and β in the first two equations we get

$$\frac{\mu}{D} = \frac{\beta}{D}\sqrt{u^2 - 1},$$

and

$$p = \mu - \sqrt{u^2 - 1},$$

where

$$u = 1 + \frac{\alpha}{\beta} = 1 + \frac{c_1}{c_2}\frac{(a\sqrt[3]{A_x^2} + b\sqrt[3]{A_y^2} + c\sqrt[3]{A_z^2} + \cdots)}{(aA_x^2 + bA_y^2 + cA_z^2 + \cdots)}.$$

Thus we see, on this simple theory of addition, that μ/D and p depend only on the universal constants c_1 and c_2, and on the atomic weights of the atoms of the molecules of the compound. Example, water, (H_2O).

$$c_1 = 7.66; \quad c_2 = 0.113; \quad A_x = 1; \quad A_y = 16; \quad a = 2; \quad b = 1;$$

$$\sqrt[3]{A_x^2} = 1; \quad \sqrt[3]{A_y^2} = 6.35; \quad A_x^2 = 1; \quad A_y^2 = 256.$$

Then,

$$\mu = 1 + \frac{7.66(2 + 6.35)}{0.113(2 + 256)} = 3.194,$$

$$\sqrt{u^2 - 1} = 3.035,$$

$$\frac{\beta}{D} = \frac{0.113}{18}(2 + 256) = 1.619,$$

$$\frac{\mu}{D} = 1.619 \times 3.035 = 4.91,$$

$$p = 3.194 - 3.035 = 0.159.$$

Schmidt[1] has made a number of careful tests on the quantity μ/D for both solids and liquids, which show good agreement with the values as calculated from the above equations. In the latter part of his paper he devotes considerable space to a discussion of my results, comparing the calculated values with my observed ones. The agreement is better in the case of the solid compounds, such as the bichloride, than in the liquids, and especially so for those compounds which have atoms of high atomic weight. He finally concludes that, within the errors of observation, my results form a confirmation of the correctness of his own theoretical assumptions and theories.

The series of experiments in the present investigation were carried out in the same manner, and with almost the same apparatus as already described in my previous paper, and so a detailed description will not be necessary here. The ionization chamber consisted of two hemispherical electrodes, the upper one made of very thin aluminium foil through which the rays could pass with very little absorption.

The radium was placed in a small lead block, which could be so arranged that the primary β rays could fall upon the substance to be investigated at mean angles of incidence varying from 0° to about 60°. In the former work the angle of incidence was approximately 70°. The ionization, as before, was balanced through the electrometer by means of an equal amount of opposite sign furnished by the standard radium preparation. Only a limited number of

[1] *Loc. cit.*

representative substances were tested, from the groups of solids, saturated aqueous solutions and pure liquids. The relative secondary radiations were tested through the thin aluminium foil, and also through an additional layer of tin foil 0.001 mm. thick.

The results obtained are expressed in Tables I.–III. Three angles of incidence were chosen, viz., 0°, 30° and 50°. The first column for each angle of incidence gives the value of the secondary radiation through the aluminium, expressed in the arbitrary units of the electrometer, the second column the same through .001 mm. tinfoil, while the third column gives the per cent. of the primary beam transmitted through the tinfoil.

TABLE I.

Scattered β Radiation from Solids.

Substance.	Radiation at Angle of 0°.			Radiation at Angle of 30°.			Radiation at Angle of 50°.		
	Through Thin Aluminum.	Through .001 mm. Tinfoil.	Per Cent. Transmission.	Through Thin Aluminum.	Through .001 mm. Tinfoil.	Per Cent. Transmission.	Through Thin Aluminum.	Through .001 mm. Aluminum.	Per Cent. Transmission.
Lead..........	600	400	66.7	665	440	66.2	688	490	71.0
Bismuth.......	615	405	65.8	683	448	65.8	696	503	71.8
Cadmium......	443	288	65.0	558	340	——	614	412	67.1
Copper........	354	227	64.0	400	256	64.0	496	335	67.5
Iron..........	304	193	63.4	366	239	65.3	471	317	67.3
Sulphur........	245	139	56.7	324	198	61.0	404	282	69.8
Aluminum	205	115	56.1	260	153	58.8	370	248	67.2
Paper	134	64	47.7	180	100	55.5	292	203	69.5

TABLE II.

Scattered β Radiation from Aqueous Solutions.

Substance.	Radiation at Angle of 0°.			Radiation at Angle of 30°.			Radiation at Angle of 50°.		
	Through Thin Aluminum.	Through .001 mm. Tinfoil.	Per Cent. Transmission.	Through Thin Aluminum.	Through .001 mm. Tinfoil.	Per Cent. Transmission.	Through Thin Aluminum.	Through .001 mm. Tinfoil.	Per Cent. Transmission.
Potassium iodide	315	192	61.0	381	231	60.7	471	312	66.4
Lead nitrate....	277	172	62.1	327	196	60.0	421	279	66.2
Cadmium iodide	222	138	62.1	284	164	58.0	367	241	65.7
Copper chloride.	190	114	60.0	259	133	53.2	357	229	65.0

TABLE III.

Scattered β Radiation from Pure Liquids.

Substance.	Radiation at Angle of 0°.			Radiation at Angle of 30°.			Radiation at Angle of 50°.		
	Through Thin Aluminum.	Through .oor mm. Tinfoil.	Transmission in Per Cent.	Through Thin Aluminum.	Through .oor mm. Tinfoil.	Transmission in Per Cent.	Through Thin Aluminum.	Through .oor mm. Tinfoil.	Transmission in Per Cent.
Methyl iodide ..	511	—	—	564	340	60.3	631	—	—
Ethyl iodide....	481	307	64.0	541	—	—	600	397	66.2
Tetra brom acetelene.....	388	257	66.0	481	281	58.6	530	347	65.8
Iod benzine	377	238	63.1	458	272	59.4	536	348	65.0
Iso anyl bromide	368	230	62.5	446	266	59.6	515	340	66.0
Ethelene bromide	358	228	63.7	441	258	58.5	503	333	66.2
Ethel bromide ..	323	207	64.0	382	223	58.4	493	—	—
Brom benzene..	310	—	—	344	197	58.0	443	292	65.9
Propyl bromide.	298	183	61.5	362	212	58.6	433	292	67.3
Amyl bromide..	284	—	—	333	182	56.0	433	285	65.8
Meta brom aniline.......	265	162	61.1	320	187	58.4	408	273	66.9
Chloroform	262	154	58.6	304	182	60.2	419	273	65.9
Water.........	114	69	61.1	170	96	56.5	280	182	67.5
Ethyl alcohol...	91	55	60.4	159	82	51.0	270	170	65.4
Ester.........	74	46	62.0	150	77	51.0	235	154	65.6

An examination of the results therein contained will show clearly I think the following facts.

Pure Solids.—The elements of high atomic weight show very small change in their radiating powers with increasing angle of incidence, there being, in the case of lead, only an increase of about 7 per cent. for a change of angle from 0° to 50°. As the atomic weight decreases, the angle of incidence has a large effect upon the secondary radiation. In the case of aluminium the radiation increases about 80 per cent., and for carbon over 110 per cent., between 0° and 50°.

If we consider the radiation after passing through the tinfoil the change is even more marked amounting to as much as 210 per cent. in the case of carbon. The transmission of rays through the tinfoil at angle 0° is considerably greater for elements of high atomic weight than for those of low atomic weight, while

at 30° the difference is small, and at 50° practically the same for all atoms. This would seem to indicate that the velocity of the returned rays is not greatly affected by the nature of the radiating substance, except at small angles of incidence.

Aqueous Solutions.—In this table are four representative salt solutions, with elements of both high and low atomic weight. They show the same general characteristics as the pure solids; those containing elements of low atomic weight increasing more rapidly with increasing angle of incidence than those of high atomic weight.

TABLE IV.

Substance.	0°.	30°.	50°.	Former Values.	Calculated (Schmidt).
Lead	719	719	719	719	719
Bismuth	738	739	730	723	721
Cadmium	531	604	614	625	611
Copper	424	433	496	526	488
Iron	365	396	471	474	462
Sulphur	294	351	404	408	336
Aluminium	245	281	370	359	296
Methyl iodide	613	610	631	658	593
Ethyl iodide	577	586	600	652	566
Tetra brom acetelene	476	520	530	580	518
Iod benzene	452	495	536	588	502
Iso amyl bromide	441	483	515	560	373
Ethelene bromide	426	476	503	568	491
Ethyl bromide	387	413	493	490	487
Propyl bromide	358	392	433	460	416
Brom benzene	371	372	443	382	376
Amyl bromide	340	360	433	384	368
Meta brom aniline	318	346	408	378	362
Chloroform	315	329	419	380	335
Water	137	184	280	200	159
Ethyl alcohol	109	173	270	170	131

The scattered radiation from a salt solution will depend to a great extent upon two factors; the number of atoms of the dissolved substance, and the number of atoms of the solvent. If the atoms of the solute are of high atomic weight we would expect very little increase in that part of the radiation due to them, and the great part of the increase would be due to the atoms of the solvent (in this case H_2O). This is clearly seen in the case of cadmium

iodide, and copper chloride, where in the former, the increased radiation is only about 68 per cent., while in the latter it is about 88 per cent.

The penetrating power of the rays from the aqueous solutions is of the same general nature as for the pure solids, and at an angle of 50° is practically the same for all solutions.

Pure Liquids.—The results as indicated in Table III. show that those liquids which contain heavy atoms have the greatest radiation, and the least increase with increase of angle of incidence. We also see that in any group containing the same kind of atoms the radiation in general is governed by the relative number of heavy atoms present. The penetrating power of the rays at 50° angle of incidence is practically the same for all substances.

In Table IV. the results for the pure solids, and pure liquids, at the three angles of incidence, are shown expressed in terms of lead (719). The last column shows the theoretical values as calculated from Schmidt's formulæ, already referred to at the beginning of this paper. It can be seen by an examination of this table that the experimental values for the radiation agree very well down to an atomic weight of about 50. Below 50 the experimental values for large angles of incidence are too large, and for normal incidence they are too small. At an angle of incidence of 30°, the agreement is very good throughout.

In conclusion it can be said that the results expressed in this paper for compounds are in general agreement with those already found for pure solids by other experimenters, and show that the scattered β radiations from chemical atoms in compounds is governed by the same factors as in the pure atoms, viz., atomic weight, and relative number of heavy atoms present; also, that the change of secondary radiation with angle of incidence in compounds is the same as in pure elements, being small for heavy atoms and large for light ones.

There seems no longer any reason for believing that the secondary β radiations are other than the primary rays bent out of their path by collision with the atoms of the radiator. Bragg, in the paper already referred to, states that the chance of a ray being deflected in any one direction by an atom can be represented by a

radius vector drawn in the given direction. The loci of all these radii vectores gives the complete case. Madsen has done some work to show that these loci are nearly centric with the atom in the case of heavy atoms but eccentric for light atoms. This would suffice to explain, in the case of the present results, why it is that the radiation for light atoms increases so rapidly with increasing angle of incidence, whereas the increase for heavy atoms is so small. For example, in the case of a light atom lying at the surface of the radiator, at normal incidence the radii vectores drawn outward from the surface representing the chance of rays being deflected in those directions would be short, and consequently the secondary radiations small. As the angle of incidence increased the radii vectores outside of the surface would be increased.

UNIVERSITY OF CINCINNATI,
    June, 1910.

A SIMPLE METHOD FOR DETERMINING NOCTURNAL RADIATION[1] PROPOSED BY K. ÅNGSTRÖM

A. K:son Ångström.

IN a recent note on an application of the electrical method of compensation to the determination of nocturnal radiation[2] published in 1905, K. Ångström has described a method for determining nocturnal radiation by means of a compensating instrument of his own construction. This compensating method which had been found so advantageous for the study of solar radiation was here applied to an instrument for determining terrestrial radiation. With this instrument one may determine at a given moment the absolute value of terrestrial radiation, and when it is a question of an absolute and exact determination the superiority of the instrument is unquestionable.

The relation between radiation received by the earth (insolation) and radiation from the earth (irradiation) as well as the absolute values of these quantities is of prime importance alike for the determination of climatic conditions in various localities and for the study of the calorific economy of the earth in general. It is important therefore to facilitate as much as possible the study of these phenomena. For this purpose a simple method is necessary which will permit of the making of measurements as easily as is done in the case of ordinary meteorological observations.

On account of the great cost of construction the compensating instrument does not lend itself, however, to the service of climatology and it demands at the same time an observer of considerable training. From the point of view of climatology it is not necessary moreover to determine the exact value of the irradiation or of the insolation at a given moment but to determine the integral radiation for a certain time. One might indeed apply the compensating

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[1] Translated for the Physical Review from the Transactions of the Royal Society of Sciences in Upsala by E. L. Nichols.

[2] Nova Acta Reg. Soc. Sc. Upsaliensis (4), I., No. 2.

instrument for this purpose by changing it into a registering instrument but that would be too costly and very complicated.

To satisfy the conditions of moderate price, simplicity of construction and ease of handling, while obtaining at the same time an integral value of the radiation, my father Prof. K. Ångström whose death has postponed the publication of his latest work in this field, devised the instrument which I am going to describe.

The principle of this instrument is as follows: A black surface radiates heat, it is thereby cooled and this cooling is compensated by the condensing of ether upon the surface. One determines the quantity of ether condensed and this quantity affords a measure of the intensity of radiation during a given time.

The instrument is constructed of glass and consists of an upper reservoir e, Fig. 2, which serves as a distilling vessel and an interior reservoir L which terminates below in a vertical graduated tube bb about 15 cm. in length. Above the interior reservoir is the blackened surface s which is simply the concave cover of the upper reservoir e drawn down to a point. The apparatus constitutes a closed vessel containing ether, the air within which has been driven off by ebullition under the air pump. The glass parts are surrounded by an envelop of polished metal which enclose them on all sides, leaving only a circular hole above the blackened surface s. The glass is insulated from the metal by means of ebonite.

The radiation from the blackened surface will evidently be greater than that from the other parts. It therefore becomes cooler than the surrounding parts but this cooling is compensated progressively in that the ether condenses upon the surface s from which it falls drop by drop into the graduated tube. To return the ether into the vessel e the apparatus is quietly tipped over after which it is returned to its upright position.

By a comparison of the observations obtained with this apparatus with those made with the compensating instrument it is found that one may consider as a sufficiently good approximation:

$$i_t = k L_t,$$

where I_t is the integral radiation for the time t and L_t is the increase in the height of the column of ether during the same time. The

If then i is the radiation in gr. cal. per cm.^{2} per min. and $i = F(t)$, where t is the time we shall evidently have

$$I_t = \int_0^t i\,dt = \int_0^t F(t)\,dt = kL_t.$$

If i is continually equal to i_0 during the time t, which is commonly
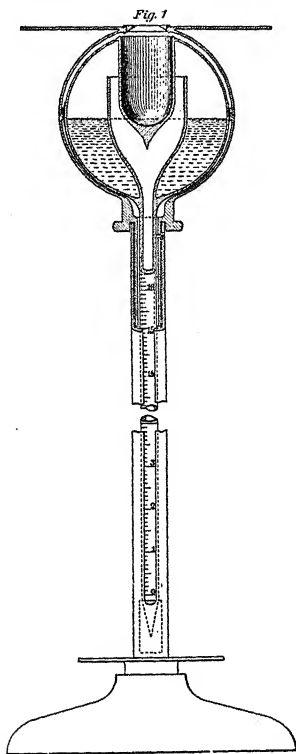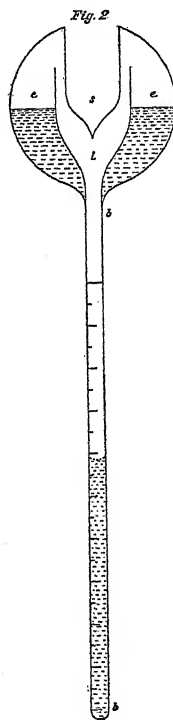
Fig. 1.   Fig. 2.

the case with nocturnal radiation during the greater part of the night, when the sky is clear, we get

$$i_0 = k\,\frac{L_t}{t}.$$

By means of the compensating instrument one may determine i_0 with sufficient accuracy. One may thus ascertain k once for all by simple comparison and the apparatus will serve for the measure-

ment of irradiation.　By multiplying the height of the column of ether by the constant k and by dividing the product by the time t one gets a mean value of the irradiation.　The following table shows good agreement between simultaneous observations made with the compensating instrument and with this condensing actinometer.　The observations were made in the Physical Laboratory at Upsala by Mr. F. Lindholm of that institution and by the author.

Date.	Temperature.	Instrument Used.	
		Condensing.	Compensating.
14 Sept.	$+10°$	8.2	8.4
27 "	$- 8°$	2.0	2.2
6 Oct.	$+10°$	8.9	9.0
30 "	$- 7°.5$	8.6	8.3
5 Nov.	$+ 3°.5$	7.3	6.9
7 "	$+ 2°$	8.3	8.2
8 "	$0°$	7.3	7.5
14 "	$- 5°$	7.6	7.9
15 "	$- 6°$	7.5	7.5
16 "	$-10°$	6.3	6.4
17 "	$- 9°$	7.0	7.0
27 "	$-13°$	6.1	6.1
21 Dec.	$-13°$	6.4	6.2

The influence of temperature is a question of great importance. One may see nevertheless from the table that for from $+ 10°$ to $- 13°$ this influence is not large.　It is clear from the foregoing that the apparatus possesses the following advantages:

1. It is of the simplest possible construction which permits of its being handled by everybody.

2. The influence of the heat by convection is reduced to a minimum; the lowering of the temperature of the surface being very small and the surface being protected from the direct action of the wind.

3. The irradiation takes place from a surface which approaches as nearly as possible what we understand as an absolute black body.

The instrument appears then to satisfy well all the conditions which we have spoken of above.　It is the first device which has been constructed for the determination of integral radiation and by its supreme simplicity it may prove of great use to aeronauts as well as in meteorology and in agriculture.

FORMATION OF OZONE.

REMARKS ON THE PAPER OF D. H. KABAKJIAN.

BY ARTHUR W. EWELL.

D. H. KABAKJIAN has an interesting article upon the formation of ozone in the August, 1910, number of this journal. He found that the yield of ozone per coulomb was practically independent of the current density. As he implies, this result appears contradictory to the observations of the writer, who found an optimum value of the current density, beyond which the yield decreased.[1]

This apparent discrepancy disappears, however, if one compares the current densities and polar distances employed by the two observers. Kabakjian used comparatively low current densities and small polar distances. The writer showed that the optimum current depended upon the polar distance and occurred at increasing current densities as the polar distance was decreased. The minimum air space employed by the writer was 4.5 mm., while the maximum of Kabakjian was 3.2 mm. If one compares the observations for these two distances (allowance being made for the greater area of the writer's electrodes), it is seen that even for the greater distance, the writer also found approximately constant yields of ozone per coulomb up to the maximum current density employed by Kabakjian. If allowance is made for the difference in air space, the two yields are practically identical. The rapid increase of yield, followed by a decrease, is only found for such small polar distances by increasing the current density beyond the maximum employed by Kabakjian.

As the writer has shown,[2] the electromotive force which is plotted in Fig. 5 of Kabakjian's paper and discussed on page 133 is very different from the actual electromotive force impressed upon the gas. It is difficult to apply his explanation of the influence of ultra-violet light to the great yield of ozone *outside* a quartz discharge tube.

WORCESTER, November 10, 1910.

[1] PHYS. REV., 1906, XXII., p. 232.

[2] Am. Jour. of Science, 1906, p. 368; Phys. Zeit., 1906, p. 927.

THE KINETIC PRESSURE-DROP CORRECTION IN THE TRANSPIRATION METHOD FOR GAS-VISCOSITY.

By WILLARD J. FISHER.

THE writer has elsewhere,[1] following the lead of workers in the subject of liquid viscosity, indicated the need of at least determining the order of magnitude of the pressure fall due to the velocity of the entering and emerging gas streams at the ends of a capillary tube, in the transpiration method of determining the viscosity of a gas. M. Brillouin[2] has given a formula for this correction, which the writer has criticised[3] as involving an incorrect view of the relation of the velocity to the initial pressure. The following is presented as an approximation to the correct method. and the formula is evaluated from data of the writer to find whether the correction is sensible as compared with the inevitable errors of observation.

In O. E. Meyer's[4] integration of the equations of the gas-capillary assumptions are made which lead to this formula for the velocity at any point in the tube:

$$u = \frac{p_2^2 - p_1^2}{2\eta pl}\left(\frac{r^2 - a^2}{4} - \frac{\zeta a}{2}\right),$$

in which p_1 and p_2 are the entrance and exit pressures, p the pressure at the point where the velocity is measured, η the viscosity coefficient, ζ the slip coefficient, l the length, a the radius of the tube, r the radius vector from the axis to the point in question. The mean value of u^2 for any normal cross-section is then

$$(\overline{u^2}) = \frac{1}{192}\left(\frac{p_2^2 - p_1^2}{\eta pl}\right)^2 [a^4 + 12\zeta^2 a^2 + 6\zeta a^3].$$

Applying this to the generalized form of Bernoulli's equation,[5]

[1] PHYS. REV., 29, p. 151, 1909.
[2] Leçons sur la Viscosité, II., p. 37.
[3] PHYS. REV., 29, p. 152, 1909.
[4] O. E. Meyer, Pogg. Ann., 127, p. 269, 1866.

$$V + P + q^2/2 = \text{const.},$$

in which V is the potential of external forces acting on the fluid, $P = \int dp/\rho = RT \log p + \text{const.}$ (if Boyle's law is assumed with isothermal expansion) and q^2 is the velocity squared, to be replaced in our case by $(\overline{u^2})$. The equations expressing the correction then become

$$RT \log (p/p') + \tfrac{1}{2}(\overline{u^2}) = 0,$$

with subscripts 1 or 2, for entrance or exit end of the tube. p' is the pressure at a point in the gas where the velocity is negligible, *i. e.*, the measured pressure given by the manometers.

In the expression for mean velocity the terms in brackets are of very unequal value. Using the dimensions of the writer's capillary, and known data for air at ordinary temperature and pressure, we have:

$$a = 5.5 \cdot 10^{-3}, \quad \zeta = 9.5 \cdot 10^{-6}.$$

The terms in brackets then become

$$a^4 = 9.2 \cdot 10^{-10}, \quad 12\zeta^2 a^2 = 3.3 \cdot 10^{-14}, \quad 6\zeta a^3 = 9.5 \cdot 10^{-13},$$

so that under the conditions of use of the writer's apparatus, and in general for similar apparatus, the first term is the governing one, or

$$RT \log (p_1/p_1') = -\frac{a^4}{384} [(p_1^2 - p_2^2)/\eta p_1 l]^2,$$

with a second similar equation with subscripts 2. Using this we may compute the value of $\log (p/p')$, taking as experimental data the values given in the author's tests on air, run no. 2.[1] In this p_1' was 96.457 cm. Hg, p_2' was 62.273, the temperature was 24° C., the logarithm of the pressure factor for converting centimeters of mercury into dynes per square centimeter was 4.1229, η was 0.000184, the length of the tube 49.6 cm., and R was taken $2.86 \cdot 10^6$. From these is found

$$\log_e (p_2'/p_2) = 0.000044, \quad \log_e (p_1'/p_1) = 0.000019.$$

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[1] PHYS. REV., 28, p. 104, 1909.

Hence p_2 is about 1/200 per cent. less than p_2', and p_1 is about 1/400 per cent. less than p_1'.   This is a first approximation; a second approximation is not necessary, as these values are smaller than the errors of observation.   With similar pressures, etc., the corrections for hydrogen gas would be about 56 times as large, and so not to be neglected.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE FIFTY-FIFTH MEETING.

THE annual meeting of the Physical Society was held in the physical laboratory of the University of Minnesota, Minneapolis, on December 28, 29 and 30, 1910, the meeting consisting of a series of joint sessions with Section B of the American Association for the Advancement of Science. During the morning session of December 28, and both sessions on December 29, the meeting was in charge of Section B, E. B. Rosa, vice-president of Section B, presiding. The afternoon session of December 28, and both sessions on December 30, were in charge of the Physical Society, President Henry Crew presiding, except during the afternoon session of December 30, when Professor B. W. Snow acted as temporary chairman.

The annual business meeting of the Physical Society was held during the afternoon session of December 28. In the absence of the Secretary M. G. Lloyd was elected secretary *pro tem.* Messrs. J. F. Hayford and W. J. Humphreys were appointed tellers to canvass the vote for officers. The following were elected:

President: W. F. Magie.

Vice-president : B. O. Peirce.

Secretary : Ernest Merritt.

Treasurer : J. S. Ames.

Members of the Council : R. A. Millikan and John Zeleny.

The president-elect and the secretary *pro tem.* were elected representatives of the Physical Society on the Council of the American Association.

During the afternoon session of December 29, Section D of the American Association met with Section B and the Physical Society and the addresses of the retiring vice-presidents of the two sections were read, the titles being:

The Broader Aspects of Research in Terrestrial Magnetism. By L. A. BAUER, vice-president of Section B.

The Relation of Isostasy to Geodesy, Geology, and Geophysics.   By J. F. Hayford, vice-president of Section D.

The morning session of December 29, in charge of Section B, was devoted to addresses on subjects of interest to the members of more than one section of the American Association.   The program was as follows:

Recent Advances in Phosphorescence and Fluorescence.   Edw. L. Nichols.

The Isolation of Ions.   R. A. Millikan.

The International Electrical Units.   (Report on changes to go into effect January, 1911.)   E. B. Rosa.

Osborne-Reynolds' Theory of Gravitation.   John Mackenzie.

In addition to the addresses mentioned above the following papers were presented during the course of the meeting:

The Ratio of the Two Heat Capacities of Carbon Dioxide as a Function of the Pressure and the Temperature.   A. G. Worthing.

On the Adiabatic Expansion and Porous Plug Effect in Water between 0° and 40°.   John R. Roebuck.

Ocean Currents and Barometric Highs.   W. J. Humphreys.

The Acoustic Shadow of a Sphere; the Theory and Certain Practical Applications.   G. W. Stewart.

On the Function of Rest in Restoring a Platinum-iridium Wire to its Annealed Condition.   L. P. Sieg.

The Nature of the Recovery of Light-positive and Light-negative Selenium.   F. C. Brown.

On the Fluorescence Spectra of the Uranyl Salts and the Structure of Luminescence Spectra in General.   Edward L. Nichols and Ernest Merritt.

The Effect of Red and Infra-red Rays on the Decay of Phosphorescence in Zinc Sulphide.   Herbert E. Ives and M. Luckiesh.

Experimental Indications of the Nature of Magnetism.   S. R. Williams.

The Effect of Magnetization on the Optical Constants of the Magnetic Metals.   L. R. Ingersoll.

A Preliminary Report of Experiments on the Effect of Changes in the Pressure of Gases upon the Mobility of Positive and Negative Ions Produced by the Discharge of Electricity from a Fine Point.   E. J. Moore.

On the Presence in Point Discharge of Ions of Opposite Sign.   John Zeleny.

The Coefficient of Recombination of Ions in Carbon-dioxid and Hydrogen.   Harry A. Erikson.

The Light-emission of Tungsten, Tantalum, and Carbon as a Function of their True Temperature.   C. E. Mendenhall and W. E. Forsythe.

The Pentane Lamp as a Primary Light Standard. E. B. Rosa and E. C. CRITTENDEN.

The Second Postulate of Relativity. O. M. STEWART.

A Method of Measuring the Fluctuations in a Rapidly Varying Resistance. F. C. BROWN and W. H. CLARK.

The Causes of Zero Displacement and Deflection Hysteresis in Moving-coil Galvanometers. ANTHONY ZELENY.

A Probable Explanation of the Irregularities Obtained by Ehrenhaft and Przibram in their Work on the Value of the Elementary Electrical Charge. HARVEY FLETCHER.

Diffraction and Secondary Radiation with Short Electric Waves. A. D. COLE.

Notes on the Discharge of Electricity through Gases. G. S. FULCHER.

The Free-expansion and Joule-Kelvin Effects in Air and in Carbon-dioxide. A. G. WORTHING.

Ocean Currents and Barometric Lows. W. J. HUMPHREYS.

Note on a Form of Spectrophotometer. EDWARD L. NICHOLS and ERNEST MERRITT.

On the Temperature Coefficients of the Free and the Absorbed Charges in Electric Condensers. ANTHONY ZELENY.

The Effect of Wave Form upon Incandescent Lamps. M. G. LLOYD.

The Question of Valency in Gaseous Ionization. R. A. MILLIKAN and HARVEY FLETCHER.

A Kinetic Theory of Gravitation. CHARLES F. BRUSH.

On a Variation in the Intensity of the Penetrating Radiation at the Earth's Surface Observed during the Passage of Halley's Comet. A. THOMSON.

On the Resolution of the Spectral Lines of Mercury. J. C. McLENNAN and E. N. MACALLUM.

Terminal Velocity of Fall of Small Spheres in Air at Reduced Pressure. L. W. McKEEHAN.

The Effect of Distance upon the Electrical Discharge between a Point and a Plane. O. HOVDA.

A Lecture Electroscope for Radioactivity. JOHN ZELENY.

A Variable High Resistance of India Ink on Paper. FRANZ A. AUST.

The Electrical Discharge between a Pointed Conductor and a Hemispherical Surface in Gases at Different Pressures. FRANZ A. AUST.

The Transmission of Excited Radioactivity. E. M. WELLISCH.

Infra-red Gratings. C. F. BRACKETT and A. TROWBRIDGE.

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M. G. LLOYD,
Secretary pro tem.

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On the Absorption of the γ Rays of Radium by Solids.[1]

By S. J. Allen.

THERE is considerable discussion at the present time as to whether the γ rays are homogeneous or not. Work done by Soddy and Russell seems to point to the conclusion that after the secondary disturbing effect has been eliminated the γ rays are homogeneous. The correctness of this view has been questioned by Kleeman. In the past, experimenters on this subject have differed quite widely in their results, some reaching the conclusion that the rays after having passed through a sufficient layer of material are homogeneous, and that λ/d is a constant, whereas others obtained results showing that λ/d is not a constant, even after very thick layers have been traversed.

The difficulty in interpreting the various results evidently arises from the fact that it is hard to decide just what the effect of secondary rays is.

During the past year and a half the author has worked from time to time on the subject, and the results obtained are incorporated in the present paper.

The apparatus and the method of procedure used is very simple, and is illustrated in the accompanying figure. *A* represents a brass box, open at the top and supporting a gold leaf electroscope *B*. The electroscope is of the usual type, with guard ring, and sulphur insulation, the fall of the gold leaf through a fixed distance being observed by means of a reading microscope.
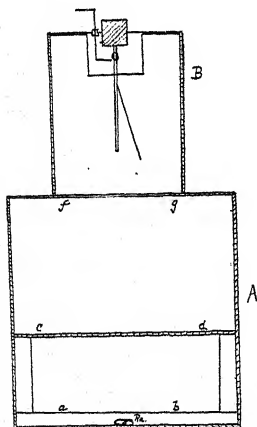
Fig. 1.

The radium was placed at the bottom of the brass box and covered with a sheet of lead of about 7 mm. thickness. At a variable distance was placed a horizontal platform of thin aluminum, *cd*. The entrance to the electroscope *fg* was covered in some cases with a layer of thin paper, and in others by a lead plate of about 7 mm. thickness.

The readings of the electroscope were correct to at least one half of one per cent. Since the value of λ was generally calculated from two readings of the ionization not differing by more than 10 per cent., the error in it might be as great as five per cent. In a great many cases it would be less.

A series of tests under various conditions were made for both solids and liquids. These tests were made under the three following conditions:

1. When the absorbing layer was placed at *ab* directly over the radium and gradually increased in thickness, values of λ being calculated for each succeeding layer.

2. When the absorbing layer was placed at *cd*, about 5 cm. from *ab*, and gradually increased in thickness, as in (1).

3. When both (1) and (2) were repeated with a layer of lead of 7 mm. placed over the electroscope opening *fg*.

Also in (2) and (3) the absorbing layer was kept constant and gradually increasing layers of various materials was placed under it.

A typical series of results under these three conditions is given in the following Table I.

An examination of these results leads to the following conclusions:

(*a*) Under condition (1) the values of λ/density are very irregular and far from constant. λ itself for any substance is not constant but varies both with the thickness and the position of the absorbing layer. For low atomic weight substances λ increases with the thickness, but for those of high atomic weight it decreases. For substances having an atomic weight of about 100 there is very little change.

When the radium is covered with a thick layer of lead the emerging rays are hardened (λ smaller) but when they pass through a thick layer of iron they are softened (λ greater).

(*b*) Under condition (2) the values of λ/density are more regular, and for those substances having atomic weights up to about 100 this ratio is nearly a constant. λ still decreases with thickness for high atomic weight substances, but more slowly than before, while for low atomic weight substances it is nearly constant.

(*c*) Under condition (3), the values of λ/density are still better, though those for atoms under 100 do not differ much from those of (2). The values for lead, mercury and bismuth are much lower than those of (2). It was found that the values of λ for lead at different thicknesses still decreased; for example, after passing through 50 mm. of lead the value of λ had changed from .554 to .398. On the other hand, the value of λ for lead, after the rays had passed through 60 mm. of iron, was .539 or about the same as when the iron was not present.

(*d*) Bismuth always gives unusual values; and λ/density for it is in general greater than due to lead, which fact must certainly be accounted for by its high atomic weight.

(*e*) If we leave out the high atomic weight elements, lead, mercury and bismuth, we find that the average value of λ/density is .0382. The value given by Soddy and Russell is .0399. It is thus seen that the apparent value of λ depends greatly on the experimental disposition used, and on the atomic weight of the absorbing substance. The density has no apparent effect upon the value of λ.

Table I.

Ra Covered with 7 mm. Lead.

Substance.	Thickness in mm.	λ	$\frac{\lambda \times 100}{\text{Density}}$
Platinum	0– 1.0	1.043	4.85
Mercury.....	0– 4.0	.669	4.92
Mercury.....	16–20	.466	3.43
Palladium ...	0– 1.0	.471	4.09
Bismuth.....	0– 3.25	.515	5.25
Lead........	0– 3.0	.621	5.44
Lead........	18–21	.501	4.39
Silver	0– 3.2	.344	3.27
Copper......	0– 6.0	.242	2.72
Copper......	15–18	.302	3.39
Wrought iron	0– 3.0	.2.6	2.64
Wrought iron	15–18	.291	3.73
Nickel.......	0– 4.0	.252	2.95
Sulphur	0– 6.9	.0774	3.85
Aluminum ...	0– 8.7	.0736	2.60

Ra Covered with 24.5 mm. Lead.

Substance.	Thickness in mm.	λ	$\frac{\lambda \times 100}{\text{Density}}$
Bismuth.....	0– 3.25	.464	4.73
Silver	0– 3.2	.344	3.27
Copper......	0– 2.15	.323	3.63
Wrought iron	0– 6.1	.245	3.14
Wrought iron	18–24	.249	3.19
Lead........	0– 3.36	.494	4.33

Ra Covered with 7 mm. Lead + 15 mm. Iron.

Substance.	Thickness in mm.	λ	$\frac{\lambda \times 100}{\text{Density}}$
Lead	0– 3.36	.769	6.75
Silver........	0– 3.2	.508	4.84
Copper	0– 2.15	.448	5.03

Ra Covered with 7 mm. Lead and Screen 5 cm. Away.

Substance.	Thickness in mm.	λ	$\frac{\lambda \times 100}{\text{Density}}$
Mercury	0– 4.08	.660	4.89
Bismuth	0– 3.25	.607	6.19
Lead	0– 3.36	.664	5.82
Silver........	0– 3.2	.442	4.21
Copper	0– 3.35	.331	3.72
Copper	10–13.4	.313	3.52
Wrought iron .	0– 6.1	.296	3.79
Cast iron.....	0–10.6	.259	3.67
Sulphur.......	0– 6.9	.0897	4.46
Nickel........	0– 4.0	.357	4.49
Aluminum	0– 6.83	.110	3.88

Same with Electroscope Covered by 6.6 mm. Lead.

Substance.	Thickness in mm.	λ	$\frac{\lambda \times 100}{\text{Density}}$
Lead	0– 3.36	.554	4.86
Mercury	0– 2.04	.621	4.58
Mercury	6.12–12.24	.554	4.07
Bismuth	0– 3.25	.496	5.06
Silver........	0– 3.2	.421	4.00
Wrought iron .	0– 6.1	.278	3.58
Cast iron.....	0–10.1	.262	3.72
Copper	0– 3.33	.333	3.74
Copper	10.1–13.4	.331	3.72
Sulphur.......	0– 6.9	.0790	3.93

It is difficult to estimate the part played by the secondary rays, but in many cases it is large. Whether this effect can be entirely eliminated by any special experimental disposition is still open to question.

University of Cincinnati,
　　November 15, 1910.

On the Absorption of γ Rays of Radium by Liquids.[1]

By S. J. Allen.

THESE experiments were made in exactly the same manner as those for solids, described in this same issue. The liquid was placed in a cylindrical glass cell, and the cell situated at a distance of about 5 cm. from the radium. The radium was covered with 7 mm. of lead.

Before a reading was taken enough liquid was poured into the cell to cover the glass bottom and thus ensure a level surface to measure the thickness from. The thickness was measured both directly and by means of a calibrated pipette.

Two series of tests were made: (1) When the bottom of the electroscope was covered with a very thin layer of paper. (2) When the bottom was covered with 7 mm. of lead. The results of these two tests are shown in the following table. Both pure liquids and saturated aqueous solutions of the salts were tested.

Table I.

Cell Placed 5 cm. from Ra.				Electroscope Covered with 7 mm. Lead.			
Substance.	Thickness in mm.	λ	$\frac{\lambda \times 100}{\text{Density}}$	Substance.	Thickness in mm.	λ	$\frac{\lambda \times 100}{\text{Density}}$
Water	0–20.4	.0462	4.62	Water	0–18.4	.0465	4.65
Ethyl alcohol	0–20.0	.0352	4.40	Amyl alcohol	0–18.3	.0355	4.37
Amyl bromide	0–20.0	.0544	4.50	Amyl bromide . . .	0–18.3	.0497	4.11
Carbon tetra bromide	0–14.8	.1144	3.73	Iod benzene	0– 6.5	.074	4.05
Iod benzene	0–20.0	.0783	4.28	Chloroform	0–18.4	.0603	4.06
Ether	0–20.0	.0326	4.46	Sulphuric acid . . .	0–18.4	.0757	4.11
Ethyl iodide	0–12.0	.0860	4.52	Lead nitrate	0–18.4	.0652	4.82
Methyl iodide	0– 8.0	.0907	4.12	Lead nitrate	0–32.7	.0636	4.68
Brom benzene	0–20.0	.0607	4.04	Potassium iodide	0–18.4	.0658	4.17
Ethelene bromide .	0–18.0	.0870	3.95	Potassium iodide	0–26.5	.0653	4.21
Meta brom anilene	0–20.0	.0731	4.56	Copper chloride .	0–18.4	.0609	4.22
Propyl bromide . . .	0– 9.0	.0617	4.54	Copper chloride .	0–32.7	.0612	4.25
Iso amyl bromide .	0–19.0	.0685	4.10	Cadmium iodide .	0–18.4	.0549	4.28
				Bismuth tri chl. .	0–18.4	.0465	4.33
Lead nitrate	0–20.0	.0754	5.57				
Potassium iodide .	0–20.0	.0760	4.81				
Copper chloride . . .	0–20.0	.0594	4.12				

From an examination of this table it can be seen that the results for liquids are very similar to those for solids. For those liquids not containing

atoms of high atomic weight λ/density is nearly constant, but for the liquids of high atomic weight this ratio is much larger. The value for lead nitrate is practically the same as for solid lead. Excluding lead nitrate and water the ratio λ/density in the (2) case is 4.18.

A New Formula for the Vapor Tension of Water between 0° and 200° C.[1]

By K. E. Guthe and A. G. Worthing.

THE pressure of saturated water vapor has recently been very accurately determined for the interval from 50° to 200° C. by Holborn and Henning,[2] and for the interval from 0° to 50° C. by Scheel and Heuse.[3] Thiesen's empirical formula

$$(t + 273) \log \frac{p}{760} = 5.409(t - 100) - 0.508 \times 10^{-8}(365 - t)^4 - 265^4 \quad (1)$$

represents very closely the results between 0° and 100°, but leads to increasing differences at higher temperatures, reaching the large value of 0°.35 C. at 200°. Holborn and Baumann[4] propose the following formula for the interval between 100° and 200°:

$$(t + 273) \log \frac{p}{760} = 5.3867(t - 100) - 0.5262 \times 10^{-8}(365 - t)^4 - 265^4. \quad (2)$$

This gives a very good agreement between observed and calculated values for the range of temperature in question, but the equation does not fit as well as Thiesen's formula for the lower temperatures. Nernst[5] proposes a still more complicated equation, which, though very interesting from a theoretical point of view, can hardly be considered as of value for rapid calculation.

We have found the following simple formula to be very convenient and of great accuracy

$$\log p = 7.39992 - \frac{\log^{-1} 3.949046}{\theta^{1.2806}}, \quad (3)$$

where p is the pressure of the saturated vapor and θ the absolute temperature ($= 273°.1 + t$). This equation has not only the great advantage of simplicity

[1]Abstract of a paper presented at the Chicago meeting of the Physical Society, November 26, 1910.

[2]Holborn and Henning, Ann. Phys., 26, 833, 1908.

[3]Scheel and Heuse, Ann. Phys., 31, 715, 1910.

[4]Holborn and Baumann, Ann. Phys., 31, 945, 1910.

[5]Nernst, Verh. d. d. Phys. Ges., 12, 565, 1910.

of form, as compared with those mentioned, but it also fits the observed vapor-tension curve in the whole interval between 0° and 200° C. more closely than any of the others.

Table I. allows a comparison between the three equations. Under observed values are given those of Scheel and Heuse from 0° to 50°. But since the value of the pressure at 50°, found by them, namely 92.54 mm. of mercury, exceeds the corresponding value given by Holborn and Henning, by 0.24 mm., it seemed best to increase the values, given by these experimenters for 60° and 70°, by 0.16 mm. and 0.08 mm. respectively. For 80° and 90° Holborn and Henning's values were assumed to be correct.

TABLE I.

Temp.	*p*, obs.	Thiesen.		Holborn a. Baumann.		Guthe a. Worthing.	
		p, calc.	Δ*p*	*p*, calc.	Δ*p*	*p*, calc.	Δ*p*
	mm.	mm.				mm.	
0	4.579	4.586	0.007	4.577	−0.002	4.579	0
10	9.210	9.214	0.004	9.207	−0.003	9.207	−0.003
20	17.539	17.544	0.005	17.546	0.007	17.539	0
30	31.834	31.841	0.007	31.839	0.005	31.835	0.001
40	55.341	55.350	0.009	55.406	0.065	55.341	0
50	92.54	92.55	0.01	92.66	0.12	92.54	0
60	149.35	149.42	0.07	149.60	0.15	149.41	0.06
70	233.61	233.71	0.10	233.96	0.35	233.74	0.13
80	355.1	355.15	0.05	355.45	0.35	355.14	0.04
90	525.8	525.79	−0.01	526.02	0.22	525.76	−0.04

In Table II. the differences between the observed values of *t* for the interval from 0° to 200° and the values calculated from our formula are given.

TABLE II.

Temperature.	0°	10°	20°	30°	40°	50°	60°
$t_{obs.} - t_{calc.}$	0	−0°.005	0	+0°.001	0	0	+0°.009

Temperature.	70°	80°	90°	100°	110°	120°	130°
$t_{obs.} - t_{calc.}$	+0°.012	+0°.003	−0°.002	0	+0°.01	+0°.02	+0°.03

Temperature.	140°	150°	160°	170°	180°	190°	200°
$t_{obs.} - t_{calc.}$	+0°.05	+0°.05	+0°.06	+0°.05	+0°.04	+0°.01	−0°.03

Only at one temperature, 160°, the difference between the observed and calculated values reaches 0°.06; at all other temperatures it is not larger

than 0°.05, and between 0° and 100° the maximum difference is only a little over 0°.01 at 70°. Our formula represents therefore very closely the experimental results.

ELASTIC PROPERTIES OF BISMUTH WIRES.[1]

BY K. E. GUTHE AND J. E. HARRIS.

A FEW years ago one of the authors found that platinum-iridium wires, containing a large percentage of iridium, show an abnormal behavior when subjected to a torsional strain. Further experiments with wires of different material have shown that this exceptional behavior is not restricted to platinum-iridium wires, but that it is found in wires of other material, and that it is especially pronounced in the case of bismuth. The results of a preliminary set of readings, taken with a bismuth wire, 458 mm. long and 0.28 mm. in diameter, are given in the following table.

TABLE I.

No. of Swings.	Amplitude.	Amplitude.	Period.
1	91.7	79.9	3.950
5	58.2	74.1	.948
10	48.9	68.2	.947
15	35.6	55.7	.937
20	26.6	48.9	.932
25	19.7	35.6	.919
30	15.0	26.6	.909
35	11.9	19.7	.896
40	9.2	15.0	.891
45	7.2	11.9	.886
50	5.6	9.2	.879
55	4.5	7.2	.872
60	3.4	5.6	.870
65	2.6	4.5	.857
70	2.0	3.4	.853
75	1.5	2.6	.850

As in the case of platinum-iridium wire, reported on by Guthe and Sieg[2] the period decreases considerably with decrease of amplitude, the rate of decrease being most pronounced at small amplitudes. In the case of bismuth wire the internal friction is, however, very much larger. During 75 vibrations the amplitude decreased from 91.7 to 1.5 degrees, while in the case of the 40 per cent. platinum-iridium wire[3] it required nearly 400 vibrations, before

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, November 26, 1910.

[2] Guthe and Sieg, PHYS. REV., 30, 610, 1910; Sieg, PHYS. REV., 31, 421, 1910.

[3] PHYS. REV., 30, 624, 1910.

the amplitude fell from 83.1 to 3.93 degrees.   The logarithmic decrement for the bismuth wire is between 0.03 and 0.02, or six times larger than that of the platinum-iridium wire.

On account of the small number of swings it is impossible to calculate the period from a set of 20 or 40 swings, as was done in the former case, and therefore, no great accuracy can be claimed for the periods, as given in the above table.   We hope to overcome the difficulty by employing in our further work with bismuth wires an automatic device for marking the exact time of the passage of the moving system through the position of rest.

The Spectrum of the Spark under Pressure and an Application of the Results to the Spectrum of the Chromosphere.[1]

By Henry G. Gale and Walter S. Adams.

A CONSIDERABLE number of photographs of spark spectra under pressure were taken by us last spring at the spectroscopic laboratory of the Mt. Wilson Solar Observatory.

The spark was generated by a 5-kilowatt transformer, and differences of potential up to 60,000 volts were available.   No self induction was employed, and a considerable amount of capacity was used in the secondary circuit.   Most of the photographs were taken in the second order of a 7-inch Michelson grating, mounted in a 30-ft. Littrow spectrograph.   The pressure varied from 2 to 15 atmospheres.

The first photographs obtained were of titanium in the region from $\lambda 4250$ to $\lambda 4650$.   At pressures of 5 or 6 atmospheres practically all of the arc lines were reversed, and all of the enhanced lines remained bright, or showed only a faint trace of reversal which did not increase appreciably up to 15 atmospheres.   A similar result was obtained to the violet of $\lambda 4250$.   The enhanced lines were practically the only ones to remain bright.   In the green, yellow and red, however, a number of arc lines were found which did not reverse.   This is in agreement with the well-known general result that the tendency of arc lines to reverse increases toward the violet.

A similar series of photographs with iron and chromium gave in general the same result, except that the arc lines did not reverse so readily at the lower pressures.   In no case were the enhanced lines reversed at the pressures used.

In the case of cobalt, a plate from $\lambda 5000$ to $\lambda 5400$ taken at a pressure of 8 atmospheres showed no reversed lines, although iron, titanium and chromium showed many reversed arc lines in this region.

It has long been known that the enhanced lines are particularly prominent

[1]Abstract of a paper presented at the Chicago meeting of the Physical Society, November, 26, 1910.

in the bright line spectrum of the chromosphere, and the explanation of this has been one of the most difficult problems connected with the commonly accepted views of the nature of the sun's reversing layer. The laboratory results suggest a possible explanation.

As Fabry and Buisson have recently shown, the pressure in the reversing layer is about 5 or 6 atmospheres. In view of the laboratory results we should expect the enhanced lines to be more conspicuous as bright lines in the chromospheric spectrum, than as dark lines in the Fraunhofer spectrum, *i. e.*, they should appear as bright lines when a majority of the arc lines are still dark. This appears to be the case. Moreover, in the regions of longer wave-length many arc lines appear as bright in both laboratory and chromospheric spectra. Cobalt is remarkable for the number of bright lines present in the chromosphere, and the lines were not reversed on the laboratory plates under conditions which gave many reversals for titanium, iron and chromium.

This investigation is as yet incomplete but the results so far obtained point toward the conclusion that the conspicuousness of the enhanced lines in the chromospheric spectrum is intimately connected with the fact that they do not reverse so readily as the arc lines. It seems probable also that cobalt is conspicuous for the reason that its lines are not so readily reversed as the lines of many other elements.

Distribution of Discharge between a Point and Plane under Varying Pressures.[1]

By Robert F. Earhart and F. W. Pote.

AN electrified point, which may be either positive or negative, is discharged against a plane one centimeter distant. The plane is formed by ten concentric, insulated rings of brass. Proper connections permit the current discharging through any ring to be measured without interfering with the discharge through the remaining rings. The electrical system is contained in a vessel which may be evacuated, in order that the influence of pressure as well as voltage may be determined. Curves indicate the total current under varying conditions of potential differences and pressures as well as the distribution of the discharge.

Physical Laboratory,
Ohio State University.

A Comparison of the Influence of Planes of Transverse Section on the Magnetic Properties of Iron and of Nickel Bars.[1]

By E. H. Williams.

T HE results of Maurain[2] indicate that, for very thin layers, the magnetic properties of iron and of nickel are different. Ewing and Low[3] have shown that the influence of a plane of transverse section on the magnetic induction of an iron bar is to produce a large magnetic resistance. Now, if the surface layers of nickel have different magnetic properties from those of iron, it would appear that the effect of transverse joints would be different in the two metals.

In this investigation, the effects of surface layers in bar magnets of iron and of nickel, produced by cutting the bar in planes of transverse section, have been studied and compared under various conditions of field strength and longitudinal pressure.

The final results are in agreement with what one should expect from the observations of Maurain on the magnetic properties of very thin layers of iron and nickel. Whereas the effect of transverse joints in iron is always to produce a decrease in the magnetic induction whatever the value of the field strength or of the pressure, in case of nickel the results are very different. In this metal, beyond magnetic fields of 25, the effect of transverse joints is to *increase* the magnetic induction. The paper contains data and curves showing the effect of transverse joints under various conditions of pressure and field strength.

University of Illinois,
Urbana, Ill.,
November 2, 1910.

Echoes in an Auditorium.[1]

By F. R. Watson.

A N investigation of the acoustics of the University of Illinois Auditorium has been carried on for more than two years. The auditorium is shaped nearly like a hemisphere, with several large arches breaking the regularity of the surface. The dimensions of the building are great enough to allow echoes to exist in addition to a reverberation.

By Sabine's method, the time of duration of the residual sound was found to be a little more than six seconds. The absorbing power of audiences

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, November 26, 1910.

[2] Maurain, Jour. de Phys. (4), Vol. 1, p. 151, 1902..

[3] Ewing and Low, Phil. Mag. (5), Vol. 26, p. 274, 1888.

was also determined, and a calculation made to determine the amount of absorbing material needed to remedy the reverberation. There still remained the problem of curing the echo.

In the dearth of definite information about echoes, several methods were used to trace the path of sound in the auditorium. An attempt to locate echoes by generating a sound and listening with the ear met with only partial success. The ear is sensitive enough, but is confused when many echoes are present, so that the evidence obtained is not altogether conclusive.

The next step was to use a faint sound which could not be heard at any great distance unless reënforced by reflected sound. The ticks of a small clock were directed, by means of a reflector, to certain walls suspected of giving echoes. Using the relation that the angle of incidence equals the angle of reflection, the reflected sound was readily located, and the clock ticks heard distinctly at points as far as seventy feet from the clock.

In another method, a louder source of sound, namely, the ticks of a metronome, was used but enclosed in a sound-proof structure except for one opening, so that this sound could be directed by means of a horn. The method was successful and verified the observations taken previously.

The most satisfactory method, however, involved the use of an alternating arc light at the focus of a parabolic reflector. In addition to the light, the arc gave two sets of sounds; one sound being a hum due to the alternations of the current, and the second sound a successive "spitting" of the arc. This latter sound was of short wave-length and therefore experienced little diffraction. It was also of suitable intensity to allow the reflected sound to be heard easily. The bundle of light rays included also a bundle of sound rays, the sources of both being at the same place and subject to the same law of reflection. The path of the rays was easily found. The observer could see where the sound rays struck by noting the position of the spot of light. To trace successive reflections, small mirrors were fastened to the walls and the path of the reflected light followed. This method allowed a complete "diagnosis" of the acoustic troubles of the auditorium to be made, so that methods of cure may now be applied intelligently and with confidence.

Several methods of cure are under consideration, and this phase of the work is being actively pushed. The author acknowledges the valuable assistance given by Mr. Otto Stuhlmann, now fellow at Princeton University, during the early stages of the investigation.

UNIVERSITY OF ILLINOIS,
  URBANA, ILL.,
    October 26, 1910.

LIMITATIONS IMPOSED BY SLIP AND INERTIA TERMS UPON STOKES'S LAW FOR THE MOTION OF SPHERES THROUGH LIQUIDS.[1]

BY H. D. ARNOLD.

THE production of small metal spheres has made possible the verification of Stokes's law for the terminal velocities of spheres in liquids of less viscosity than those with which consistent results have hitherto been obtained. Experiments with a series of spheres of different radii give the rate of departure of the observed velocities from those computed, thus defining experimentally the range within which the formula may be considered valid.

From similar observations has been demonstrated the validity of Ladenberg's correction for the influence of the surrounding walls, and its limits of applicability have been determined.

Viscosity determinations with these spheres and by Poiseuille's method give values whose differences are smaller than the probable error involved in either of the methods.

The coefficient of sliding friction at the interface of the spheres and the oils used is shown to be greater than 5,000, and with the more viscous oils greater than 20,000.

Observations on steadily diminishing air bubbles in various liquids have shown the dependence of the apparent coefficient of sliding friction on the nature of the liquid and the radius of the bubble. It has been found possible to obtain conditions of approximately infinite surface slip and thus to verify the more general form of Stokes's law in which the slip terms are not neglected. An explanation of the irregular results hitherto obtained has been found in the film forming properties of the liquids used.

RYERSON PHYSICAL LABORATORY,
CHICAGO, ILL.

ON THE FREE VIBRATIONS OF A LECHER SYSTEM USING A BLONDLOT OSCILLATOR.[1]

BY F. C. BLAKE AND MISS E. A. RUPPERSBERG.

IT was shown in this paper how it is possible on a set of parallel wires to get a system of free vibrations uncontaminated by oscillator influence except that of coupling. It was found that the unmodified theory fitted even the case of close coupling to a first approximation. The manner in which the theory has to be modified for a second approximation was pointed out. The conditions for freedom from contamination were found to be two, the introduction of suitable air gaps into the oscillator circuit and suitable

[1]Abstract of a paper presented at the Chicago meeting of the Physical Society, November 26, 1910.

water-resistance between the induction coil and oscillator. The results of varying these conditions were studied. A ratio between the fundamental maximum and minimum equal to 125 to 1 was experimentally obtained and it was thought that this ratio could be materially increased by observing the best conditions for controlling the various factors that enter into the problem.

The ratio between Lecher wire length and wave length was found to be 2.07 and the difference between this value and the theoretical value 2.00 was shown to be due to an "end correction" which is necessarily always present.

PHYSICS LABORATORY,
OHIO STATE UNIVERSITY,
November 3, 1910.

THE PRODUCTION OF LIGHT BY CANAL RAYS.[1]

BY GORDON SCOTT FULCHER.

A COMPARISON of the results of the simultaneous eloctrostatic and magnetic deflection of the canal rays with the details of the Stark effect leads to the conclusion that the canal rays cannot be the sources of the light showing this effect. An alternative hypothesis was suggested by the author two years ago, that the light in question is emitted by gas molecules hit and ionized by the canal rays. Recently more direct experimental evidence for this conjecture has been obtained. A discharge tube was so designed that the pressure of the gas through which the canal rays pass can be varied in the ratio of 1 to 20 without changing the number or the velocity of the canal rays. Photographic measurements of the intensity of the light from the path of the rays showed it to vary directly as the pressure. Also, measurements of the energy transported by the rays (by means of their heating effect) and of the intensity of the light emitted, showed that for a cathode fall of potential between 1,500 and 5,000 volts the relation $I = kEp$ holds within experimental errors. That is, the intensity of the light emitted is proved to be proportional to the number of collisions times the mean energy transmitted at each collision. A statistical calculation has been made which tends to show that the intensity minimum observed between the displaced and rest lines in the Stark effect can be explained on the basis of the emission of the series line spectrum as a result of molecular collision only if we assume that no light is emitted unless the hit molecule is ionized by the collision and that the bombarding molecules do not emit the series line spectrum. This is equivalent to assuming that the series spectrum lines which show the Stark effect are emitted only by positively charged molecules or atoms—a law for which there is other evidence but which has been much disputed.

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, November 26, 1910.

On the Free Vibrations of a Lecher System Using a Lecher Oscillator.[1]

By F. C. Blake and Charles Sheard.

TWO forms of Lecher system were employed; one after the manner of Lecher, using 5 cm. plates attached to the oscillator circuit and 3 cm. plates connected to the wire system proper. A second form, consisting of cylinders, cone-shaped at one end, was used as a part of the oscillator circuit; this allowed the coupling of the two circuits to be readily changed. A similar device was made use of at the bolometer end of the circuit. A check receiver, of constantan-iron wires, shunted through a high resistance across the spark-gap in oil, was employed for the purpose of checking the energy delivered to the Lecher circuit. By its use it was possible to check readings at any given point on the system to within 1 per cent. at any time.

The main points of the work as presented were:

1. A first approximation of the ideal case as to the distribution of energy as given by Abraham[2] was obtained. The theory was found to hold for the first, the third (2 per cent. error due to experimental errors) and the second harmonics, these last being in error by 8 per cent. due to the influence of the oscillator vibrations also imposed upon the system.

2. The ideal case ceases to be such just in proportion as the bridge length is more and more comparable to the Lecher wire length. Curves for 2, 5 and 11.5 cm. distance between parallel wires were shown; for this last distance the vibrations ceased to be of a simple nature.

3. It is known theoretically[3] that such a system has two superimposed oscillations; those due to the oscillator and those which are the free vibrations proper. The effect of coupling (using the second form of coupling and check receiver) on each of the sets of vibrations was investigated. It was found that any particular overtone could be strengthened by so varying the coupling that the oscillator vibrations fell in unison with it.

4. It was found that for bridge lengths small in comparison to the Lecher wire length the currents behaved *as if* they were longitudinal and not transverse. The proof of this is based upon an experimental verification of Abraham's[4] theoretical considerations that for longitudinal currents the potential varies as the logarithm of the distance from the wire.

5. The Kirchhoff-Abraham generalization of the Thomson formula[5] was verified to within 1 per cent. in the cases where the tones were strictly harmonic.

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, November 26, 1910.

[2] Theorie der Elektrizität, Vol. 1, second edition, equation 224, p. 352.

[3] Bjerknes, Wied. Ann., 1895, Vol. 55, p. 121.

[4] Theorie der Elektrizität, Vol. 1, p. 346.

[5] See Theorie der Elektrizität, Vol. 1, equation 225d, p. 354.

6. It was found that there is a "back action" of the Lecher circuit upon the oscillator circuit. Curves were shown of results obtained proving such a back action. Since the coupling was electrostatic, the need seems apparent for the introduction of the term "mutual capacity" into such electrical work.

Physics Laboratory,
Ohio State University,
November 3, 1910.

On the Ionization of Gases by the Alpha Particles from Polonium.[1]

By T. S. Taylor.

IN an article on the "Retardation of Alpha Rays by Metals and Gases,"[2] the writer pointed out some differences that had been observed between the Bragg ionization curves obtained in atmospheres of air and hydrogen respectively. The observation of these differences suggested that a careful delineation and comparison of the Bragg ionization curves in other gases, when polonium was used as the source of rays, might lead to interesting results. The same apparatus that had been used in the previous experiment[3] was used, and with polonium as the source of rays, Bragg ionization curves were obtained in atmospheres of air, H_2, O_2, N_2, CO_2, CH_3I, C_2H_5Cl, CH_4, $C_4H_{10}O$, SO_2, HCl, HBr and HI. In each case the pressure the gas or vapor supported was always such as to have the range of the alpha particle 11.1 cm. By taking the areas under the curves as the measures of the ionization in the gases respectively and comparing the areas separately with that of the air curve, ratios of the ionization produced in the different gases to that produced in air were obtained which agree for the most part with similar ratios found by Bragg[4] by a less direct process.

A study of these ratios together with the form of the ionization curve in each gas makes it possible to obtain a relation between the ionization in a gas and the constants of the gas.

University of Illinois,
Urbana, Ill.,
November 2, 1910.

[1]Abstract of a paper presented at the Chicago meeting of the Physical Society, November 26, 1910.
[2]Taylor, Amer. Jour. of Science, October, 1909; Phil. Mag., October, 1909.
[3]*Loc. cit.*
[4]Bragg, Phil. Mag., March, 1907.

The Electrical Properties of Light-Positive and Light-Negative Selenium.[1]

By F. C. Brown.

1. By light-positive selenium is meant that variety of selenium which increases its conductivity when it is illuminated.

2. By light-negative selenium is meant that variety of selenium which decreases its conductivity when it is illuminated.

3. It is considered that both the increase and the decrease of conductivity of selenium by illumination are genuine changes in the selenium tself.

4. The various conditions which change the conductivity of selenium are considered, such as light, mechanical pressure, electrical potential, temperature, moisture and radium, and it is pointed out that the experimental facts observed by the different investigators seem to show only disagreement and confusion.

5. It is then pointed out that the various observed properties of the light-positive and the light-negative selenium can be explained on the view that every light sensitive variety of selenium contains one or all of three kinds of selenium, A, B and C, which can coexist in various states of equilibrium depending on the quantities of A, B and C present and on the condition of light, pressure, temperature, electrical pressure, etc.

6. In view of the extremely high resistance of so called high sensibility selenium cells,[2] it is assumed to start that the A kind has zero conductivity. It is further assumed that the B kind has a specific conductivity k_1 approaching in magnitude that of the metals, and that the C kind has a specific conductivity between that of A and B, and equal to K_2. The light is supposed to act on the selenium so as to change A into B and B into C, according to the reaction,

$$A \rightleftarrows B \rightleftarrows C.$$
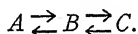
The rate of change is considered a function of the number of A, B and C present and of the intensity and wave-length of the light when the other conditions are unchanging. In order to simplify the problem so that it can be easily handled theoretically, it is first assumed tentatively that the wave-length and intensity and the light is kept constant and that at all times the change of A into B and of B into C is, under the action of light, large compared to the corresponding changes of C and B and of

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[1] Abstract of paper presented at the Chicago meeting of the Physical Society, November 26, 1910.

[2] Paper by F. C. Brown, Phys. Zeits., 11, p. 481, 1910.

B into A. This leads to the following equation for the conductivity of various mixtures of A, B and C, for any duration of exposure to light:

$$i = k_1 = \left\{ \frac{A_0\alpha}{\alpha-\beta}(e^{-\beta t} - e^{-at}) + B_0 e^{-\beta t} \right\} + k_2 \left\{ \frac{A_0}{\alpha-\beta}[\alpha(1 - e^{-\beta t}) - \beta(1 - e^{-at})] \right.$$
$$\left. + B_0(1 - e^{-\beta t}) + C_0 \right\},$$

where α and β are the respective rates of change, and A_0, B_0 and C_0 are the initial quantities of the three kinds of selenium.

7. The theory here given and observed facts are in agreement in the following:

(a) The selenium of highest sensibility is the light-positive variety and in the dark it is composed of almost entirely one kind, called A.

(b) The light-positive selenium consists of various proportions of the A, B and C kinds.[1]

(c) The light-negative selenium which decreases its conductivity from the instant it is illuminated consists of various proportions of B and C in the dark.[2]

(d) The selenium whose conductivity first increases and then decreases with duration of illumination consists of from one to ten per cent. of the A kind, before exposure to light, and most of the remainder is of the B kind. Some instances show that the decrease of conductivity is greater than the increase.[3] However the decrease is usually less than the increase.

(e) In the Giltay selenium cell the maximum conductivity occurs more quickly with intense illumination than it does for weak illumination. The constants, in the formula, which show the rates of change must be made larger to show this fact.[4]

(f) The curve for the conductivity of the A and B mixture at different times of exposure is convex upward before the maximum conductivity is reached and concave upward after the maximum is reached.[5]

(g) The specific conductivity of the light-negative selenium is extremely high, indicating a preponderance of the B kind.[6]

[1] The Giltay selenium cell is a good illustration of a mixture of A and B kinds, with the amount of A more than one hundred times greater than the amount of B. This cell was purchased of J. W. Giltay, of Delft, Holland.

[2] This is the kind of selenium mentioned in Phys. Zeits., 11, p. 482.

[3] Ries discusses a number of such mixtures of selenium produced by himself, and which he terms abnormal selenium cells. Phys. Zeits., 9, p. 569, 1908.

[4] In one instance the maximum was reached in 50 seconds and with an intensity about 100 times as great the maximum was reached in 1.8 seconds.

[5] All samples of selenium show this, providing there is a maximum.

[6] Samples of selenium now in my possession show this.

Conclusions.

That the assumption that all light-sensitive selenium is made up of three components, *A*, *B* and *C*, is probably sufficient to explain all the observed phenomena concerning the change of conductivity of selenium.

That the formula given will probably prove satisfactory except when the amount of *C* becomes considerably larger than the amount of *B*.

The indication is that the rate of change of *A* into *B* is some larger than the rate of change of *B* into *C*, but not as much as twenty times larger.   The writer believes, in view of previous results[1] and recent observations, that changes of *A* into *B* and of *B* into *C* are produced by mechanical pressure, electrical potential, temperature and other agencies, and that through selenium we will be able to deduce quite directly fundamental relations existing between the units of these quantities.

In view of his own observations and those of Ries[2] and others the writer suggests that even moisture by some mechanical means may change *B* into *C*.

The recovery of selenium from light effects is being investigated from a similar viewpoint to the one here proposed for the light effect.

The Question of Valency in Gaseous Ionization.[3]

By R. A. Millikan and Harvey Fletcher.

BY directly catching upon oil drops, at the instant of formation, the ions produced in air by γ rays of radium, *X* rays of varying hardness and β rays of radium and measuring the charges carried by these captured ions it has been shown that the process of ionization by any of these agencies consists in the detachment from a neutral molecule of one single, elementary, electrical charge.   If doubly or trebly charged ions are ever formed by the detachment from a molecule of two or three elementary charges, the number of such bi- or trivalent ions cannot, under any conditions thus far experimented upon, be more than one or two per cent. of the number of univalent ions formed.

It is probable therefore, despite the contrary evidence brought forward by Townsend and Franck and Westphall, that the process of ionization in gases never gives rise to ions of either positive or negative sign, which carry more than a single elementary electrical charge.

Ryerson Physical Laboratory,
    Chicago, Ill.

[1] Paper by Brown and Stebbins, Phys. Rev., 26, p. 273, 1908.

[2] Paper by Ries, Phys. Zeits., 9, p. 569.

[3] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 27–30, 1910.

The Effect of Red and Infra-Red on the Decay of Phosphorescence in Zinc Sulphide.[1]

By Herbert E. Ives and M. Luckiesh.

THE effect of red and infra-red in suppressing phosphorescence has long been known. The more usual mode of action consists in a flash or increase of phosphorescent light immediately upon exposure to long waves, followed by a rapid decay. Sidot blende or phosphorescent zinc sulphide has been considered an exception to the general rule, in that the preliminary flash before decay has not been noted. In the present communication it is shown that zinc sulphide under certain conditions does show this preliminary flash.

In performing some experiments to test the applicability of phosphor-photography to a search for faint infra-red radiation the observation was made that the ordinary immediate drop in intensity following a brief exposure to red light was much delayed if the exposure to red occurred a minute or more after excitation. This has been followed up by accurate photometric tests and it appears that there are two stages in the decay of phosphorescence, merging gradually one into the other. In the first stage the effect of long waves is to cause a quick drop in intensity; in the second stage the long waves cause a preliminary increase of brightness before decay. In the intermediate stage the accelerated decay appears to the unaided eye to be delayed in starting.

Observations have been made on this phenomenon using both red and infra-red light; varying the intensity and time of excitation, and color of the exciting light; and also the intensity of the extinguishing light and the time of its application. It appears that with great intensity of red or infra-red the preliminary flash may be made very to occur very close to the beginning of decay, having been obtained at 30 seconds.

With lower intensity the flashing up effect can be produced only at a later period in the decay. Neither less intense nor shorter excitation will cause the flashing up phenomenon to occur closer to the beginning of decay, even although the intensity of phosphorescence in these cases is far below that at which the flash occurs in the strongly excited substance. On the other hand, brief exposure to red or infra-red immediately after excitation causes the substance quickly to come to the stage where flashing occurs under subsequent exposure to long waves. No difference in the behavior of the flash-up phenomenon has been noted when the exciting light has been entirely visible or entirely ultra-violet, or if the extinguishing light is red, infra-red, or both together.

Measurements of the actual increase of intensity under infra-red

radiation show the increase to vary from zero to a maximum and then again decrease, as the total brightness of the phosphorescence decreases. Measured in percentage of the total brightness the increase of light rises from zero to about one hundred per cent. at 45 minutes, under the conditions of the experiment.

The zinc sulphide was prepared by Verneuil. The decay curves plotted in terms of $1/\sqrt{I}$ against time do not give straight lines but are concave toward the time axis. By plotting all the observed curves on logarithmic paper parallel straight lines were obtained from one minute on; before one minute the lines curve. The law of decay after one minute seems to be

$$I = \frac{1}{(a + bt)^m},$$

where $m = 1.03$.

They may therefore be plotted in terms of $1/I^{.97}$ against time. The effect of different lengths or intensities of excitation or of a brief exposure to long waves after excitation is to produce decay curves with different values of a and b. The decay curve after brief action of red or infrared does not correspond to the original curve with origin shifted.

The Pentane Lamp as a Primary Light Standard.[1]

By E. B. Rosa and E. C. Crittenden.

THE primary photometric standard of Great Britain is the 10 candlepower Harcourt pentane lamp. This lamp has been investigated at the National Physical Laboratory and the Reichsanstalt, and the ratio of light given by it to that of the Hefner has been many times determined. The Bureau of Standards has been carrying on an investigation for some time on primary flame standards, including the study of the pentane lamp. We have found that pentane lamps as made from the same specification by different manufacturers, differ by several per cent. in candlepower, and hence any determination of the ratio of the pentane to the Hefner, or other primary standard, amounts only to a determination of the ratio of particular lamps. In this investigation we have studied not only the difference in lamps but also the difference in different kinds of fuels, and have devised a new form of pentane lamp which has some advantages over the Harcourt form. Results of measurements on various lamps, and with different fuels, under varying atmospheric conditions will be communicated, and suggestions made as to the important characteristics of a primary standard, these suggestions being

[1] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 27–30, 1910.

based upon a considerable experience with lamps of different types, and also with lamps of different makes of the same type.

On the Function of Rest in Restoring a Platinum-Iridium Wire to its Annealed Condition.[1]

By L. P. Sieg.

THE peculiar elastic properties of platinum-iridium wire have been discussed by Guthe and Sieg,[2] and by Sieg.[3] Among the experiments described in the latter paper were many devoted to a study of the relation between period and amplitude of vibration of the wire when used as the support of a torsion pendulum. It appeared that the relation between these two quantities was a variable one and depended largely upon the previous treatment of the wire. For example with a freshly annealed 40 per cent. platinum-iridium wire, the period changed by 0.19 per cent. while the amplitude of vibration passed over the range from 47° to 18°. On the other hand if the wire was vibrated through a large amplitude (360° from the center), allowed to die down, and then without annealing was set in vibration, it was found that the period changed by 3.46 per cent. while the amplitude passed over the range from 47° to 18°.

Certain results obtained at the time of the earlier experiments pointed to the conclusion that the condition of the wire in the second case described above was unstable, and that the wire tended to attain the annealed condition (described in the first case above) as a result of continued rest under the load used in the experiments; and the prediction was made that, with a sufficient rest, the wire might ultimately reach the annealed condition. This prediction has been verified in an observation recently made. The wire, which had been twisted through a large amplitude, and not subsequently annealed, had been hanging practically at rest for eight months. The period in the first experiment following this long rest was found to change by only 0.36 per cent. while the amplitude passed over the range from 47° to 18°, thus showing that the wire had practically reached the annealed condition. To make certain that this last result was relatively correct the wire was again annealed and vibrated at once through an amplitude of about 50°. In passing over the range from 47° to 18° the period was found to change by 0.18 per cent., thus practically agreeing with the result obtained eight months previously under similar conditions.

A second point is that the wire seems to have had its coefficient of rigidity (measured kinetically for small amplitude) increased by 2 per cent. as a result of the long rest. The increase was from 10.5×10^{11}

[1] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 27–30, 1910.

[2] K. E. Guthe and L. P. Sieg, Phys. Rev., 30, p. 610, 1910.

[3] L. P. Sieg, Phys. Rev., 31, p. 421, 1910.

dynes/cm². for the freshly annealed wire, to 10.7×10^{11} dynes/cm². for the same wire after an eight months' rest.

PHOTO-ELECTRIC EFFECTS IN ALUMINUM AS A FUNCTION OF THE WAVE-LENGTH OF THE INCIDENT LIGHT.[1]

BY J. R. WRIGHT.

RECENTLY R. A. Millikan[2] has shown that the values of the positive potential assumed by a metal in a vacuum when illuminated by ultra-violet light increase enormously with long exposure to a very intense light. The increase for five metals investigated was from ten to thirty times the values obtained for the same metals before illumination.

The relation between these new positive potentials and the wavelength of the incident light has been investigated. An aluminum disk which showed an initial positive potential of 0.25 volt was illuminated intermittently for a period of three weeks, the positive potential gradually increasing to a maximum value of 14.1 volts. Tests were then made on the relation between this high positive potential and the wave-length of the incident light. Contrary to the results obtained by E. Ladenburg[3] and by A. W. Hull,[4] who worked with the low values of the positive potentials, the positive potential was found to reach a definite maximum at λ2166. Changes in the intensity of the light used was without effect on the maximum value of the potential. Using different sources of light was also without effect either on the value or the position in the spectrum of the maximum point.

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CHICAGO, ILL.

THE RATIO OF THE TWO HEAT CAPACITIES OF CARBON DIOXIDE AS A FUNCTION OF THE PRESSURE AND THE TEMPERATURE.[5]

BY A. G. WORTHING.

THE method employed is the null method first described and used by Maneuvrier.[6] It is based on the perfectly general theorem due to Reech. The ratio of the two heat capacities of a substance is expressed as follows:

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, November 26, 1910.

[2] Paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

[3] E. Ladenburg, Phys. Zeits., VIII., p. 590, 1900.

[4] A. W. Hull, Am. Jour. of Sc., XXVIII., p. 251, 1909.

[5] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 27–30, 1910.

[6] Ann. de Chim. et de Phys. (7), 6, 321, 1895.

$$\gamma = \frac{\left(\dfrac{\partial P}{\partial v}\right)_Q}{\left(\dfrac{\partial P}{\partial v}\right)_\theta}.$$

Q and θ represent respectively quantity of heat and temperature, and as subscripts indicate respectively a reversible adiabatic process and an isothermal process. Technically the expression used in this work differed somewhat from the above. It was

$$\gamma = \frac{\Delta P_Q}{\Delta P_\theta},$$

where ΔP_Q and ΔP_θ are the reversible adiabatic and the isothermal changes in pressure which result from the same small change in specific volume. ΔP_Q was measured experimentally, being read directly on a differential mercury manometer which also served as the manoscope. ΔP_θ was computed with the aid of the empirical equation given for carbon dioxide by Kamerlingh Onnes.[1] Some values obtained from the smoothed curves are indicated in the following table.

γ for Carbon Dioxide.

	10 Atmos.	20 Atmos.	30 Atmos.	40 Atmos.	50 Atmos.	57 Atmos.
.2° C.	1.362	1.502	1.75			
31.0° C.	1.330	1.409	1.536	1.733	2.01	2.37
50.0° C.	1.314	1.375	1.462	1.580		
98.5° C.	1.265	1.288	1.329	1.383	1.45	

The following interesting table was obtained by assuming that an equation of the form $Pv^{\gamma'} = $ constant holds for the reversible adiabatic processes made use of in this work.

γ' of Equation $Pv^{\gamma'} = $ Constant for CO_2.

Pressures in Atmos.	7.78	14.62	21.52	28.52	35.52	42.34	49.36	57.0
0.0° C.	1.263		1.232	1.238				
31.0° C.	$\left\{\begin{array}{c}1.268\\1.258\end{array}\right\}$	1.255	1.262	1.256		1.259		1.282
50.0° C.	1.252	1.261	1.261	1.262	1.266			
98.5° C.	1.241	1.230		1.236	1.219	1.260	1.250	

[1] Comm. No. 74 from Phys. Lab. of Univ. of Leiden, 1901.

The approximate constancy of γ' for a given temperature is remarkable and is worthy of further consideration. A γ', of course, is not to be interpreted as a ratio of the two heat capacities of the gas excepting for the gas under zero pressure in a condition indicated, perhaps, by the same adiabat or perhaps by the same temperature. These suppositions would lead to the holding of the following relation generally:

$$\frac{\gamma - \gamma'}{\gamma'} = -\frac{P}{v}\left(\frac{\partial v}{\partial P}\right)_\theta - 1 = \text{departure from Boyle's law.}$$

PHYSICAL LABORATORY,
    UNIVERSITY OF MICHIGAN.

THE FREE-EXPANSION AND JOULE-KELVIN EFFECTS IN AIR AND IN CARBON-DIOXIDE.[1]

BY A. G. WORTHING.

THE two following perfectly general formulæ for the free-expansion and Joule-Kelvin effects have been used. For the former,

$$\eta = \frac{\theta - P\left(\dfrac{\partial \theta}{\partial P}\right)_v}{\dfrac{\gamma}{\gamma - 1}\theta\left(\dfrac{\partial P}{\partial \theta}\right)_v - P},$$

for the latter,

$$\mu = \frac{\gamma - 1}{\gamma}\frac{\theta - v\left(\dfrac{\partial \theta}{\partial v}\right)_P}{\theta\left(\dfrac{\partial P}{\partial \theta}\right)_v}.$$

θ and γ respectively represent the thermodynamic temperature and the ratio of the two heat capacities of a substance.

For air, compressibility data by Witkowski,[2] γ data by Koch,[3] and an empirical equation of state given by Kamerlingh Onnes[4] were used. The computed Joule-Kelvin effects for air under low pressures at 0° C. based on Witkowski's data do not agree with experimental data, those based on Kamerlingh Onnes's equation agree very well with experimental data. The trend of the curve obtained using Kamerlingh Onnes's equation indicates that the results based on Witkowski's data for the higher pressures are nearly correct. The writer knows of no experiments with which

[1] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 27–30, 1910.
    [2] Phil. Mag. (5), 41, 309, 1896.
    [3] Ann. d. Phys. (4), 26, 551, and 27, 311, 1908.
    [4] Comm. No. 109c from the Phys. Lab. of Univ. of Leiden.

to check the free-expansion calculations. At $0°$ C. the two effects are represented well up to 125 atmos. by the following empirical equations,

$$\eta = .320 - .00018\,P - .0000032\,P^2,$$
$$\mu = .273 - .00076\,P - .0000012\,P^2,$$

where P is expressed in atmos. and η and μ in degrees/atmo. For $-79.3°$ C. the following values are taken from the smoothed curves:

	0 Atmos.	25 Atmos.	50 Atmos.	75 Atmos.	100 Atmos.	125 Atmos.
η	.51	.50	.49	.44	.37	.32
μ	.50	.48	.45	.38	.29	.23

The units for η and μ are as before.

For carbon-dioxide Kamerlingh Onnes's[1] equation of state and the writer's data on γ were used. The results indicate very little if any effect of pressure on η and μ up to pressures of about 40 atmos. for a temperature range from $0°$ C. to $100°$ C., excepting the region near where liquefaction takes place. The computed values for μ do not agree well with Kester's experimental values, but do agree very well with the experimental values of Joule and Kelvin. The following table gives approximate results.

t	$0°$ C.	$30°$ C.	$50°$ C.	$100°$ C.
η	1.26	.99	.84	.54
μ	1.35	1.06	.90	.59

η and μ are expressed as before in deg./atmo.

With increasing pressures beyond 40 atmos. both η and μ decrease. Theoretically at the critical point (72.9 atmos., $31.0°$ C.) there is but a single value for $d\theta/dP$. Hence there

$$\eta_c = \mu_c = \left(\frac{d\theta}{dP}\right)_c.$$

$(d\theta/dP)_c$ has been determined by Keesom[2] to be .62 deg./atmo. The results of the present work are in good agreement with this.

Physical Laboratory,
    University of Michigan.

[1] Comm. No. 74 from the Phys. Lab. of Univ. of Leiden.
[2] Comm. No. 88 from the Phys. Lab. of Univ. of Leiden.

The Coefficient of Recombination of Ions in Carbon-Dioxid and Hydrogen.[1]

By Henry A. Erikson.

THIS is a continuation of the work on recombination carried on by the author part of which has been published.[2] The method employed is practically the same as there described, all observations being obtained at constant gas density. In the case of carbon-dioxid the coefficient of recombination is found to increase at first slowly as the temperature is lowered from $+100°$ C. The maximum increase takes place between $+70°$ and $-10°$ C. Below $-10°$ it remains practically constant. The following are the average values obtained on an arbitrary scale:

Temp.	a
100° C.	2.68
68	3.17
52	3.66
23	4.65
0	5.48
–18	5.44
–64	5.89

In the case of hydrogen the coefficient increases at first slowly as the temperature is lowered from $+23°$. At about $-64°$ it begins to increase more rapidly and at $-179°$ the increase is very rapid. The average relative values obtained are as follows:

Temp.	a
23° C.	4.23
– 64	7.10
–179	40.08

The results obtained may be quite satisfactorily explained by assuming that there is a velocity above which the ions cannot recombine, the magnitude of this velocity being 53,000 cm. per sec.

Physical Laboratory,
University of Minnesota.
December 8, 1910.

[1] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 27–30, 1910.
[2] Phil. Mag., vol. 18, p. 328.

THE ACOUSTIC SHADOW OF A RIGID SPHERE.[1]

BY G. W. STEWART.

THIS paper discusses the acoustic shadow of a rigid sphere, with the source located on the surface, and points out several practical applications of the theory.

Lord Rayleigh has considered this problem and has obtained a solution for an indefinitely great distance from the center of the sphere. The expression he derives for the velocity potential is

$$\psi = -\frac{e^{i\kappa(at-r+c)}}{2\pi r} \int \int U ds \times \sum \frac{2n+1}{2} P_n(\mu) \frac{f_n(i\kappa r)}{F_n(i\kappa c)}.$$

This paper considers the sound intensity and the mean flow of energy at various points of a sphere of finite radius, r. By "sound intensity" is meant the mean intensity due to pressure, and by "mean flow of energy" is meant the mean flow normal to the sphere of radius r. It is shown that the relative intensities in different directions can be obtained by the expression

$$F^2 + G^2,$$

where

$$F = \sum \frac{2n+1}{2} P_n(\mu) \frac{\alpha\alpha' + \beta\beta'}{\alpha^2 + \beta^2},$$

$$G = \sum \frac{2n+1}{2} P_n(\mu) \frac{\alpha\beta' - \alpha'\beta}{\alpha^2 + \beta^2},$$

and where the following relations hold:

$$f_n(i\kappa r) = \alpha' + i\beta', \quad F_n(i\kappa c) = \alpha + i\beta.$$

To the above is added the following relation:

$$F_n(i\kappa r) = \alpha'' + i\beta''.$$

It is then shown that the relative values of the mean flow of energy at different points at a distance r is proportional to

$$F'G - FG',$$

where

$$F' = \sum \frac{2n+1}{2} P_n(\mu) \frac{\alpha\alpha'' + \beta\beta''}{\alpha^2 + \beta^2},$$

$$G' = \sum \frac{2n+1}{2} P_n(\mu) \frac{\alpha\beta'' - \alpha''\beta}{\alpha^2 + \beta^2}.$$

[1] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 27-30, 1910.

The theory shows that, in general, the relative values of intensity and flow of energy in different directions are not the same. When r is very great, however, the relative values of intensity in different directions are the same as those of the flow of energy.

An application is made to audition, assuming that the ear detects changes in pressure. It is shown that (assuming the head to be a rigid sphere and the ears on a line passing through the center) a source at any distance can best be heard when a line joining the ears points toward the source. The advantage of this position of the ears decreases with increasing distance of the source.

A further application is found in the consideration of the importance of the reflecting surfaces in various directions from a speaker.

EXPERIMENTAL INDICATIONS OF THE NATURE OF MAGNETIZATION.[1]

BY S. R. WILLIAMS.

A STUDY of the Joule magnetostrictive effect in steel tubes, which may be described as a change in length due to a longitudinal magnetic field, has led to a working hypothesis as to the nature of the processes which go on in a magnetic substance when introduced into a magnetic field.

The fact that an iron rod first lengthens and then shortens as one increases the external magnetic field leads one to believe that this mechanical effect due to magnetization may be described by some mechanical orientation of particles, inside the substance, whose dimensions vary in different directions.

To account for the above and allied phenomena, it is assumed that the magnetic atom is composed of a positive nucleus which is an oblate spheroid to which are attached one or more negative electrons which are revolving about the positive nucleus in such a manner that the plane of revolution of the electrons and the equatorial plane of the nucleus are coincident or nearly so. This magnetic atom may be compared to one of the planets, being flattened at the poles, and having its attendant satellites. For want of a better name I have called it the planetesimal theory of magnetism.

Assuming this shape of the magnetic atom there will be two forces acting upon it when introduced into a magnetic field, first, that due to the nuclei having a tendency to set so that the magnetic resistance is a minimum, and second, that due to the magnetic fields of the revolving electrons striving to set themselves parallel to the external field. In the

[1] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 27–30, 1910.

first case the tendency is for the equatorial plane of the magnetic atom to become parallel to the imposed field while in the second case the axis of revolution attempts the same position. For brevity's sake we may call these two forces the equatorial and axial forces respectively. Two points are to be emphasized in this theory which are not common to others; first, making the property of permeability dependent upon the nature of the positive nuclei, secondly, giving definite form to the magnetic atom.

For iron, the permeability is a maximum in weak magnetic fields, therefore the nuclei orient themselves so that the equatorial planes are parallel to the field; this produces a lengthening. As the external field increases the axial force becomes stronger and stronger and finally is predominant, which causes the magnetic atoms to set with axes of revolution parallel to the field, and thus shortening occurs. This is the Joule magnetostrictive effect in iron bars.

If such a process goes on in an iron bar then by the theory proposed we may expect a transverse effect, viz., a shortening for weak fields and a lengthening for strong. This effect has been found and is a very striking phenomenon.

In nickel we have only a shortening occurring as field is increased; this indicates that the equatorial force is not sufficient to orient the magnetic atoms in weak fields and only the axial force is predominant. We can predict then a transverse effect of lengthening. This is now being looked for.

Similar reasoning shows this theory accounts for the Wiedemann magnetostrictive effect and its reciprocal relations. An important test for this theory is that it accounts for the fact that in the Wiedemann effect the direction of initial twist is independent of the sequence of applying the circular and longitudinal fields. The theory of molecular magnets will not do this as Knott has pointed out.

This theory accounts for the Smith effect, viz., a twist due to a longitudinal field. It accounts for the behavior of certain crystals in magnetic fields which Faraday described as magne-crystallic action.

The Kerr magnetic phenomenon and the Faraday effect in thin iron films indicate that the relation between the rotation of the plane of polarization and field strength must depend upon some similar process as has been described.

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    OBERLIN, OHIO.

A Contribution to the Theory of Brownian Movements with Experimental Applications.[1]

By Harvey Fletcher.

AN extension of Einstein's formula for the displacement of a particle due to Brownian movements has made possible the computation of the distribution of times of fall and ascent in gravitational and electrical fields respectively, which ought to have been observed in the experiments of Ehrenhaft and Przibram.[2] The predicted distribution checks with the distribution actually shown in their observations. It also checks with the results of new experiments made under conditions similar to those under which the above mentioned experiments were performed. This not only explains all of the irregularities found by Ehrenhaft and Przibram in the apparent value of *e*, but it brings to light a new and powerful method of studying Brownian movements.

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Chicago, Ill.

A Method of Measuring the Fluctuations in a Rapidly Varying Resistance.[1]

By F. C. Brown and W. H. Clark.

WHEN it becomes necessary to measure a variable resistance which changes during short intervals of time, we have found that by placing the variable resistance in one arm of a Wheatstone bridge circuit we can express its average value for the interval in terms of the throw of a ballistic galvanometer, where the galvanometer is thrown into the circuit by means of a pendulum for an interval of time which is small compared with the period of the galvanometer. How small this interval should be is obviously also determined by the rapidity with which the resistance changes. If the time is made sufficiently short the average resistance practically becomes the actual resistance at the middle of the period.

If the capacity and self-induction of the system are very small, the quantity of electricity passing through the galvanometer is a function of the constant of the galvanometer, of the resistance in the arms of the bridge, and of the length of time the galvanometer is in the circuit. If the battery resistance is small in comparison with the other resistances, the fluctuation of the variable resistance from the equilibrium value is calculated to be

[1] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 27–30, 1910.
[2] Phys. Zeit., July, 1910.

$$\Delta x = \frac{K(k'x + k'')^2}{\Delta t \times E(bk'' + k'aR)}, \tag{1}$$

where K is the constant of the galvanometer; a, b, R and x are the resistances in the bridge circuit, G is the galvanometer resistance, E is the E.M.F. of the battery.

$$k' = Gb + aG + bR + aR + ab,$$
$$k'' = aRG + RGb + abR,$$

Δt is the time the galvanometer is in circuit. Where Δx is small compared with x,

$$\Delta x = \frac{Cd}{\Delta t}, \tag{2}$$

where C is a new constant.

This last equation was tested experimentally with several galvanometers, using known values for the variation Δx in the fluctuating resistance. It was found that if the Δt was left constant that for a large range of values of Δx the deflection varied as Δx, and if Δx was kept constant, then the deflection varied as Δt, the length of time that the galvanometer was in circuit. Then knowing the value of the constant C in equation (2) it should be possible to calculate Δx, where Δx is small. But if Δx is large then equation (1) should be used.

However we have found it most convenient in practice to merely substitute known varying resistances in place of the fluctuating resistance and to so adjust the known resistances as to get a range of deflection covering those obtained with the fluctuating resistance. We have thus obtained the average resistance of a selenium cell for a .02 sec. interval at 0.01 sec. to 20 min. after illumination, as desired. This method works well even when Δx is .99 of the value of x.

The Nature of the Recovery of Light-Positive and Light-Negative Selenium.[1]

By F. C. Brown.

IN a recent paper before the American Physical Society I proposed an explanation for the various kinds of behavior of selenium under the action of light, pressure, temperature and other agencies. This explanation involved the presence of three kinds of selenium, A, B and C, changing under the influence of light according to the reaction,
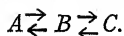
$$A \rightleftarrows B \rightleftarrows C.$$

[1] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 27–30, 1910.

The conductivity was expressed by the equation,

$$i = k_1 B + k_2 C,$$

where k_1 was much larger than k_2.   As far as could be seen from a some-what general study of the observed phenomena, the theory explained satisfactorily the characteristic behavior of the several varieties of sele-nium.   The indications were that the rates of change between A and B were large compared to the corresponding rates of change between B and C, and that the initial quantities of A, B and C in equilibrium in the dark determines how the conductivity changes when the selenium is exposed to light.

However no accurate notion of the magnitude of the rates of change was obtained.   The investigation described in this paper was undertaken in order to obtain a closer approximation to these rates and thereby subject the theory to a more rigid test.   The recovery of selenium from light effects suggested itself as the easiest case for theoretical treatment. The outline of the argument is about as follows:

Consider a variety of selenium in equilibrium in the dark under certain fixed internal and external conditions such that there are of the three kinds amounts A_0, B_0 and C_0.   Next expose the selenium to the light for any length of time, and there will then be the respective amounts A_1, B_1 and C_1 of the three kinds.   When the selenium is removed to the dark again it is at first distorted from its equilibrium condition and it at once begins to recover to its condition before exposure to light, pro-viding of course that it was then in stable equilibrium.   The equations of change during recovery are

$$\frac{dA}{dt} = \alpha_2 B - \alpha_1 A, \tag{1}$$

$$\frac{dB}{dt} = \alpha_1 A - \alpha_2 B + \beta_2 C - \beta_1 B, \tag{2}$$

$$\frac{dC}{dt} = \beta_1 B - \beta_2 C, \tag{3}$$

$$A + B + C = K. \tag{4}$$

The solution of these equations gives

$$B = \frac{L}{N} + c_1 e^{-m_1 t} + c_2 e^{-m_2 t},$$

$$C = \frac{L\beta_1}{N\beta_2} + \frac{\beta_1}{\beta_2 m_1} c_1 e^{-m_1 t} + \frac{\beta_1}{\beta_2 - m_2} c_2 e^{-m_2 t},$$

where

$$L = \alpha_1\beta_2 K,$$

$$N = \alpha_1\beta_2 + \alpha_1\beta_1 + \alpha_2\beta_2,$$

$$c_2 = \frac{B_1 - \dfrac{L}{N} - \dfrac{\beta_2 - m_1}{\beta_1} c_1 + \dfrac{L}{N} \dfrac{(\beta_2 - m_1)}{\beta_2}}{1 - \dfrac{\beta_2 - m_1}{\beta_2 - m_2}},$$

$$c_1 = B_1 - \frac{L}{N} - c_2.$$

It is found that by what seems a consistent choice of rates of change the characteristic recovery of the several varieties of selenium can be explained by assuming that the specific conductivity of the C kind as well as of the A kind is at all times small compared to the conductivity of the B kind. The problem of the conductivity then resolves itself merely into the problem of the determination of the B kind of selenium present. This conclusion was further verified by a careful study of the recovery of a Giltay selenium cell.

The constants in the above equations were approximated for a Giltay cell as follows: $m_1 = 4.2$, $m_2 = 1.2 \times 10^{-2}$, $\alpha_2 = 4.2$, $\alpha_1 = 4.2 \times 10^{-3}$, $\beta_1 = 5 \times 10^{-2}$, $\beta_2 = 10^{-2} \times 1.1$. My observations and the results of Miss McDowell's work[1] point toward the conclusion that all these constants decrease in value as the temperature is lowered. Before an accurate determination of the constants can be made it will be necessary to know what is the average penetration of selenium by light. This and other questions I shall consider in later papers.

The Electrical Discharge between a Pointed Conductor and a Hemispherical Surface in Gases at Different Pressures.[2]

By Franz A. Aust.

WHEN a point situated at the center of curvature of a hemispherical surface discharges to that surface the current density was found to be nearly constant over the whole surface of the hemisphere, at each of the pressures used, which ranged from 5 to 750 millimeters.

The currents corresponding to various voltages were measured in air and oxygen for the different pressures. The starting potentials found for the positive discharge can be expressed for all of the pressures used by the relation,

[1] Phys. Rev., 31, p. 524, 1910.

[2] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 27–30, 1910.

$$M = a\sqrt{p} + b,$$

where a and b are constants and p is the pressure.

No simple relation has been found connecting the currents at all of the pressures.

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THE EFFECT OF DISTANCE UPON THE ELECTRICAL DISCHARGE BETWEEN A POINT AND A PLANE.[1]

By O. HOVDA.

THE potentials required to start a current between a point and a plane, and the currents flowing for different voltages up to 20,000 volts, were measured for distances ranging from 5 mm. to 500 mm., the diameter of the plane used being 330 cm.

The starting potentials, while increasing quite rapidly with distance for the shortest distances used, were found to approach a constant value for the longest distances. For the large distances, too, the current increases very slowly with increase of voltage.

The results for all of the distances can be expressed quite well by Warburg's formula,

$$C = \frac{k}{d^{1.14}} V(V - M),$$

where k is a constant, d the distance between point and plane, V the potential, and M the starting potential.

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A LECTURE ELECTROSCOPE FOR RADIOACTIVITY.[1]

By JOHN ZELENY.

THE electroscope consists of a vertical charged plate near which hangs a gold-leaf suspended from one of two horizontal plates, the other of which is earthed. The space between the two plates forms the ionization chamber. The radiations may enter this chamber from the side or the radioactive substance may be placed in a shallow dish on the lower plate in the usual way. The vertical charged plate is pushed so near to the gold-leaf that the latter is attracted to it and after receiving a charge from the plate is repelled. The charge given to the leaf produces a field between the two plates and when an ionization current flows the leaf gradually sinks until it is again quite suddenly pulled to the plate, again repelled and so on in pendular fashion. Between two contacts with the

[1] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 27–30, 1910.

plate the leaf system has lost a definite quantity of electricity, and the times between such contacts under different conditions are a measure of the corresponding ionizations. When the whole is projected upon a screen the effects are readily followed by a class.

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A VARIABLE HIGH RESISTANCE OF INDIA INK ON PAPER.[1]

By FRANZ A. AUST.

INDIA ink rulings on a strip of paper make a convenient form of resistance for regulating the voltage or current from a static machine and for other small current high potential work. The maximum resistance is limited by that of the paper used. A range from .01 megohm to 20,000 megohms is readily obtained on a single strip of paper 3 by 45 cm. The paper strip is stretched between two insulating supports at one of which the high potential terminal is attached. A slide giving a metallic contact with the paper strip forms the second terminal. When the strip is made of drawing paper the current which may be passed through it is limited to about .2 milliampere, but if asbestos paper is used currents as high as 25 milliamperes may be employed with strips only 3 cm. wide.

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EFFECT OF WAVE FORM UPON INCANDESCENT LAMPS.[1]

By M. G. LLOYD.

IT is shown on theoretical grounds that the power consumption, the candlepower and the life of incandescent lamps depends upon the wave form of the current or voltage used to operate them. The magnitude of the effect depends upon the heat capacity of the filament and upon the frequency, as well as upon the wave form. A typical case chosen for illustration was between sinusoidal current and continuous current. With metal filaments the power consumed is less with alternating current, while with carbon it is greater. In both cases, however, the candlepower is greater with alternating current, and the life is shorter. Experimental investigation indicates that at frequencies of sixty cycles or higher the effects will not be appreciable with commercial lamps, and even at twenty-five cycles the heat capacity of carbon is sufficiently high to prevent observable effects. With a twenty-five watt, 120-volt tungsten lamp at twenty-five cycles the power consumption was found to be 0.16 per cent. less with alternating current than with direct current, whereas the candlepower was 1.6 per cent. greater.

[1] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 27–30, 1910.

THE

PHYSICAL REVIEW.

THE CONSTRUCTION OF STANDARD CELLS AND
A CONSTANT TEMPERATURE BATH.

By G. A. Hulett.

FOR exact electrical measurements a known electromotive force is indispensable and at present we rely on standard cells. From our knowledge of the source of the energy of a voltaic cell we might expect that any combination made up of two metallic electrodes surrounded by solutions of their respective salts would give a reproducible electromotive force which would be a function of the temperature only, but experience shows that this is not the case even when the two electrolytes are not allowed to mix by diffusion. The difficulty seems to be in the inhomogeneity of the surface of a solid metal, since solid metallic surfaces give variable potentials even against solutions of their own salts while metals in the liquid state do show the desired constancy and reproducibility.

The two combinations which have most nearly answered the requirements of a standard are the Clark cell—mercury, mercurous sulphate, zinc sulphate, zinc amalgam—and the Weston standard cell which is the same as the Clark cell except that the zinc is replaced by cadmium throughout. In these cells the cathodes are liquid mercury and each anode amalgam is made up of two phases, a liquid amalgam and a solid amalgam. The composition of the liquid amalgam, which determines the potential of the electrode, depends on its temperature only, while the composition of the electrode as a whole may vary over a considerable range. All these electrodes function as liquids.

In the Clark cell the zinc amalgam is surrounded by a saturated solution of zinc sulphate with an excess of the solid ($ZnSO_47H_2O$) which furnishes a definite concentration of zinc ions in the electrolyte for each temperature. Since the electrode is made up of two metals, mercury as well as zinc, the electrolyte in equilibrium with it must also contain some mercury ions. The relation between the zinc and mercury ions in the electrolyte is expressed by the Nernst equation

$$\sqrt[n_1]{\frac{P_1}{p_1}} = \sqrt[n_2]{\frac{P_2}{p_2}},$$

where P_1 and p_1 refer to the solution pressure of zinc and osmotic pressure of the zinc ions and these letters with the subscript (2) refer to these magnitudes of mercury, while n_1 and n_2 represent the valences of the metals. In our case the concentration of the mercury ions in the electrolyte (p_2) is exceedingly small but there is evidence that these ions are actually present and that they come from the electrode with an equivalent deposition of zinc in the amalgam.[1] The positive pole or cathode of this cell is mercury surrounded by an electrolyte saturated with both zinc sulphate and mercurous sulphate with an excess of both salts in the crystalline state. Here again the mercury will not remain pure mercury but gains a small amount of zinc from the electrolyte, sufficient to satisfy the equilibrium conditions of the equation. The zinc sulphate has a concentration of 3.36 molecular weights to a liter and the mercurous sulphate one of only .00168 or a .056 per cent. solution while the concentration of the zinc in the mercury electrode is something of the order of 10^{-10}.

The cathode and anode legs of these cells represent two different systems each of which comes to equilibrium at a given temperature, but the cell as a whole is not a system in equilibrium and cannot be as long as it shows an E.M.F. The dissolved mercurous sulphate in the cathode leg slowly diffuses over to the anode amalgam where it is at once reduced to the equilibrium concentration which was already present before it arrived. This is the function of the excess of zinc in the anode, but since the concentration of the mercury in

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[1]Crenshaw, Jour. Phys. Chem., 14, 158.

the cathode leg is small and the electrolyte a very viscous solution, the diffusion of mercury to the anode is a slow process and the life of the cell is very great. These same relations obtain in the Weston standard cell where the zinc is replaced by the cadmium and it would seem that these two combinations were ideal from the standpoint of constancy and reproducibility. The one drawback seems to be in the mercury salt. It is a characteristic of mercury salts to hydrolyze, that is, to interact with water and form a basic salt and acid, and mercurous sulphate is not an exception to this rule.[1] Sulphuric acid and sulphate solutions prevent or decrease this hydrolysis, and the hydrolysis of mercurous sulphate in a zinc sulphate solution has not been detected but it is surely present in a cadmium sulphate solution. Such a reaction need not detract from the value of the cell for equilibrium is established as soon as the basic salt and a definite acid concentration are present and then the potential is again a function of the temperature only. In the Clark cell the reaction has not been detected but in the Weston standard cell the hydrolysis takes place, although it is a very slow change which does not come to equilibrium.[2] However this change is generally small as well as slow and the cell has other qualities which make it valuable.

Materials.—Any amount of work may be done in purifying the materials used in constructing standard cells, but only a few foreign substances (in traces) measurably affect the E.M.F. of these standards and for our purpose it seems best to go only so far as to insure the desired reproducibility since the simpler the operations are the greater will be the agreement between the cells constructed at different times and by different investigators.

Mercury.—Pure redistilled mercury may be obtained from reliable dealers and it answers every purpose. In lieu of this, ordinary mercury is chemically purified and then distilled.[3]

Mercurous Sulphate.—The depolarizer is chiefly responsible for variations in the reproducibility and constancy of standard cells and the preparation of the mercurous sulphate and the making

[1] Zeit. Phys. Chem., 49, 493.
[2] PHYS. REV., 30, 648.
[3] PHYS. REV., 21, 388.

of the paste is still the most difficult and important part of the construction of these standards. In 1884 Lord Rayleigh called attention to this difficulty and the point was emphasized by Kahle[1] and later by Jaeger and Lindeck,[2] but the first explanation of the changes which caused the trouble was offered by Carhart and Hulett.[3] Evidence was given to show that the irregularities in the standard cells were due to the presence of a basic mercurous sulphate which had been formed in preparing the mercurous sulphate or in making the paste. It was also shown[4] that mercurous sulphate is stable in a sulphuric acid solution when the concentration of the acid is molecular (98 grams H_2SO_4 to a liter) or greater, but when the acid strength drops below this value the mercurous sulphate begins to hydrolyze with the formation of a difficultly soluble basic salt—$Hg_2(OH)_2 . Hg_2SO_2$. After this information was obtained it was found possible to suggest conditions for preparing the mercurous sulphate and making the paste so as to exclude the basic salt and the cells made under these conditions did not require the usual aging but showed a constant value at once and were in exceptionally good agreement.[5] There are a number of methods for preparing mercurous sulphate and in such a way that it is possible to maintain the concentration of the sulphuric acid of molecular strength (or greater), at the point where the mercurous sulphate is forming and under such conditions only the normal mercurous sulphate appears. All such preparations give essentially the same value to standard cells.[6]

It is easy to prepare mercurous sulphate chemically and with sufficient sulphuric acid present to prevent hydrolysis, but there is the inclusion and isomorphism of nitric acid, nitrates or other substances to be considered and while the depolarizer made by these chemical methods does, with the proper precautions, give correct values to the standard cells, it is our experience that the electrolytic mercurous sulphate gives on the whole the most uniform and reproducible values.

[1] Wied. Ann., 51, 225.
[2] Zeit. Instk., 21, 33.
[3] Trans. Amer. Electrochem. Soc., 5, 63.
[4] Zeit. Phys. Chem., 49, 483.
[5] Trans. Amer. Electrochem. Soc., 5, 71.
[6] F. E. Smith, Phil. Trans., 207 A, 395.

The electrolytic method[1] has the distinct advantage that only mercury and moderately dilute sulphuric acid are used and the only foreign substance to be looked after is sulphuric acid which may be effectively removed as indicated below. We use an acid having a density of 1.15, made by pouring one volume of concentrated (density 1.84) sulphuric acid into six volumes of water. When this acid is electrolyzed between a mercury anode and a platinum cathode, mercurous sulphate is formed and goes into solution at the anode, but when the acid is saturated (.2 grm. Hg_2SO_4 to the liter) the solid mercurous sulphate appears and covers the anode. At the cathode, hydrogen is liberated and a little mercury but the efficiency is very great, forming about 9 grams of the salt per ampere hour. If the mercurous sulphate is allowed to collect on the anode to any great extent, a secondary reaction may take place but this is to be avoided and a stirrer has been employed to keep the mercurous sulphate in suspension in the electrolyte while the current was passing,[2] but only a limited amount of mercurous sulphate could be formed at a time. Lately we have improved the method by using an inner dish to hold the mercury and so arranged that the mercurous sulphate passed over the rim of the inner dish and collected in the space between it and the outer dish. The accompanying sketch, Fig. 1, shows the arrangement. A large deep crystallizing dish was nearly filled with the acid and in this was placed a somewhat smaller and very much lower crystallizing dish. Mercury was poured into the inner dish to a depth of about one centimeter and electrical connection was made through a stiff platinum wire fused into the end of a glass tube so that 4 or 5 cm. of the wire extended out into the mercury. The glass tube was bent to fit into the inner dish and over the edge of the outer dish as indicated in the figure. A short spiral of platinum wire was suspended in the sulphuric acid to serve as cathode, the platinum wires used were 0.5 mm. as a current of over two amperes was generally used with the mercury anode surface of 175 cm. (The inner dish has a diameter of 15 cm.) A motor-driven stirrer was

[1] Carhart and Hulett, Trans. Amer. Electrochem. Soc., V., 59. F. A. Wolff, Trans. Amer. Electrochem. Soc., V., 56, and M. Coste et M. Etaix, Bull. de la Soc. des Electriciens, 2 Serie, VIII., 1908.

[2] Trans. Amer. Electrochem. Soc., V., 63. PHYS. REV., 22, 334.

used and was made from a glass rod which was bent at a right angle and so arranged that the *L* part of the stirrer passed over and near to the surface of the mercury. This stirrer was held in a brass tube *bb* which also carried the pulley *p*. This brass tube turned in bronze bearings in the tube *hh* which was firmly held by clamps (not shown). It was found to be important to have the stirrer well made so that it worked smoothly and with certainty.
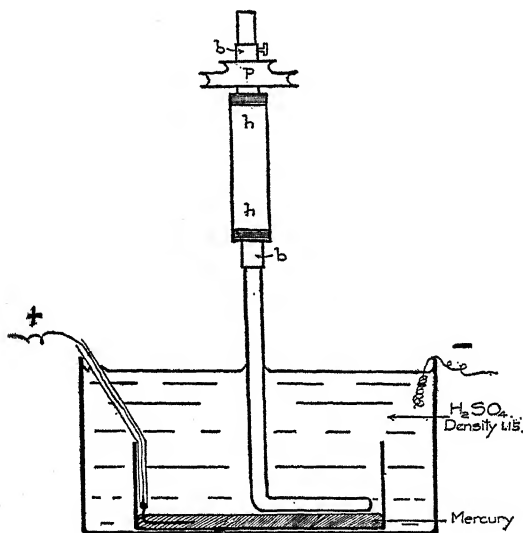

Fig. 1.

After the stirrer was running uniformly at a rate of some 200 revolutions per minute, the current was turned on. With fresh acid a skin formed on the surface of the mercury but by breaking and making the current several times or until the acid became saturated with the mercurous sulphate and the solid sulphate appeared throughout the acid, then there was no further trouble. The position of the stirrer was such that it did not unduly agitate the mercury. The mercurous sulphate formed was carried up by the rotating liquid and settled in the space between the two dishes and much more readily when the rotation of the liquid outside of the inner dish was retarded. This was easily accomplished by inserting glass plates edgewise down into the acid and allowing them to rest

on the edge of the inner dish. With a current density of from 1 to 2 amperes per 100 cm.[2] of mercury anode surface the product obtained was gray, due to finely divided mercury. The presence of this finely divided mercury is an advantage in checking any tendency to oxidation. Generally 50 or 60 grams were prepared at a run and after the stirrer had been removed and the mercurous sulphate had settled, most of the acid was removed, the contents of the inner dish were poured into the outer dish and all well stirred for some time, then the acid, with the suspended sulphate, was poured into a clean dish and after the sulphate had settled the acid was returned to the mercury, stirred and again decanted and this was repeated until the mercury and mercurous sulphate were separated. The product was transferred to a glass stoppered bottle, covered with a little of the acid and kept in a dark place until needed. Some of the acid, diluted with two volumes of water, was brought into an ordinary washing bottle and the stream blown from this bottle aided materially in transferring the sulphate from one dish to another.

After this apparatus was once perfected and assembled it has been a simple matter to prepare the depolarizer whenever needed. Our experience is that this gray electrolytic mercurous sulphate, prepared as just described, is the most reliable and reproducible depolarizer for standard cells and that the grains are sufficiently large to avoid all effects of surface tension.[1]

The Flowing Anode Method.—Lately we have also used an interesting and very simple method for preparing electrolytic mercurous sulphate. A fine stream of mercury flowing from a funnel into the sulphuric acid is made the anode and no stirring is required. The accompanying sketch, Fig. 2, illustrates the apparatus. An ordinary funnel was used, the stem was warmed in a flame and drawn down to a capillary which was about 10 cm. in length and of such a diameter that 10 c.c. of mercury were delivered in about five minutes. An ordinary liter beaker glass was filled nearly full with the sulphuric acid (density 1.15) and the funnel adjusted so that the tip of the capillary was just under the surface of the acid. A short platinum spiral was hung in the acid

[1] PHYS. REV., 22, 326–328.

and served as cathode, mercury was poured into the funnel and it ran through the capillary and formed a spray as it entered the acid but when contact was made with the mercury in the funnel and a current of from 2 to 3 amperes passed through this mercury, the electrolyte, and out through the platinum spiral, the spray changed to a cylinder of flowing mercury which extended to the bottom of the beaker and lo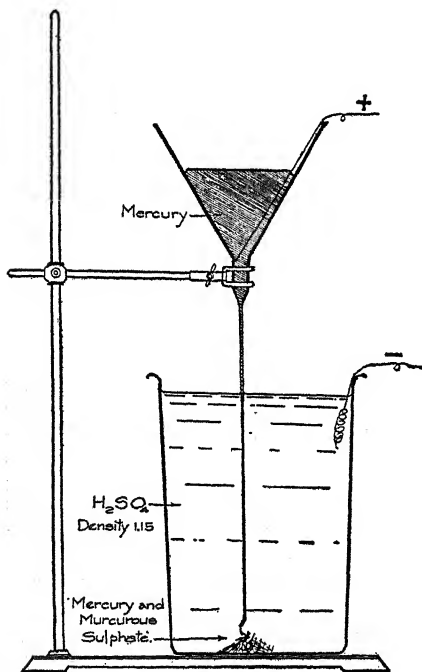oked like a wire. It was distinctly gray save for a short distance at the top which was bright mercury. M. Coste et M. Etaix[1] have used an alternating current in preparing mercurous sulphate. The flowing electrode method also permits us to use an alternating current and very simply. Two funnels were prepared with the same length of capillary stems and of the same diameter. These funnels dipped into the same beaker of sulphuric acid and the two streams of mercury were used as the electrodes, a 60-cycle alternating current was used and so regulated that there was no arcing between electrode and electrolyte and thus

Fig. 2.

a current of about 5 amperes was used. The efficiency here is as great as with the direct current, about 9 grams of mercurous sulphate per ampere hour. With this flowing anode method the preparation is white, even with large current densities, and the sulphate is easily separated from the mercury by decanting it with the electrolyte. Cells made with these preparations as depolarizers show a slightly higher value than with the gray electrolytic previously described. The alternating current preparation seems to

[1] Bull. de la Soc. des Electriciens, 2 Serie, VIII., 1908.

be the better of the two but they have not been tested for a suffi-
cient length of time if we are concerned with the fifth decimal place
in the E.M.F. of the cells.

The Paste.—The problem is to prepare a mixture of mercurous
sulphate, zinc or cadmium sulphate and saturated solution, but to
avoid the presence of sulphuric acid and basic mercurous sulphate.
The only impurity in the electrolytic mercurous sulphate is the
sulphuric acid in which it is made and preserved but any attempt
to remove this acid by washing with water introduces the basic
salt. Formerly we used alcohol which was subsequently removed
by washing with the saturated zinc or cadmium sulphate solution,
but we have found that this preliminary washing with alcohol is
not necessary and that really only three washings with the saturated
sulphate solution are necessary, when certain precautions are ob-
served. A Gooch crucible, filtering apparatus and good suction
were employed. A disk was cut from filter paper and fitted nicely
to the bottom of the crucible and then the mercurous sulphate
was transferred to the crucible and washed down with the acid.
The layer of solid sulphate was about 15 mm. thick and the acid
was quite completely removed by suction. By shaking off the drops
which formed on the under side of the crucible it was determined
when no more of the acid was being removed by suction. The
inside of the crucible down to the sulphate was wiped with a bit
of filter paper, to remove any drops of acid, and then the sulphate
was washed directly with the saturated zinc or cadmium sulphate
solution, using one to two cubic centimeters each time and taking
care to remove this solution quite completely by suction before a
second portion was added. It had been our practice to wash five
or six times in this way but some experiments have shown that two
washings gave the same value to the cells as was obtained when
the sulphate had been washed five or six times. Now we wash only
three times with the saturated zinc or cadmium sulphate solution
but with due attention to the details just given. The washed
sulphate was quite compact and the top layer with any dust was
easily scraped off, then the solid cake was removed by tapping
the inverted crucible on a piece of filter paper. A large agate
mortar was used in mixing the components of the paste. Zinc or

cadmium sulphate crystals, about equal in volume to the mercurous sulphate, were crushed but a few cubic centimeters of the saturated solution were first added to the crystals to prevent partial dehydration by the crushing.[1]   After the crystals had been ground to a fine powder, the mercurous sulphate was added and all thoroughly mixed with enough of the saturated solution to make a thin paste and of such a consistency that it readily flowed from a 5-mm. tube.   Unless the mercurous sulphate was gray a little mercury was ground up with the crystals and mixed with the paste.   In transferring the paste to the cells a thin-walled glass tube 30 cm. long, 5 mm. in diameter was slightly contracted at the lower end and used as a pipette.   By gentle suction, and stirring the paste with the end of the pipette, the paste was drawn up into the pipette and easily transferred to the cell and added until it covered the mercury electrode to a depth of about 15 mm.   In all the operations of preparing the depolarizer and making the paste, direct sunlight or undue exposure to light was avoided.   Excessive exposure to light darkens mercurous sulphate and the product then gives a higher value to the E.M.F. of the standard cells.

Zinc Sulphate. — $ZnSO_47H_2O$ is one of our more soluble salts. The solubility increases rapidly with the temperature and at 39° the solid phase changes to the hexahydrate which is to be avoided in making the Clark cells.   E. Cohen gives the following relation between solubility and temperature; $L_t = 41.80 + .522t + .00496t^2$. From this we find that 100 c.c. of water dissolves:

At	$ZnSO_4$. Grams.	$ZnSO_47H_2O$. Grams.
0°	41.80	110.6
0°	41.80	110.6
15°	50.78	154.9
20°	54.22	167.6
25°	57.92	188.5
30°	60.92	207.3
39°	70.66	275.3

Zinc blend, from which this salt is obtained, generally contains cadmium and manganese and often lead and tin.   The salt is

[1] W. F. Hildebrand, J. Amer. Chem. Soc., 30, 1120.

isomorphous with the other vitriols—iron, magnesium, cobalt' nickel and copper, but none of these metals, in the small amount usually found present in zinc sulphate, seems to affect the E.M.F. of the Clark cells.   Sulphuric acid is to be excluded as .027 per cent. of this acid in the electrolyte of the cell reduces the E.M.F. measurably.[1]   Zinc sulphate, as obtained in the trade, generally contains less than the theoretical amount of water of crystallization due to efflorescence, so it is best to recrystallize the salt and at room temperature in order to insure the heptahydrate.   Starting with "chemically pure" zinc sulphate we made a saturated solution at room temperature, the solution showed a density of 1.45 and was brought into a flask with a handful of granulated zinc, warmed to 50° and frequently shaken for an hour or so in order to remove less positive metals than zinc and also any excess of acid.   The solution was now filtered until perfectly clear.   It was then poured into a large crystallizing dish, but not over a centimeter in depth. The dish was placed where the temperature was fairly constant and covered with filter paper.   Due to the shallowness of the solution the crystals formed on the bottom of the dish and in a day or so a good crop of perfectly clear crystals one to two centimeters long were obtained.   If a crust began to form on the solution or around the edges, a few drops of water were carefully run on to correct the difficulty and with little attention only clear well formed crystals were obtained. The solution was poured off and the crystals brought onto sheets of filter paper to absorb the excess of solution and then they were preserved in a well stoppered bottle to prevent efflorescence.   A saturated solution is easily made from these crystals; the preceding table will indicate the amounts to use.

Zinc Amalgam.—Zinc is one of the more soluble metals.   With an excess of zinc we have, for each temperature, a liquid amalgam of definite composition and in equilibrium with a solid phase which is pure zinc.

At 0° the liquid amalgam contains 1.35 per cent. of zinc.
　　15° "   "   "   "   1.74 "   "   "
　　20° "   "   "   "   1.99 "   "   "
　　25° "   "   "   "   2.18 "   "   "
　　39° "   "   "   "   2.85 "   "   "

[1]Phys. Rev., 27, 352.

For our purpose it is seen that a 3 per cent. amalgam would serve every purpose since for the range of the Clark cell, such an amalgam is always two phase, but a 3 per cent. amalgam is rather too liquid while the 10 per cent. amalgam commonly used is too hard.. We have found a 7 per cent. amalgam to be plastic at ordinary temperatures and most satisfactory. There seems to be no difference between the potentials of amalgams made from "chemically pure" zinc and mercury and amalgams made from the most highly purified metals so the "chemically pure" materials serve every purpose. Zinc dissolves but slowly in mercury and the amalgam oxidizes readily when hot but we have had no trouble with this amalgam since using the following method: A 2.5 cm. hole was cut in a piece of asbestos cardboard and an ordinary porcelain crucible was pushed into this hole so that it was about half way through, the board was placed on a tripod and the crucible was charged with about 7 grams of chemically pure zinc (weighed to .010 gram) and then thirteen times the weight of mercury was added. A small adjustable bunsen flame was placed under the crucible and the contents heated, but without stirring, until the zinc had all dissolved. This takes a temperature near to the boiling point of mercury and the crucible was covered with a little watch glass. The flame was now pushed a few centimeters to one side and when the crucible cooled down to about 100° the amalgam was readily transferred to the cells with a pipette. The pipette was made from a thin-walled glass tube 5 or 6 mm. in diameter with the tip drawn down in the flame to about 3 mm. The amalgam as prepared had only a slight coat, which was readily brushed to one side and the clean amalgam was sucked up into the pipette and transferred to the (warmed) anode leg of the cell without getting it on the sides of the glass. The amalgam was generally run in to a depth of about 10 mm. It soon cooled and solidified and then was ready for the zinc sulphate crystals and the saturated solution.

Cadmium Sulphate. — $CdSO_4 8/3H_2O$ is a very soluble salt but is peculiar in that the *rate* of solution is exceptionally slow so that considerable attention is needed to prepare a saturated solution. The solubility changes only slightly with the temperature and the 8/3 hydrate is stable up to 74° where it changes to the monohydrate.

100 c.c. of water dissolve the following amounts at the indicated temperatures:

At	$CdSO_4$.	$CdSO_4 8/3 H_2O$.
0°	75.5	112.5
15°	76.1	113.4
25°	76.8	114.7
40°	78.5	117.9
74°	78.6	133.8

Cadmium sulphate does not seem to be isomorphous with any known salt and consequently is obtained in a sufficiently pure state in the trade but one may obtain such beautiful crystals that it is worth while to recrystallize the salt. We dissolve a kilogram of the salt in a liter of water, using a mechanical stirrer, and filter the liquid into large crystallizing dishes. The depth of the liquid in the dishes is generally about one centimeter and the dishes are placed where the temperature is quite constant and covered with filterpaper. As the solution slowly evaporates perfect crystals appear on the bottom of the dish and most of them are water clear but some are cloudy, due to minute liquid inclusions. We have not found the cells made with the cloudy crystals to differ in E.M.F. from those made with the perfectly clear crystals; the cloudy crystals may of course be redissolved and recrystallized. Attention is to be given to the preparation of a saturated solution of cadmium sulphate on account of the slow rate of solution of these crystals; 100 c.c. of the saturated solution requires 75 c.c. of water and 86 grams of the crystals but a considerable excess of the crystals are to be used and stirred over night with a motor-driven stirrer.

Cadmium Amalgam. — Cadmium is the most soluble metal in mercury; the saturated amalgam contains the following percentages of cadmium:

At	Per Cent. Cadmium.
0°	2.5
15°	4.4
25°	5.6
35°	7.1

With an excess of cadmium the solid phase is not cadmium but an isomorphous mixture of cadmium and mercury.[1] The composition

[1] Bijl, Zeit. Phys. Chem., 41, 641.

of this solid phase also varies with the temperature but within
certain limits the potential of this amalgam is definite and repro-
ducible for a given temperature. In view of the great solubility
of cadmium at ordinary temperatures it does not seem well to use
less than 8 per cent. of cadmium in the amalgam. $12\frac{1}{2}$ per cent.
has been commonly used but Cohen and Krypt[1] have shown this
amalgam to be unreliable below 14° and there is a question as to
whether the solid phase of these amalgams are homogeneous and
in equilibrium with the liquid phase when the amalgam is cooled
from a higher to a lower temperature.[2] The question of equi-
librium in this amalgam evidently needs further investigation; it is a
much more complicated system than is the zinc amalgam.

We have used for some time a 10 per cent. amalgam for our cells
and make it quite accurately and uniformly as follows: 99.00 grams
of mercury were placed in a little crystallizing dish, 5 or 6 cm.
in diameter, and 25.00 grams of the clear cadmium sulphate crystals
(43.82 per cent. Cd) were placed on this mercury and then about
50 c.c. of distilled water was carefully added and made acid with
a drop of sulphuric acid. A flat platinum spiral was so adjusted
that the spiral was just beneath the surface of the water and con-
tact was made with the mercury which was cathode. When 2
or 3 amperes were used the cadmium was deposited in the mercury
about as fast as the sulphate dissolved, then when the crystals
had all disappeared the current was increased to 4 or 5 amperes
for half an hour. This insured a complete deposition of the cad-
mium and also liberated enough heat to melt the amalgam. The
acid was finally removed by a pipette or syphon and at the same
time distilled water was run in. This washing continued until the
current dropped to zero. The amalgam may be left under water
with the "current on" as long as one pleases as there is no possibility
of loss or oxidation as long as the amalgam is cathode. When
needed the amalgam was easily melted by removing the water
and adding hot water and the hot water was finally removed quite
completely by a pipette and then the liquid amalgam transferred

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[1] Zeit. Phys. Chem., **65**, 359.

[2] F. E. Smith, Phil. Mag., **19**, 250. S. W. J. Smith, Phil. Mag., **20**, 206. G. A.
Hulett, Trans. Amer. Electrochochem. Soc., **15**, 438.

to the cells as described under "zinc amalgams." These quantities suffice for 5 or 6 cells in all cases.

We have prepared cadmium sulphate and cadmium of the highest degree of purity,[1] also zinc sulphate and zinc[2] and constructed cells with these materials but the E.M.F. of these cells did not differ from the cells made with materials prepared as described above and this result is the one we would expect when we consider the factors controlling the potentials at the anodes and cathodes of these standard cells. For example a trace of iron sulphate could not affect the zinc amalgam nor measurably decrease the solubility of the zinc sulphate crystals, and the potential here depends on the ratio of the concentrations of the zinc in the amalgam to that in the electrolyte, nor would a trace of iron affect the mercury of the cathode or the solubility of mercurous sulphate. On the other hand a trace of basic mercurous sulphate would directly increase the concentration of the mercury at the cathode and since

Fig. 3.

this concentration is small, the effect on the E.M.F. would be relatively large. Sulphuric acid also directly and measurably affects the concentration of the mercury in the catholyte and so the greatest attention was given to these points.

The Glass Parts.—The "H" cell proposed by Lord Rayleigh has proved to be the most practical form of cell. It is easily filled and allows the contents of each electrode to rapidly take up the temperature of the bath. We use thin-walled tubing about 10 mm. in diameter made into the form indicated in Fig. 3. The .2 mm. platinum wire leads are sealed through the glass so that all but the tip end is covered with a sheath of glass. The wire is first sealed in so that the end is only just through the glass and then, while the glass is still soft, the wire is pushed in about 5 mm. and covers itself with a sheath of glass leaving only the tip exposed. Wires sealed in this way give uniform contact with the mercury
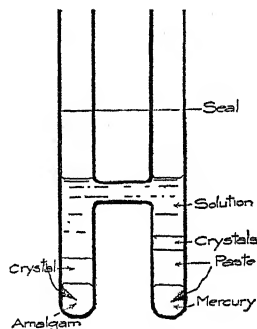
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[1]Jour. Amer. Chem. Soc., 30, 1808, 10.
[2]Jour. Phys. Chem., 14, 161–164.

or amalgams and are less liable to cause subsequent cracking of the glass especially when a fine wire is used.

The amalgam and mercury are introduced by a 5-mm. thin-walled pipette made of tubing some 30 cm. long with the lower end drawn down in the flame to about 3 mm. A good piece of rubber tubing attached to the upper end of this pipette makes it more convenient to manipulate. This pipette is also used for introducing the paste. The mercury, amalgam and paste are each from 10 to 15 mm. in depth. A few crystals are placed on top of the paste and on the amalgam a layer of crystals not over 10 mm. in depth. Only a small amount of the crystals are necessary for either the Clark or Weston cell, while a large excess may grow together and cause trouble. The cell is filled to the top of the cross tube with the saturated solution and then the glass parts are sealed off 2 or 3 cm. above the liquid by using two small blast flames which impinge on opposite sides of the point to be sealed. The part to be sealed is gently warmed at first to prevent cracking and after sealing, by properly heating a bit lower down, the expanding gas inside the cell will cause the seal to round out perfectly. The cells are arranged in supports and provided with lead wires so that they may be readily compared when in a bath.

The difficulties we have encountered with these cells are, the tendency of the glass parts to crack which is most pronounced in the amalgam leg of the Clark cells, and the fact that the contents of the cell are not accessible. This difficulty was recognized by Lord Rayleigh in recommending the cork seal.[1] It is often desirable to get at the contents of a cell to loosen a crust which has formed or to find a poor contact and while the glass seal may be broken it is seldom possible to reseal it without ruining the cell. The trouble with the cork seal has been that the cork was generally in contact with the liquid and so leaked sooner or later. We have used for some time a form of cell which avoids these difficulties; the glass part is the simplest possible (Fig. 4). Thin-walled tubes, 20 cm. long and 12 mm. in diameter, are closed at one end and blown out

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[1]Phil. Trans., 176, 42, 1886. Cells were first hermetically sealed by Wright, Phil. Mag., 16, 25, 1883.

at the side about 5 cm. above the closed end.   Two of these tubes
are joined directly together giving the advantages of the test-tube
form.   No platinum wires are fused into this part of the cell.   These
cells are closed by corks which are 15 cm. above the liquid of the
cell and are never wet and do not need to be covered with wax.
These corks carry the long narrow tubes which inclose the contact
wires and these may be removed and replaced at any time.   This
long form of cell is found to have many advantages in handling in
the bath; several may be bound together in a very compact form
and the contents of these cells most readily take on the bath tem-
perature while the insulation is perfect in any kind of a bath liquid.

The details of the contact wire and pro-
tecting tubes are also shown in Fig. 4.   A
piece of tubing 3 mm. in diameter is softened
in the flame and drawn down so that the part
which is to pass through the contents of the
cell is not over 2 mm. in diameter.   This
narrow part is about 6 cm. long while the
total length is about 22 cm.   A piece of .1
mm. platinum wire about 10 cm. long is sol-
dered to a 20-cm. piece of silk-covered copper
wire (no. 32).   This wire is passed into the
protecting tube until the platinum wire pro-
jects one or two millimeters from the narrow
end of the tube and this end is then sealed
in the flame.   A bit of wax is run into the
upper end to hold the wire securely.   Six cells
are made at one time and all bound together
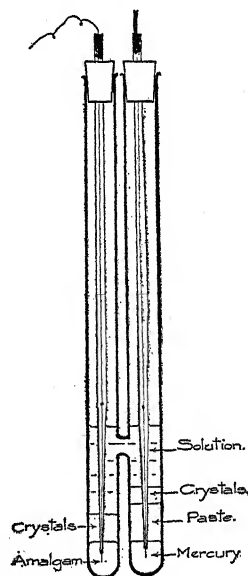and then the negative or anode wires are

Fig. 4.

twisted together and the exposed ends soldered so that only one
contact need be made for the anodes of all six cells.   The posi-
tive or cathode wires are scraped to remove the covering and then
each one is wound about a small piece of millimeter copper wire
which is forced into the warmed wax in the top of the tube leaving
about a centimeter exposed and here a good contact is easily and
rapidly made with a pinch connector.   It is well to amalgamate
the platinum tip of the contact wire just before it is inserted into

the contents of the cell.   This is readily done by holding it for a few minutes in boiling mercury (in a test-tube), a suggestion of Mr. J. S. Laird of this laboratory.   The numeral of the cell is always etched on the glass part and also written on a little paste-board disk which is pierced with a small hole and put down over the lead wire and onto the cork so that it is easily read from above when the cell is in the bath.   These cells are filled in the same way as the old form but instead of sealing the tops the contact wires with the protecting tubes and corks are inserted.[1]

In the old form of cell a crack was liable to develop at the point where the platinum lead wires passed through the glass.   This was most noticeable in the amalgam leg of the Clark cell but would often happen in both legs of either cell.   Sometimes the crack was hardly perceptible but sufficient to admit some of the bath oil in time while occasionally it was large enough to allow some of the liquid contents of the cell to escape and thus many cells were rendered worthless.   The new form of cell entirely obviates this difficulty.   Even if a crack develops where the platinum contact wire is sealed in the protecting tube no damage is done to the cell for this wire and tube is easily removed, repaired and replaced without injury to the cell.   It frequently happens that the crystals over the amalgam cake and often a little gas forms between the amalgam and the liquid and crystals, sufficient to break the contact. By removing the contact wire and protecting tube, this gas escapes through the hole left by the tube and is replaced by the saturated solution and then the contact wire may again be inserted, or the crystals may be broken and stirred.   This difficulty has been most noticeable in the Clark cells but also happened with some Westons.

The Electromotive Force of Standard Cells.—We have made both Clark and Weston cells at intervals since 1903 and according to the preceding specifications.   A dozen Weston cells, made at one time, often agree among themselves to one part in 100,000 but in time the agreement is not so good, in a year or so the variations may be noticeable and when compared with a freshly made set of these cells it has been found that all the older cells have decreased

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[1] Both the (H) and long form of cell have been satisfactorily made for us by Eimer & Amend, New York.

and some of them very noticeably. Clark cells have also been made at intervals during the last seven years and the agreement among themselves of any "set" of these cells is about the same as that of the Westons but the Clark cells made at different times are in better agreement and none of them have shown an E.M.F. decreasing with time such as is noticed in some of the Weston cells. A reason for this difference has been given in previous papers. It seems necessary therefore to make the Weston cells at intervals of about six months and reject those which fall to low values. We make both Clark and Weston cells at such intervals and the cells are all kept in a constant temperature bath which does not vary over .01° from 25.00°. Taken all together these two kind of cells with the aid of a thermostat, give us a standard which is independent of time and is reliable and reproducible to one or two parts in 100,000.

The value in absolute units of this standard of E.M.F. is of course not known to anything like this degree of accuracy but whenever the absolute value is more accurately determined, all work which has been based on this constant may be recalculated if necessary, so we are justified in giving the assumed value to the fifth decimal place.

We have for consideration the following determinations of the E.M.F. of the Clark cell in absolute units:

Rayleigh and Mrs. Sedgwick,	1884	1.4345 at 15°
Kahle,	1896	1.4322 " "
Carhart and Guthe,	1899	1.4333 " "
Ivanoff,	1900	1.4328 " "

The average is 1.4333 as the E.M.F. of the Clark cell at 15°, but these cells were all made in the old way and must have been about .0003 volt too high; allowing for this and calculating to 25° we get 1.42040. In 1906 Guthe,[1] working with standards constructed according to the preceding specifications and with a suitable thermostat, obtained the value 1.4204 for the Clark cell at 25° and 1.0184 for the Weston standard cell. Two determinations were made in 1908 of the value in absolute units of the Weston standard cell.

[1]Bull. Bureau of Standards, 2, 33, 1906.

Ayerton, Mather and Smith[1] obtained the figure 1.0183 at 17° and Janet, Laport and Jouaust,[2] found the value 1.01885 at 16°. Calculated to 25° these values are 1.01797 and 1.01847 respectively, but the results of these later determinations hardly justify a change in the values which have been used and it may be questioned whether the value in absolute units of our standard of electromotive force is known to better than one part in 2,000, or five parts in the fourth decimal place.

Assuming that the Clark cell is 1.4330 at 15° and using the customary temperature formula we have:

$$E_t = 1.4330 - .00119(t - 15) - .000007(t - 15)^2;$$

the value at 25° is 1.4204. Our cells numbering about 180 are maintained constantly at 25.00° and when we give this value (1.42040) to the Clark cells, the Weston standard cells, or at least those which have not decreased, are found to have the value 1.01840.

The Thermostat.—The E.M.F. of both the Clark and the Weston cells vary with the temperature. The Weston cell has only about one thirtieth of the temperature coefficient of the Clark cell and this was a decided advantage before thermostats were used but for electrical measurements which make any pretense to accuracy, an automatically controlled bath is indispensable for the Weston as well as for the Clark cell so the temperature coefficient is of no consequence. Our thermostat is reliable and constant to .01° and such variations indicate an uncertainty of about 1 in 100,000 in the E.M.F. of the Clark cell or within the variations of the cells themselves. The most reproducible temperature and the one easiest to maintain is 0° C., but the standard cells are very slow to come to their true value at this temperature, due probably to the slow rate of the reactions at low temperatures. The transition point of Glauber's salt is also available and an exceedingly well defined temperature (32.383), but it is not easy to maintain this temperature for the desired length of time. It is our experience that it takes a considerable time at a given temperature for *all* the cells of a set of standards to attain their true value. This can

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[1] Philos. Trans., A, Vol. 207, 34.
[2] Bull. de la Soc. Electriciens, 1908.

be determined by daily observations when the cells are held at a constant temperature; we generally allow a week. It is therefore necessary to have an automatically controlled bath and 25° has been chosen as sufficiently above room temperature to allow of an exact control of the temperature and also moisture may be more readily excluded than at a lower temperature. Our thermostat is an electrically heated and controlled kerosene bath which keeps well within .01° for any length of time. It consists of a tank made from galvanized sheet iron, 50 × 65 cm. and 50 cm. deep which rests in a box on legs. The space between the box and the tank (6 cm.) is packed with excelsior and the top of the tank projects about 2 cm. above the top edge of the box. A cover, 100 × 100 cm., was provided and the center (50 × 65 cm.) cut out so that the top of the tank fits into this place and is flush with the top of the table part when assembled. Plate glass strips of convenient widths cover the tank proper.

About 100 liters of kerosene serve as the bath liquid and has been found to be very clean and easily kept dry by a dish of calcium chloride which is suspended in the upper part of the tank. A motor-driven stirrer was necessary and attention was given to having it well made. The stirrer shaft turns in a bearing which is held by a cross bar near the top of the tank and a second bearing is soldered to the bottom of the tank. This shaft bears a fan wheel which is close to the bottom of the tank with its fans set rather flat so as to force the bath liquid up from the bottom rather than to rotate it. At the top of this stirrer shaft is the pulley wheel, a few centimeters above the top of the bath.

The regulator and heater have received the most attention, the aim has been to get large surfaces well distributed in the bath liquid. Both the regulator and heater are supported by a frame about 10 cm. above the bottom of the tank. This frame was made of brass tubing, 40 × 55 cm., with two cross pieces and is supported by four legs. Fastened to the under side of this frame is the glass part of the regulator (Fig. 5), which contains the toluene and mercury and the expansion or contraction of these liquids makes or breaks a contact of the relay circuit and so controls the heating current. This regulator was made of 15-mm. tubing with walls

½ mm. thick. It is entirely closed save for the outlet through the U tube *a* which is sealed to the under side, extends down and then up ending in the 5 mm. upright tube *b*. The whole system was
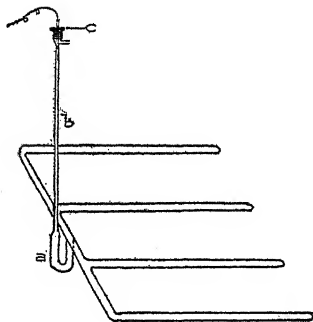

Fig. 5.

filled with toluene, except the U tube and the upright *b* which contains mercury. Air bubbles were rigidly excluded from this system. The top of the mercury column in the tube *b* makes contact with a platinum point and this surface is about 5 mm. in diameter, since it was found that a meniscus of this diameter was much better defined than the narrower ones commonly used while with the large volume of toluene well distributed in the bath liquid, the movement of this mercury surface responds rapidly and exactly to the slightest changes of the bath temperature.

Sparking between the mercury and the platinum point causes the mercury to become "dirty," changes the meniscus and thus causes a slow drift of the bath temperature. It is customary to put a condenser in parallel with this contact but we have had much better results by arranging to use a small current of low voltage in the relay circuit and find that our mercury contact surface remains perfect for months at a time and makes and breaks contact with the platinum point at exactly the same temperature. A common 150-ohm telegraph relay was rewound with no. 36 wire so that the resistance of the magnet coils was about 1,000 ohms. The arm which controls the heating circuit was lengthened and adjusted to work very easily and only required .005 ampere in the relay circuit to operate it. This arrangement, without a condenser, has given excellent results.[1] We have had the best success with a very fine platinum point for the contact with the mercury surface. A narrow glass tube was drawn down to a point and a .05-mm. platinum wire sealed in so that only ½ a millimeter projected out beyond the glass point. A small copper lead wire was passed into

[1] The Weston Electrical Instrument Co. make a relay which operates on a low voltage circuit with a current of only .001 amp. and controls a current of 1 or 2 amps.

the open end of the tube and with a drop of mercury made contact with the platinum wire. This glass tube was fastened into a little 4-mm. brass tube which was threaded and screwed into the metal cap *c*, Fig. 5, and so a very fine adjustment of the bath temperature was possible; rough adjustments were made by adding or removing mercury from the tube *b*.

With the arrangements we have, the heating current is made or broken by a change in the bath liquid of only one or two thousandths of a degree. For constancy, and to avoid oscillations of the bath temperature about the desired point, it was necessary to have the bath well insulated and this was accomplished with the 6 cm. of excelsior packing between the tank and containing box and the plate glass cover. Also it was especially important to have a small amount of heat liberated in the unit of time, and well distributed in the bath liquid. Our heating coils consist of .25 mm.
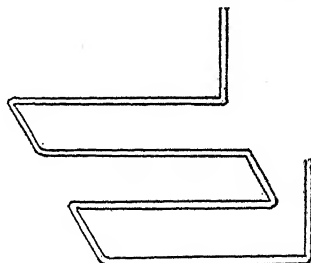

Fig. 6.

nickel wire wound on 10-mm. glass tubes. These tubes were 50 cm. long and the wire wound on with the aid of a lathe and a little guide which gave a pitch of about one millimeter to the windings. These tubes were shellacked to hold the wire in place and then four of them were fused to other tubes forming the arrangement indicated by Fig. 6. The windings were continued by hand on the new tubes and the ends soldered together so as to make one continuous coil with two free ends soldered to platinum wires which passed through the glass at the joints of the upright tubes. Contact was made with the platinum wires inside the upright tubes. The coils were now thoroughly shellacked. This heater was fastened to the lower part of the frame so that it was held in about the center of the bath liquid. The shellack insulated the wire from the bath liquid and has not been affected in the slightest degree by the kerosene while the heat generated in this coil is at once taken up by the bath liquid. The resistance of our coil is about 120 ohms and is connected with the 110-volt a.-c. lighting circuit, but we also use an external resistance as .4 ampere is sufficient to

control the bath even with very considerable variations of the room temperature. The brass frame which carries the regulator and heater is covered with coarse wire netting so that the glass parts underneath the frame are protected, while the cells, in racks, rest on this netting.

It is not only necessary to have a constant temperature for the standard cells, but it is necessary to know what that temperature is. We use a set of 1/50° calibrated thermometers and they are checked at the ice point and the transition point of Glauber's salt (32.383). Some glass tubes pass horizontally through the side of the tank and box at various distances above the bath liquid and through these tubes the thermometers are read with a telescope. We have also checked our 25.00° temperature with a special 50 ohm resistance thermometer using 0°, 32.383 and 100° as the fixed points. Our known and reliable E.M.F. with a potentiometer, standard resistances, and standard batteries make it possible to determine the resistance of the coil of a resistance thermometer with ease and accuracy.

PRINCETON UNIVERSITY,
   October, 1910.

A STUDY OF THE JOULE AND WIEDEMANN MAGNETOSTRICTIVE EFFECTS IN STEEL TUBES.

By S. R. Williams.

IN 1842 Joule[1] discovered that an iron rod changed its length when subjected to a magnetic field whose direction was parallel to the axis of the rod. This variation in length with change in magnetic field strength was found to be an increment up to a certain value of the magnetic field beyond which the rod appeared to contract. Later investigators[2] showed that if sufficiently strong magnetic fields were used the bar actually became shorter than when in its virgin state.

Some years before this interesting discovery of Joule, Wiedemann[3] found that "if a vertical wire is magnetized with its south end uppermost, and if a current is then passed downwards through the wire, the lower end of the wire, if free, twists in the direction of the hands of a watch as seen from above, or in other words, the wire becomes twisted like a right-handed screw if the relation between the longitudinal current and the magnetizing current is right-handed.

"In this case the magnetization due to the action of the current on the previously existing magnetization is in the direction of a right-handed screw around the wire. Hence the twisting would indicate that when the iron is magnetized it expands in the direction of magnetization and contracts in directions at right angles to the magnetization. This agrees with Joule's results." Thus Maxwell[4] explains the phenomenon in his second edition of Electricity and Magnetism and I have quoted directly because in the first edition it is stated oppositely and some of our recent[5] writers have persisted

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[1] Joule, Phil. Mag. (37), vol. 30, pp. 76–225, 1847.

[2] Bidwell, Proc. Roy. Soc., 38, p. 265, 1885; 40, p. 109, 1886. More, Phil. Mag., 40, p. 345, 1895. Nagaoka, Rapports, Congress Internat. de Physique de 1900.

[3] Wiedemann, Elektricität, Bd. 3.

[4] Maxwell, Electricity and Magnetism, 2d ed., vol. 2, p. 87.

[5] Auerbach, Winkelmann's Handbuch der Physik, Elek. u. Mag., 2.

in giving the direction of rotation as Wiedemann[1] gave it originally, which is also incorrect.

Nagaoka and Honda[2] in their exhaustive studies on magnetostrictive effects in iron, nickel and cobalt, have indicated the direction of rotation even more correctly when they say: "The direction of twist in iron, so long as the longitudinal magnetizing field is not strong, is such that if the current is passed down the wire from the fixed to the free end and the wire is magnetized with north pole downwards the free end, as seen from above, twists in the direction of the hands of a watch." That is to say, if one increases the longitudinal magnetizing field from zero up to about three hundred units, c.g.s., the *initial* twist of the rod will be as Nagaoka and Honda have indicated, *viz.*, clockwise; but when the field strength reaches a value between 15 and 30 units a maximum twist in this direction occurs beyond which the twist appears to take place counter clockwise, *i. e.*, in the opposite direction and in a sufficiently strong longitudinal field the twist actually carries the scale reading to the opposite side of the zero point from that on which the maximum twist occurred. It is here perhaps that much of the confusion occurs. For instance, if a strong longitudinal field be imposed suddenly upon the iron bar, instead of gradually increasing it from zero up to that value, the twist will appear to occur counter clockwise, when the directions of the two fields are in a right-handed relation. In throwing on a strong longitudinal field it must necessarily build up from zero but occurring in such a short time the rod, due to inertia effects, does not twist to its maximum value but takes a mean position and then twists from that position to the one it holds for the maximum longitudinal field which gives it the appearance of twisting counter clockwise. I mention this question of direction of twist at some length because of the apparent confusion, when in reality there is none if we are careful to state what the magnitude of the longitudinal field is and whether the longitudinal field is suddenly or gradually built up. We must also keep in mind that the above holds for iron. In nickel the initial twist takes place in just the opposite direction and if we accept Maxwell's

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[1] Wiedemann, Die Lehre vom Galvanismus, Bd. 2, p. 256, 1873.
[2] Nagaoka and Honda, Phil. Mag., 4, p. 61, 1902.

explanation that the Wiedemann effect is a special case of the Joule, then this is what we would expect, as a nickel rod shortens[1] in the Joule effect instead of lengthening as iron does for small magnetic fields.

This paper has for its object the study of these two effects (the Wiedemann and the Joule), in the same samples of steel tubing. So far as the author knows little or no comparative work has been done on the same specimens. In the Wiedemann effect the use of steel tubes enables one to thread an insulated copper wire through the tube so that the current for producing the circular field could be sent either through the wire or the tube itself as a conductor. Both cases were studied. With the current insulated from the tube, the circular field can be more readily calculated, for as Knott[2] points out we know nothing of the distribution of the circular magnetic field inside a solid iron conductor through which a current is flowing.

Description of Apparatus.

In Fig. 1 is shown a diagram of the apparatus as used in this work. C is the magnetizing coil, T, the steel tube with an insulated wire running through the center, X_1, the reversing switch for the current which flows through the steel tube, $Am.$, R_1, the ammeter and resistance in series with the circuit. For controlling the current in the solenoid, C, a reversing switch, X, was connected in series with a variable resistance, R, and ammeter, $Am.$, and a double pole double throw switch, D, whereby either alternating or direct current could be passed through C. The alternating current was used for demagnetizing the steel tubes before each set of readings. The switch, S, served as a short circuit for the ammeter when the alternating current was used. M is a tube containing mercury to serve as a connection for the lower free end of the steel tube and the wire running through them. With this arrangement of circuits the direction of the longitudinal and circular fields could be varied at will and any desired combination used in the Wiedemann effect.

The ammeters were calibrated from time to time by means of

[1] Barrett, Nature, 26, p. 585.

[2] Knott, Trans. Roy. Soc. Edinb., XXXII. (1), p. 193, 1883; XXXV, (2), p. 377, 1899; XXXVI. (2), p. 485, 1891.

a Leeds and Northrup potentiometer. The variable resistance, R, was a water resistance and was most satisfactory where one desired to vary the magnetic field continuously without any steps.
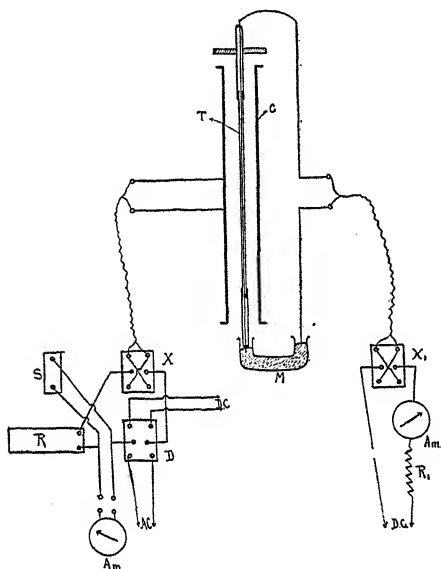

Fig. 1.

I am indebted to the Phœnix Physical Laboratory of Columbia University for the use of the solenoid, C, and wish hereby to express my appreciation of their kindness in loaning it. The coil is 100 cm. in length and wound in eight layers with 6,025 turns in all. It is wound on a very thick-walled brass tube with a slit running the full length of it. This thick-walled tube insured greater rigidity and also cut down heat changes in the space where the tubes were suspended.

I am particularly indebted to the Ellwood Ivins' Tube Works[1] for the excellent specimens of seamless steel tubing which they furnished me. There were four different sizes all of which were, according to manufacturers' notation, called "low carbon steel." For convenience I have designated these specimens, tubes A, B, C and D respectively. Their dimensions are as follows:

TABLE I.

Tube A.	Outer diameter,	.1600 cm.	Tube C.	Outer diameter,	.2447 cm.
	Inner diameter,	.0794		Inner diameter,	.0970
	Length,	79.8		Length,	79.7
Tube B.	Outer diameter,	.2386	Tube D.	Outer diameter,	.2088
	Inner diameter,	.1538		Inner diameter,	.1085
	Length,	80.2		Length,	80.0

The ends of the tubes, as the table indicates, were ten centimeters inside of the ends of the solenoid which gave fairly uniform fields.

[1] Address: Oak Lane Station, Philadelphia, Pa., U. S. A.

Heavy brass tubing was brazed to the ends of the steel tubes for supporting them and making connections. The tubes were suspended vertically in the solenoid and carefully placed along the axis of the same. The support for the tube was independent of the solenoid. A tripod support which rested directly upon the upper end of the solenoid was first tried but the results were very discordant until the supports for the tubes and solenoid were made independent. The whole system of supports was rigidly attached to a thick stone wall of the laboratory.

For a study of the Joule effect the mercury tube, M, Fig. 1, was removed and in its place was used a system of levers, shown in Fig. 2, for measuring the changes in length of the tubes. LL' is a light lever of which L is the fixed end. T is a continuation of the tube which supports the lever at B in a stirrup while S is the solenoid. At L' a silk cord drops downward and coils several times around a small roller to which is attached a mirror, M, for reading the deflection by means of a telescope and scale. A small weight, W, keeps the silk cord stretched. The multiplying power of the levers was
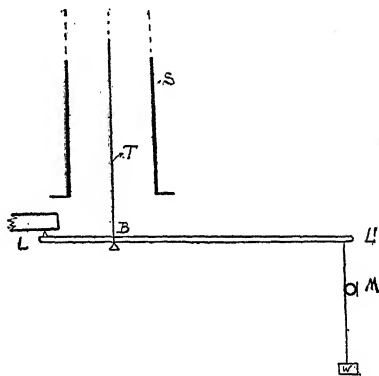

Fig. 2.

determined by attaching the table of the dividing engine to the point, B, and obtaining the deflection of the mirror when B was displaced through a measured distance. The multiplying power was found to be 11,935. This system of levers worked very satisfactorily and would return to its old zero point after the demagnetization of the tubes. Attention is called to the roller on which the mirror was mounted. It had "agate" bearings which may be made in any laboratory. A piece of capillary tubing was broken off squarely and heated in a Bunsen flame until the end was thoroughly fused and the capillary opening commenced to close. At this point it is taken from the flame and the other end treated in the same way. These fused ends of the capillary tubes make excellent bearings

on steel needle points and have very small friction. The capillary tube may be used as a roller itself or short pieces of the tubing may
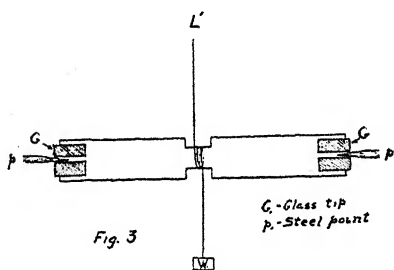

Fig. 3.

be set into the ends of larger metallic cylindrical rods. In this work the glass tips were set in the end of a brass rod which was turned down to a small diameter where the cord passed around it. Fig. 3 shows more distinctly how it worked when supported by steel points.

METHODS OF OBSERVATION.

In all of the specimens the following order of measuring the various tabulated results was used. First, the Wiedemann effect, second, the Joule effect, third, the permeability, fourth, the moments of torsion.

The values for the Wiedemann effect were obtained by keeping the current, producing the circular field, constant and by varying continuously the longitudinal field from zero up to the maximum value obtainable with the storage battery and coil which we have; this was about 275 c.g.s. units. The values thus obtained are the mean of four sets of readings taken in the following way:

TABLE II.

Vertical current flowing down.
Longitudinal field down.

Vertical current flowing up.
Longitudinal field down.

Vertical current flowing up.
Longitudinal field up.

Vertical current flowing down.
Longitudinal field up.

A reference to Fig. 1 will show that these various combinations were readily obtained by the connections and switches employed. The mean values thus obtained for the Wiedemann effect eliminated

the action of the earth's magnetic field and also the dissymmetry of field in setting up the apparatus. Before taking each set of readings the tubes were demagnetized by passing a decreasing alternating current through the solenoid. At least the scale readings returned to zero each time after such a treatment. It is evident that heating the tubes each time for demagnetization was out of the question.

Several values for the vertical current were taken for each tube and these values were repeated when the vertical current flowed along the tube itself as a conductor and not in the wire inside of the tube.

It was found that the direction of the initial twist was always the same whether the circular field was applied first or the longitudinal, but that the twist was smaller if the longitudinal was applied first. Further the two fields were made to vary simultaneously from zero upward by connecting the solenoid and vertical wire in series; the initial twist still maintained the same relation to the two directions of the circular and longitudinal fields. The following table illustrates the relation of the circular and longitudinal fields and the direction of initial twist as viewed from upper end of tube.

Table III.

Circular Field.	Longitudinal Field.	Direction of Twist.
Current down.	Down.	Clockwise.
Current up.	Down.	Counter-clockwise.
Current up.	Up.	Clockwise.
Current down.	Up.	Counter-clockwise.

The direction of the field in the solenoid was determined by a compass needle and the direction of the current in the tube by applying the terminals of a milli-voltmeter to the terminals of the tube and also by the usual test of the compass needle in the neighborhood of a conductor.

The values for the Joule effect are the means of two sets of readings, first, when the longitudinal field was up; second, when the longitudinal field was down, demagnetization occurring between each set of readings. The field was varied continuously.

The permeability of the tubes was determined by the ballistic method and the results are the mean of several sets of readings.

The moments of torsion were determined in a torsion lathe. Two mirrors were attached to the tubes and the twist determined
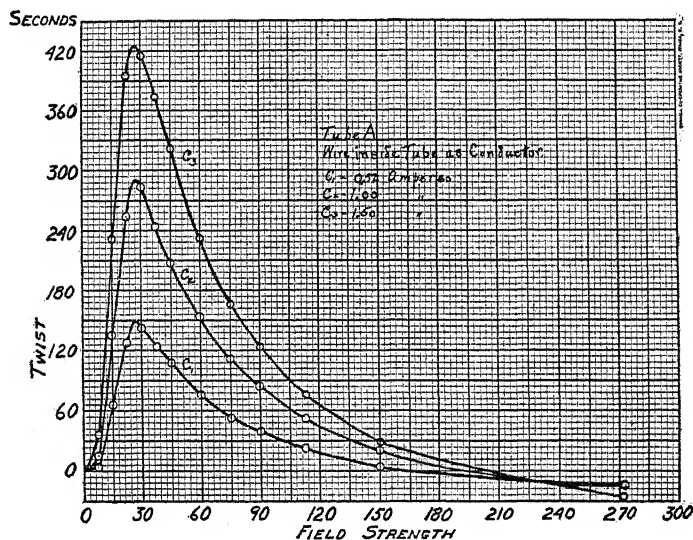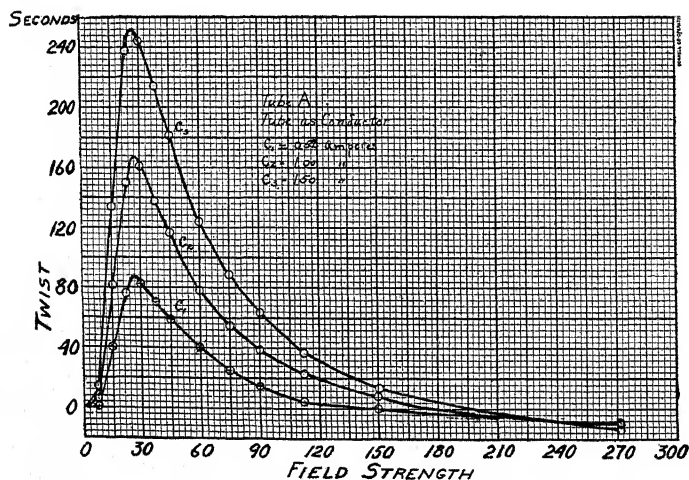

Fig. 4.

Fig. 5.

from the deflections of the mirrors as seen with two telescopes and scales. This eliminated any slipping at the chucks.

TABLE IV.
Tube A.

Longitudinal Field.	Total Twist of Tube in Seconds of Arc.						Total Lengthening in Centimeters.	Permeability.	Moment of Torsion.
	Vertical Current in Wire. Amperes.			Vertical Current in Tube. Amperes.					
	0.52	1.00	1.50	0.52	1.00	1.50			
	Twist.			Twist.					
0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
7.57	4.7	15.7	36.1	0.8	5.8	14.5	0.0	233.08	
15.14	66.3	135.5	231.8	40.0	81.2	134.2	7.46×10^{-7}	389.05	
22.71	128.4	254.1	395.6	76.1	149.2	237.5	——	443.98	
30.28	143.4	282.9	415.3	82.4	160.9	244.2	2.03×10^{-5}	409.09	615 gm. cm.
37.85	124.5	243.5	373.2	70.6	138.1	213.9	——	355.61	
45.42	108.0	209.0	322.1	59.6	116.5	180.2	2.61×10^{-5}	316.78	
60.57	76.6	154.4	232.5	40.0	78.5	125.6	2.82×10^{-5}	255.20	
75.71	53.8	111.9	167.7	25.5	55.7	89.1	2.82×10^{-5}	203.80	
90.85	39.2	84.4	123.7	15.7	39.2	63.6	2.82×10^{-5}	182.62	
106.00	——	——	——	——	——	——	——	161.74	
113.56	23.5	53.0	77.3	4.7	23.5	37.2	——	——	
151.42	4.7	21.5	29.4	0.0	8.6	13.7	2.73×10^{-5}	115.02	
272.56	−13.7	−13.7	−25.5	−8.6	−9.8	−13.7	2.06×10^{-5}	85.94	

TABLE V.
Tube B.

Longitudinal Field.	Total Twist of Tube in Seconds of Arc.						Total Lengthening in Centimeters.	Permeability.	Moment of Torsion.
	Vertical Current in Wire. Amperes.			Vertical Current in Tube. Amperes.					
	1.00	1.50	2.00	1.00	1.50	2.00			
	Twist.			Twist.					
0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.00	
7.57	53.0	57.1	113.3	4.8	14.8	18.9	0.00	706.32	
15.14	167.6	234.0	323.7	45.0	63.1	73.1	4.58×10^{-6}	739.40	
22.71	181.7	266.1	357.9	56.2	75.1	90.4	——	602.90	2,704
30.28	170.8	246.0	333.6	56.2	73.1	91.2	1.36×10^{-5}	470.61	gm. cm.
37.85	150.7	217.9	293.5	51.0	69.5	82.4	——	390.64	
45.42	130.6	189.0	256.1	46.6	64.4	75.1	1.61×10^{-5}	329.31	
60.57	101.3	144.7	197.0	40.2	53.0	63.1	1.65×10^{-5}	267.38	
75.71	77.2	110.5	151.5	34.1	45.0	54.2	1.65×10^{-5}	219.87	
90.85	63.1	88.4	119.4	28.9	37.7	45.0	1.64×10^{-5}	190.41	
106.00	——	——	——	——	——	——		164.55	
113.56	44.2	63.1	84.4	18.9	30.9	34.9	——	——	
151.42	20.9	28.9	48.2	12.5	22.1	26.9	1.56×10^{-5}	120.66	
272.56	2.8	2.0	8.8	4.0	10.8	16.8	1.25×10^{-5}	82.18	

Tables IV., V., VI. and VII. give the various results for tubes
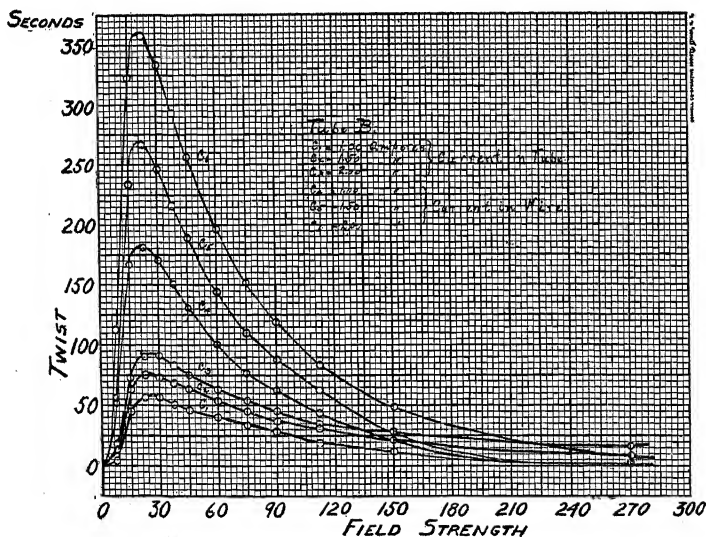A, B, C and D respectively, which are plotted in Figs. 4, 5, 6, 7, 8,
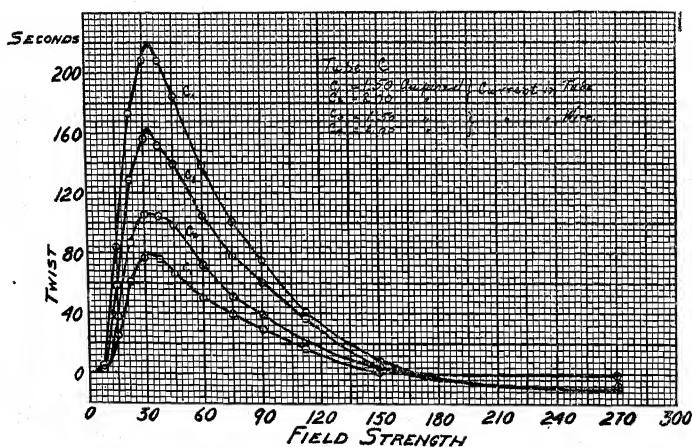

Fig. 6.

Fig. 7.

9, 10, 11 and 12. Fig. 4 shows the total twist for tube A when three
different vertical currents were used and which flowed along the
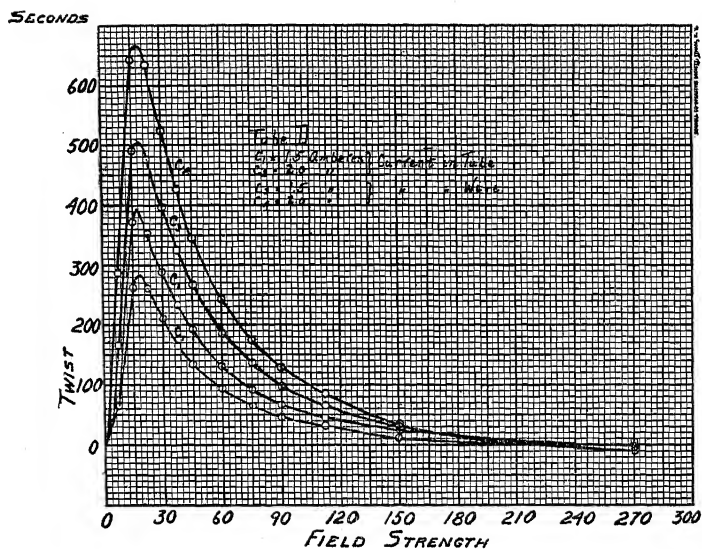insulated wire inside of the tube. Fig. 5 represents the total twist

Fig. 8.

TABLE VI.

Tube C.

Longitudinal Field.	Total Twist of Tube in Seconds of Arc.				Total Lengthening in Centimeters.	Permeability.	Moment of Torsion.
	Vertical Current in Wire. Amperes.		Vertical Current in Tube. Amperes.				
	1.50	2.00	1.50	2.00			
	Twist.		Twist.				
0.00	0.0	0.0	0.0	0.0	0.00	0.00	
7.57	2.8	4.8	0.8	0.0	0.00	133.51	
15.14	58.8	84.1	24.8	38.0	0.00	381.69	
22.71	128.2	173.1	60.9	86.9	———	440.72	3,504
30.28	156.3	208.4	76.1	106.2	0.53×10^{-5}	404.11	gm. cm.
37.85	151.1	208.4	76.1	105.0	———	362.56	
45.42	139.0	185.2	66.9	99.0	1.28×10^{-5}	322.15	
60.57	105.0	138.2	50.1	72.9	1.59×10^{-5}	265.30	
75.71	78.1	100.2	38.8	51.0	1.59×10^{-5}	220.98	
90.85	60.0	74.9	29.6	38.8	1.59×10^{-5}	191.58	
106.00	———	———			———	168.37	
113.56	36.8	40.0	16.4	22.0	———		
151.42	4.0	8.0	2.8	6.0	1.55×10^{-5}	120.83	
272.56	−8.8	−10.8	0.0	−0.8	1.34×10^{-5}	72.27	

for A when the same vertical currents as in Fig. 4 were used, only they flowed in the tube itself. Fig. 6 shows the same thing for tube B, the three upper curves for the case when the current is flowing in the wire inside of the tube and the three lower when the tube was the conductor. In all of the tubes the maximum twist was greater when the current was insulated from the tube than when flowing in the tube itself. Similarly Figs 7 and 8 show the total twist for tubes C and D respectively with the two upper curves for current in wire inside of tube and the two lower curves for the vertical current in the tube. Fig. 9 represents the total twist for the four tubes under exactly the same conditions. In these curves the vertical current was insulated from the tubes. Fig. 10 indicates the Joule effect for the four tubes under the same conditions. Figs. 11 and 12 show their permeability.

TABLE VII.

Tube D.

Longitudinal Field.	Total Twist of Tube in Seconds of Arc.				Total Lengthening in Centimeters.	Permeability.	Moment of Torsion.
	Vertical Current in Wire. Amperes.		Vertical Current in Tube. Amperes.				
	1.50	2.00	1.50	2.00			
	Twist.		Twist.				
0.00	0.0	0.0	0.0	0.0	0.00	0.00	
7.57	167.9	288.0	69.2	75.3	0.00	668.74	
15.14	490.2	642.6	263.8	372.6	0.31×10^{-5}	774.99	
22.71	477.3	634.5	262.6	352.5	———	631.90	1,876
30.28	396.8	524.5	211.5	288.8	1.56×10^{-5}	506.04	gm. cm.
37.85	324.2	427.0	165.1	234.4	———	412.47	
45.42	266.6	347.2	135.7	192.1	1.69×10^{-5}	354.87	
60.57	186.1	241.7	92.6	130.9	1.68×10^{-5}	277.21	
75.71	136.9	175.2	65.2	91.4	1.65×10^{-5}	225.10	
90.85	99.5	129.7	47.1	69.2	1.63×10^{-5}	194.43	
106.00	———	———	———	———	———	167.49	
113.56	66.4	80.5	31.0	45.1	———	———	
151.42	30.2	34.2	10.8	26.1	1.51×10^{-5}	122.07	
272.56	−2.8	−8.8	−0.8	4.8	1.17×10^{-5}	81.94	

Discussion of Results.

In Figs. 9, 10, 11 and 12 are shown the lengthening and twist and permeability of the four tubes.   It is interesting to note that
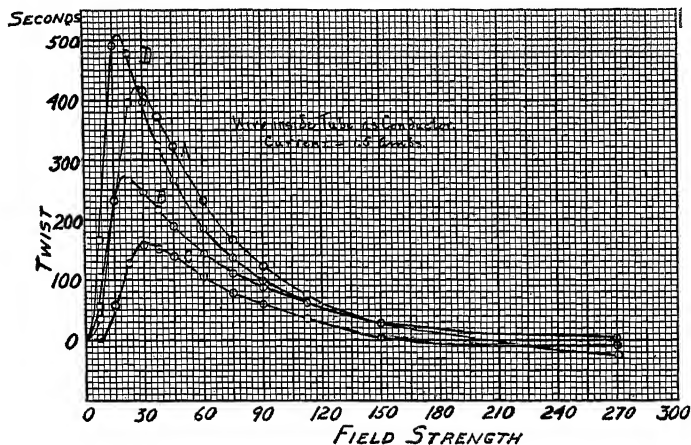

Fig. 9.

the curves for lengthening and twist of the tubes A and C have this characteristic in common with the permeability curves that thei1
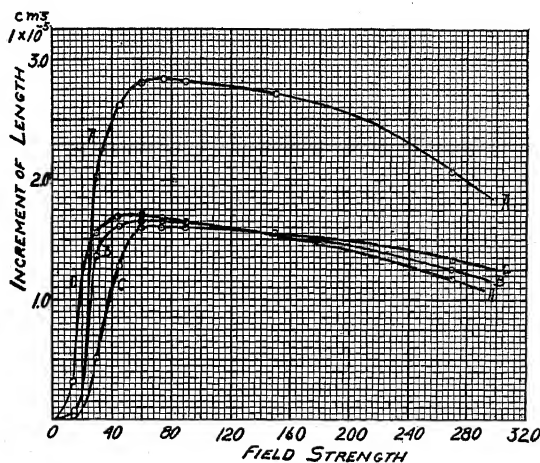

Fig. 10.

maxima occur at a higher value of the longitudinal field than do those of the tubes B and D. Apparently the permeability is the

only property of the tubes which gives a decided color to the character of the curves of twist and lengthening. From the curves
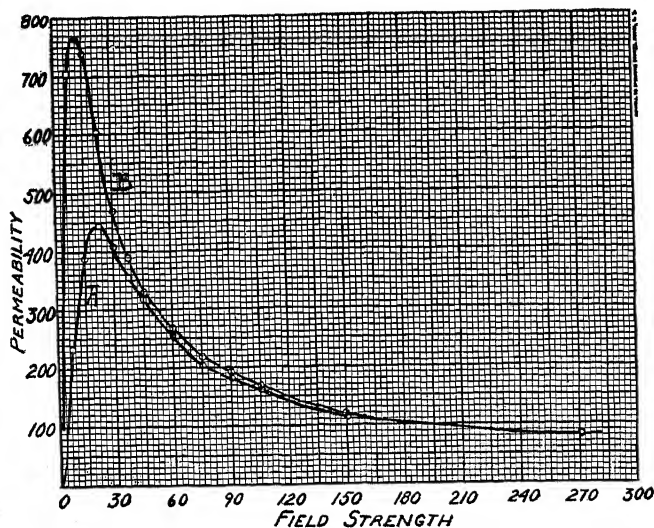
Fig. 11.
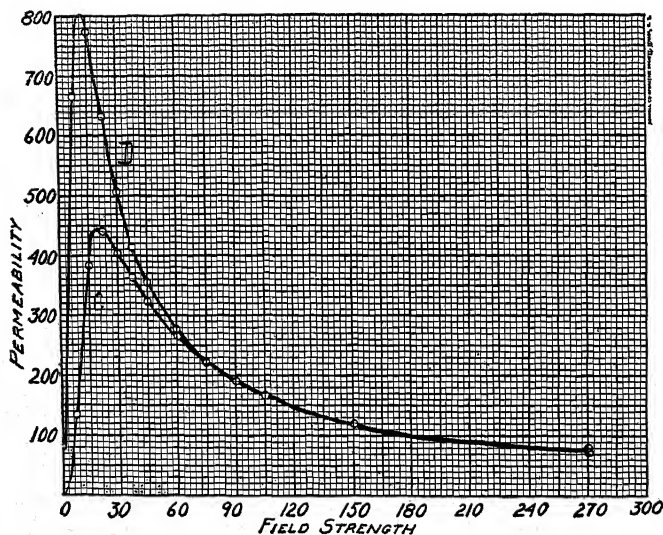
Fig. 12.

showing the Wiedemann and Joule effects it would also appear that the permeability influenced the twist more than the lengthening.

In no other way, it seems, can one account for the general results which I have tabulated in the following manner:

TABLE VIII.

Maximum twist,	$D > A > B > C.$
Maximum lengthening,	$A > D > B > C.$
Maximum permeability,	$D > B > A > C.$
Moment of torsion,	$A < D < B < C.$
Mean radius,	$A < D < C < B.$

From Maxwell's explanation one might be led to suppose that in the steel tube showing the maximum lengthening one would find a maximum twist. This does not hold for all of the tubes as shown by the curves for A and D. Their moments of torsion, mean radius and lengthening all indicate that A ought to twist more than D for a given torque, but such is not the case and although I repeated my results with numerous imposed conditions the relations still held. If the permeability is an important factor in the twist, we would expect D to twist more than A, since the maximum permeability of $D > B > A$. This reasoning however does not hold for tubes A and B when they are compared, but here either the moment of torsion or the mean radius at which the circular field is acting may be the important factor, as their difference in permeability is not as great as that of D and A. The results at least lead to the conclusion that there is no simple relation between the Joule and Wiedemann effects.

The comparative study of the twist in the Wiedemann effect when the vertical current is flowing in the wire and when flowing in the tube show that the maximum twist is from two to four times as great in the first case as in the second. I suppose this is to be ascribed to the fact that when the vertical current is flowing in the tube itself as a conductor there is not as large a circular magnetic field operative on the tube as when the same current is flowing in the wire, for there is no magnetic force in the interior of a cylindrical tube conveying a current.

As pointed out at the beginning, the use of tubes in the study of the Wiedemann effect offers some advantages over solid rods: first, one can determine the circular magnetic field when the current is confined to an insulated wire inside of the tube, secondly, for the

same vertical current the Wiedemann effect is increased when the current flows in the wire.

For the tube *A*, the moment of torsion equals 615 gram-centimeters. The maximum twist for a vertical current of 1.5 amperes is equal to 415.3 seconds. This means that a torque of about 1.24 gram-centimeters would have been necessary to have produced the same twist. If we take as our mean radius of *A* a value of .06 centimeters it would mean a force of 20 + grams weight acting at that arm's length to produce the effect. It is interesting to consider in connection with this that due to the circular magnetic field about the vertical current that the lower free end of the tube, since it is a magnetic pole in the Wiedemann effect, tends to rotate about the current with a torque equal to $2mI$ where m equals the pole strength and I is the current. The direction of this torque would be that of the initial twist in the Wiedemann effect. Computing this for the case where the permeability is a maximum in the tube *A* shows that this torque has a value of only a few dyne-centimeters and hence may be neglected.

If the equipment had been available I would have preferred to have carried out these experiments on a much larger scale, *viz.*, by using tubes several meters long and consequently of larger diameters and corresponding solenoid for producing the longitudinal fields. Limited thus I present the results obtained from the equipment I had at my disposal.

Mr. Dorsey[1] in a recent article on the Joule effect gives an excellent bibliography of the subject; most of the articles referred to were available in this work.

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    OBERLIN, OHIO.
        November 5, 1910.

[1] Dorsey, PHYS. REV., vol. 30, p. 698, 1910.

THE CAUSES OF ZERO DISPLACEMENT AND DEFLECTION HYSTERESIS IN MOVING-COIL GALVANOMETERS.[1]

By Anthony Zeleny.

§ 1. Zero Displacement after a Deflection.

IN a moving-coil galvanometer the displacement in the zero reading which takes place after a first deflection, notwithstanding the conclusion of A. Stansfield,[2] has generally been ascribed to a true set in the suspension fiber. The following investigation proves the view to be erroneous and that practically the whole effect is due to the action of magnetic impurities within the coil.

A galvanometer coil, G, was suspended within a wooden frame between the poles of a large electromagnet as shown in Fig. 1. If the zero displacement is due to a true set in the fiber its magnitude after reversed deflections of equal size would be independent of the intensity of the magnetic field, but if the effect is due to magnetic impurities within the coil the magnitude of the displacement would increase with the intensity of the field.
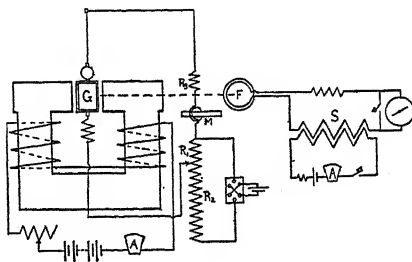
Fig. 1.

The deflecting current was varied so as to produce the same deflection with each altered intensity of the field. In all cases the coil was made to move aperiodically to its deflected position as well as to its zero or null reading, in stronger fields by placing the proper resistance in R_3 of the galvanometer circuit and in weaker ones by induced currents from an accessory coil and magnet M. The

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intensity of the field was measured by jerking the test coil F through the field by means of a spring and comparing the ballistic galvanometer throws with those obtained from a standard inductance coil[1] S. The test coil had the same area as the pole pieces and measured the intensity of the field in the immediate neighborhood of the poles. The galvanometer coil G was at some distance from the poles, and the intensity of that part of the field containing it was obtained by multiplying the values given by the test coil by the approximate, experimentally determined factor 0.768.

The galvanometer coil was taken from a Leeds and Northrup galvanometer (Type P, no. 10598), and was the one of several tested that contained the smallest amount of magnetic impurities (see § 5 and Table IV.). The coil is a hollow rectangle, 2.05 by 5.1 cm., and has a mass of 6.66 grams and a moment of inertia of 3.49 c.g.s. units. The upper suspensions employed were of phosphor-bronze strip and the lower suspension was an ordinary 1.5 mil phosphor-bronze strip spiral of 7.5 turns, each loop being 5 mm. in diameter. The torsional moment when employing a 3-mil upper suspension was 2.37 c.g.s. units and when employing a 1.5-mil suspension, 0.24 c.g.s. units. Unless otherwise stated the distance of the circular reading scale in all cases was 54.5 cm.

The observations for the zero displacement of the coil for any particular field intensity were taken at equal intervals of one minute (see § 4), after several successive deflections alternating in direction. The zero displacement after the first reversal was always considerably larger than that observed after the following ones which gave nearly equal though consistently diminishing values. The following observations give a fair illustration of the relative values of the displacement after successive reversals: 2.41, 2.08, 2.06, 2.05 cm. The displacement of the null reading in fields of different intensity, when using the 3-mil upper suspension and reversing the current from a deflection of 20 cm., is given in Table I., the displacement after the first reversal being discarded and the average of the following three taken.

[1] A. Zeleny, PHYS. REV., vol. 23, p. 411, 1906. E. B. Rosa, PHYS. REV., vol. 24, p. 241, 1907.

TABLE I.

Intensity of Field.	Zero Displacement after Reversal.
6	.015 cm.
86	.025
169	.05
220	.09
384	.24
614	.54
1,046	1.24
1,400	1.73
1,750	2.06
2,009	2.25
2,177	2.33

The data of Table I. are plotted in the curve *A* of Fig. 2. It is observed that in fields whose intensity is less than 100 the displacement curve is nearly horizontal and the point where it cuts the axis of ordinates represents the amount of displacement due to a true set in the suspension fiber. This however is so small that under ordinary conditions of use it would rarely be observable. Under the conditions given, the true set in the fiber is represented by a displacement of 0.015 cm., while the zero displacement in a field whose intensity is 400, such as is common in ordinary galvanometers, is 0.26 cm. With this coil and suspension, less than 6 per cent. of the observed zero displacement in a galvanometer is due to a true set in the suspension fiber and more than 94 per cent. to the magnetic hysteresis within the coil.
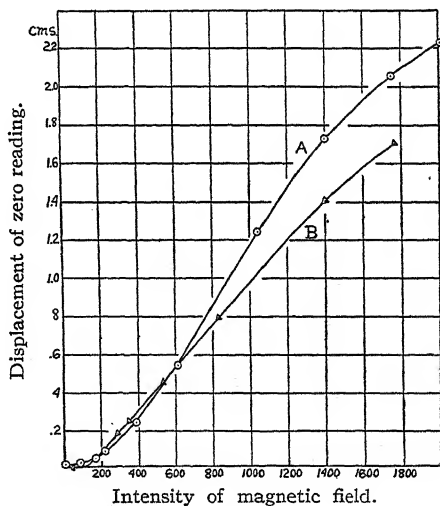

Fig. 2.

With a 1.5-mil phosphor-bronze strip suspension the zero displacement, under the same conditions as above, gave in the weakest field a displacement not exceeding 0.005 cm., or one third that

obtained with the 3-mil strip. It is seen, therefore, that the larger zero displacement observed in galvanometers with the thinner suspensions is due entirely to the magnetic properties of the coil and not at all to a greater set in the fiber. The magnitude of the torsional moment of the suspension practically alone determines the amount of the zero displacement for the same alteration in the magnetic moment produced by the reversal.

The results of a set of observations taken with a 1.5-mil phosphor-bronze strip suspension are given in Table II. and are plotted in curve *B* of Fig. 2. In this case the reversals were taken from a deflection of 15 cm. in place of 20; and the distance of the scale was 125 cm. in place of 54.5.

TABLE II.

Intensity of Field.	Zero Displacement after Reversal.
41	.000 cm.
291	.175
346	.24
538	.45
837	.79
1,402	1.40
1,766	1.70

The curves *A* and *B* are not coincident or parallel to each other because the deflections and the distances were not the same in the two cases and, in addition, the position of the zero point changed with the intensity of the field and by different amounts in the two cases, so that in fields of the same intensity the lines of force did not have the same relative direction with respect to the coil.

In the case given in Table II. the zero displacement in the weakest field was too small to be observable, and, therefore, no appreciable set existed in the suspension fiber in any of the displacements observed.

§ 2. Change in the Zero Readings with the Intensity of the Magnetic Field.

The change in the zero reading with the intensity of the magnetic field is shown for the two cases of § 1, in Fig. 3. The change is approximately proportional to the intensity of the field, and for

the 1.5-mil suspension is approximately five times that for the 3-mil suspension. The changes although observed with the scale at a distance of 125 cm. are plotted in the curve *D* from values reduced to what they would have been at the same distance as that of the similar observations with the 3-mil suspension. In each curve the change of zero is reckoned from its observed value in the weakest field.
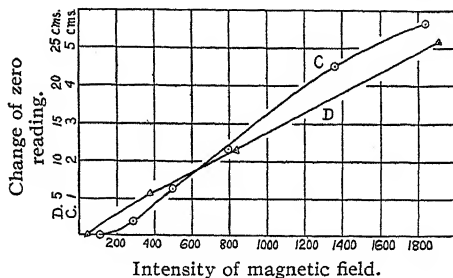

Fig. 3.   *C*, 3-mil suspension; *D*, 1.5-mil suspension.

The plane of the coil was set perpendicular to the faces of the poles of the electromagnet in the weakest field employed, and it turned visibly from this position as the field increased, suggesting that the magnetic impurities were not uniformly distributed through

Reversed deflection.
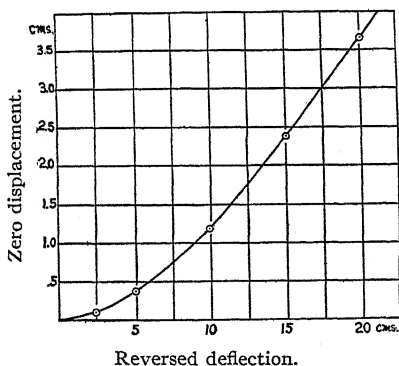
Fig. 4.

the body of the coil; but it was shown experimentally that at least a considerable part of the change was due to an alteration in the direction of the magnetic field. The position of the coil from which the same current gave equal deflections on the two sides of the zero, shifted with the change of intensity of the field.

§ 3. RELATION OF THE ZERO DISPLACEMENT TO THE MAGNITUDE OF THE REVERSED DEFLECTION.

The relation of the zero displacement to the magnitude of the reversed deflection is shown in Fig. 4. The 1.5-mil upper suspension was employed, and the intensity of the magnetic field was 1,244 c.g.s. units.

§ 4. THE CHANGE WITH TIME OF THE ZERO READING AFTER A REVERSED DEFLECTION.

The zero reading obtained after a reversed deflection changes slowly with time. This is shown in Table III. for a 1.5-mil suspension fiber and a field of 1,244 c.g.s. units, when the zero displacement at the end of one minute was 3.34 cm.

TABLE III.

Time.	Change in Zero Displacement.
10 sec.	(0.00) assumed.[1]
30 "	0.03 cm.
1 min.	0.05
5 "	0.08
60 "	0.09

§ 5. MAGNITUDE OF THE MAGNETIC MOMENT OF SOME GALVANOMETER COILS.

Several coils were suspended in succession in a magnetic field of 307 c.g.s. units. The upper suspension was a 1.5-mil phosphor-bronze strip 12.3 cm. in length, which with the lower spiral had a torsional moment of 0.171 c.g.s. units. The period of vibration in each case was determined first without and then within the field. Letting t_1 and t_2 = the periods of vibration, T = the torsional moment, and $\dot{M}$ = the magnetic moment due to the action of the field upon the impurities less the smaller moment due to the diamagnetic property of the copper, then

$$t_1 = 2\pi \sqrt{\frac{I_0}{T}} \quad \text{and} \quad t_2 = 2\pi \sqrt{\frac{I_0}{T + M}}; \quad \text{and} \quad \frac{M}{T} = \frac{t_1^2 - t_2^2}{t_2^2}.$$

Table IV. gives the results obtained for the several similar galvanometer coils. The magnetic moment in these cases had nearly the same magnitude as the torsional moment.

TABLE IV.

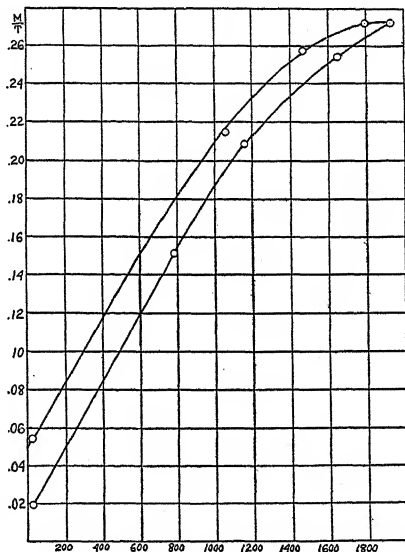| Coil Number.	$\frac{M}{T}$	M
6,788	0.87	0.144
9,127	1.31	0.217
10,597	0.82	0.136
10,598	0.78	0.129
	0.95	0.157

[1]It required 10 seconds to bring the coil to rest after a deflection.

The 3-mil suspension with the lower spiral was found to have a torsional moment $T' = 2.37$ c.g.s. units, which with the above average value of M makes $M/T' = 0.066$.

§ 6. DEFLECTION HYSTERESIS.

In a former paper[1] on Precision Measurements with the Moving-coil Galvanometer the writer observed that deflections of different magnitude were obtained with the same current, depending on whether they reached their value from a larger or a smaller one. This was explained as due to a change in the intensity or direction of magnetization of the impurities within the coil. Although the deflecting moment remained the same the restoring moment was changed by an alteration in the magnetization of the coil. In order to account for the observed changes, it was necessary to assume that the coercive power of the magnetic impurities was very large. To prove, experimentally, that this is so, the values of M/T were now obtained in various fields with the intensity increasing to a maximum and then decreasing. The hysteresis curve obtained, when using a 3-mil suspension fiber, is plotted in Fig. 5.

Intensity of magnetic field.

Fig. 5.

The coercive power is found to be sufficient to account for the observed deflection hysteresis. It will be seen, also, that the upper bends in the curve begin, as should be the case, in a field of the same intensity as the upper bends of the zero displacement curves in Fig. 2. These bends occur in a field whose intensity is more than 200 times that which produces the same effect in the case of iron.

[1] PHYS. REV., vol. 23, p. 401, 1906.

The shape of the curve is slightly affected by the change in the zero position of the coil (see § 2). The damping factor changed with the intensity of the field as given in § 7, but not enough to affect the period of vibration sufficiently to make corrections necessary for the purpose under consideration. The stray magnetism was found to have no appreciable effect upon the torsional moment of the suspension fiber.
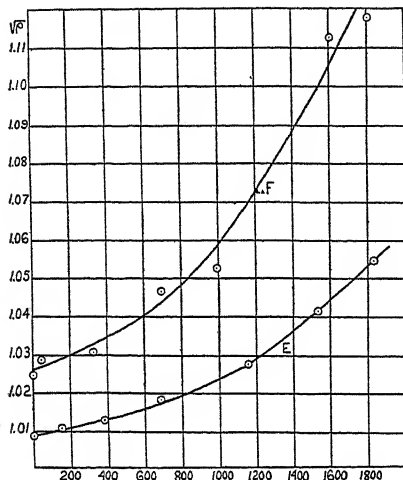
Intensity of magnetic field.

Fig. 6.

§ 7. Change in the Damping Factor with the Intensity of the Field.

It is of some interest to note the experimental values of the damping factor for the same coil in fields of different intensity. Table V. gives the observed values of $\sqrt{\rho}$ on open circuit and of M/T, for several field intensities, when using a 3-mil phosphor-bronze suspension. The resistance of the galvanometer coil with the suspension was 116 ohms.

Table V.

Field Intensity.	$\frac{M}{T}$	$\sqrt{\rho}$
0	(0.000)	1.0088±5
154	.038	1.0108
373	.075	1.0127
685	.133	1.0180
1156	.204	1.0272
1530	.250	1.0411
1832	——	1.0541

These values of $\sqrt{\rho}$ are plotted in the curve E of Fig. 6, and similar values for the 1.5-mil suspension, in the curve F of the same figure. The irregularity of the observed points is due mainly to the difference in the magnitude of the arcs of vibration employed, which varied from 6 to 20 cm. on a scale 122 cm. distant.

It is seen from the curves that in field intensities of 400, common to galvanometers, when the coil is on open circuit, the damping due to the induced currents is less than 50 per cent. of that due to the resistance of the air and the internal friction of the fiber.

§ 8. Change of Sensibility with Alteration of Intensity of the Magnetic Field.

The sensibility of the galvanometer formed by the electromagnet and the coil was found, as expected, to increase at a smaller rate than the intensity of the field. The observed values, when using a 3-mil suspension and having the reading scale at a distance of 60.5 cm., are given in Table VI. The suspension, however, was not the identical one used in the previous observations.

Table VI.

Intensity of Field.	Current per 1 mm. Deflection.	Current per 1 mm. Deflection per Field Intensity of 100.
445	1.53×10^{-8}	6.8×10^{-8}
932	$.84 \times 10^{-8}$	7.8×10^{-8}
1315	$.70 \times 10^{-8}$	9.2×10^{-8}

§ 9. Summary.

The zero displacement observed after a deflection in a moving-coil galvanometer is shown, for phosphor-bronze suspensions, to be due almost entirely to the change in the intensity and direction of magnetization of the magnetic impurities within the coil. The amount due to a true set in the fiber is negligible under ordinary conditions of use, and in the cases investigated the largest true set observed was less than 6 per cent. of the whole zero displacement.

The hysteresis curve of the magnetic impurities within the coil shows a very large coercive power sufficient to account for the observed deflection hysteresis in moving-coil galvanometers. The upper bend of the curve is at a field intensity of about 1,600, which is great compared to the corresponding intensity for iron or steel.

During the progress of the experiments several other properties of moving-coil galvanometers were investigated:

1. The relation of the zero displacement to the magnitude of the reversed deflection, and the change of the zero reading with time, were determined.

2. The moment due to the magnetism in the galvanometer coils was found to differ considerably even in instruments of the same type and from the same manufacturer. In the similar coils tested, the magnetic moment, less the diamagnetic moment, in the same field varied from 0.129 to 0.217 c.g.s. units.

3. The values of the damping factor for the same coil in fields of different intensities were determined. More than two thirds of the damping on open circuit was found, for ordinary galvanometers, to be due to the resistance of the air and the internal friction of the fiber, and less than one third to the induced currents.

4. The change of sensibility with the intensity of the field was determined with a coil of known magnetic properties and a suspension of approximately known torsional moment.

PHYSICAL LABORATORY,
    UNIVERSITY OF MINNESOTA,
        November 16, 1910.

REMARKS ON A PAPER BY J. S. STOKES ON "SOME CURIOUS PHENOMENA OBSERVED IN CONNECTION WITH MELDE'S EXPERIMENT."[1]

By C. V. Raman.

AT the conclusion of the paper referred to, the writer remarks: "As far as the demonstrator has been able to ascertain, these phenomena have not been hitherto observed or described by any one." I may therefore be permitted to observe that the phenomena described in the paper (with the exception of the effect on the rotation of the pulley of waxing the thread) were observed by me about five years ago, when working in collaboration with Mr. V. Apparao at the Presidency College, Madras, and were subsequently shown by us to a large number of others. As however, other phenomena of interest were then observed which seemed to be of greater importance and which could be explained less readily, I did not seek an opportunity of publishing my observations on the subject.[2]

I may also state that the phenomena of the rotation of the pulley was independently observed about two years ago by Mr. A. W. Porter, who published a note on the subject in Knowledge and Scientific News at that time.

My observations furnish a clue for the explanation of an effect which Mr. J. S. Stokes says he is unable to account for, *i. e.*, the increased speed of rotation of the pulley when the overhanging vertical portion of the string was vibrating and the horizontal part seemed not to be doing so. In a future paper I shall endeavor to show that with the large amplitudes of oscillation maintained in practice, Lord Rayleigh's theory of the maintenance of oscilla-

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[1] Phys. Rev., May, 1910, p. 659.

[2] For the outcome of some of the other observations mentioned, the following publications in Nature may be referred to: On the Small Motion at the Nodes of a Vibrating String, Nature, November 4, 1909; The Maintenance of Forced Oscillations of a New Type, Nature, December 9, 1909; also an addendum to the latter in Nature, February 10, 1910.

tions by forces of double frequency[1] has to be seriously modified to enable the experimentally observed phase-relations to be accounted for. It comes out that we are not dealing with one, but with two variations of tension: not merely with the variation of tension *imposed* by the motion of the prong, but also with the periodic part of the variation of tension associated with an oscillation of large amplitude. In actual practice, these two variations approach equality in amplitude and opposition in phase, with the result that when the string is maintained in vigorous oscillation, the net periodic variation of tension is very small indeed compared with what we should expect from the range of excursion of the end of the string which is attached to the prong; the torque exerted upon the pulley and the speed of rotation, if any, of the latter are quite small. But when the string between the fork and the pulley has no transverse oscillation, the full effect of the oscillation of the prong is felt in varying the tension of the string periodically, and the torque on the pulley being greater, the speed of rotation of the latter becomes very considerable. This is exactly what is observed.

[1] Phil. Mag., 1887, and Theory of Sound, 2d ed., Vol. I., pp. 82–84.

THE SMALL MOTION AT THE NODES OF A VIBRATING STRING.[1]

By C. V. Raman.

IT is generally recognized that the nodes of a string which is maintained permanently in oscillation cannot be points of absolute rest, as the energy requisite for the maintenance of the vibrations is transmitted through these points. I have not however seen anywhere a discussion or experimental demonstration of some peculiar properties of this small motion. I shall therefore endeavor to give an account of some experiments and observations of mine relating to this subject.

2. Some rather striking effects are observed when the small motion at a node is viewed stroboscopically, *i. e.*, under periodic illumination. For this purpose, the frequency of intermittence of the light should be nearly twice that of the oscillation of the string. A tuning-fork maintains the string in oscillation in any convenient number of loops, by imposing a transverse obligatory motion at one point of it. Another tuning-fork, which has nearly twice the frequency of the other, forms the interrupter of a Ruhmkorff's coil, the spark from which furnishes the periodic source of illumination. Both forks are electrically self-maintaining. The string is seen in *two* slowly-moving positions, which represent opposite phases of the actual motion. If the nodes were points of absolute rest, then the two positions seen under the periodic illumination would intersect at fixed points. On account, however, of the small transverse motion at the nodes, the points of intersection or "fictitious nodes" are seen to execute a motion of *large* amplitude parallel to the string—the range of the motion being equal to the whole length of a loop. This motion, best seen under a magnifying glass, is represented in Fig. 1, in which nine successive stages at equal intervals of a complete cycle are shown.

[1] A preliminary note on this subject was published in Nature, November 4, 1909, as a letter to the Editor.

3. The "fictitious" node is in the first stage at the center of the field. It then moves to one side of the field, first slowly, then more rapidly; at the fourth stage, it is off 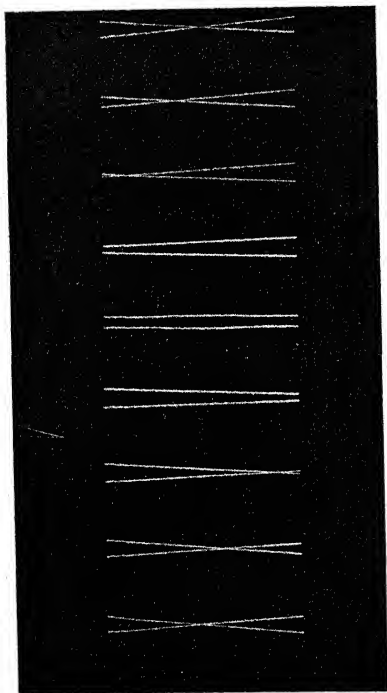the field; at the fifth, the two positions of the string are, at the center of field (*i. e.*, at the position of the node in the actual oscillation), sensibly parallel to each other. It then reappears on the other side of the field, moves in rapidly, then more slowly; at the ninth stage, it is back again at the center of the field.

Fig. 1.

3. A motion of the type shown in Fig. 1 can be represented mathematically by the expression $A \sin \alpha x \cos pt + B \cos \alpha x \sin pt$, where x is the distance from the center of the field, t is the time, the other quantities being constants or nearly so. The two terms differ in phase by quarter of an oscillation. The significance of this is that the small motion at the node and the large motion elsewhere differ in phase by quarter of an oscillation. If the expression for the displacement were of the type $A \sin \alpha x \cos pt + B \cos \alpha x \sin (pt + E)$, the two terms differing in phase by more or less than $\pi/2$, the motion would not be of the type shown in Fig. 1. It would be unsymmetrical, the velocity of the point of intersection when at a given distance from the center of the field on one side and approaching it, being much greater than its velocity when at the same distance on the other side and receding from the center.

5. Another experiment, which was first performed by me in collaboration with Mr. V. Apparao, of the Presidency College, Madras, was found later to furnish a second method of determining the phase of the small motion at the nodes. The principle of this

method was to compound the oscillation at every point on the string with another perpendicular to it of half the frequency and to observe the compound oscillation at the nodes and elsewhere. A string can be maintained permanently in a compound oscillation of this character by attaching one end of it to the prong of an electrically-maintained tuning-fork, so that it lies in a plane perpendicular to the prongs, but in a direction inclined to their line of vibration.[1]

6. A beautiful and interesting type of stationary oscillation is maintained when the tension is somewhat greater than that necessary for the most vigorous maintenance. The curves described by points on the string are then parabolic arcs.[2] As the frequency of oscillation in one plane is half that in a perpendicular plane, there are *two* vibration-loops in the latter for every *one* in the former. The consequence of this is that the parabolic arcs which form the paths of points on the string have their curvatures in opposite directions in alternate halves of a big loop, *i. e.*, in alternate small

Fig. 2.

loops. The surface generated by the moving string is one of great delicacy and purity, and an adequate idea of it can only be had on actually performing the experiment. The photograph herewith
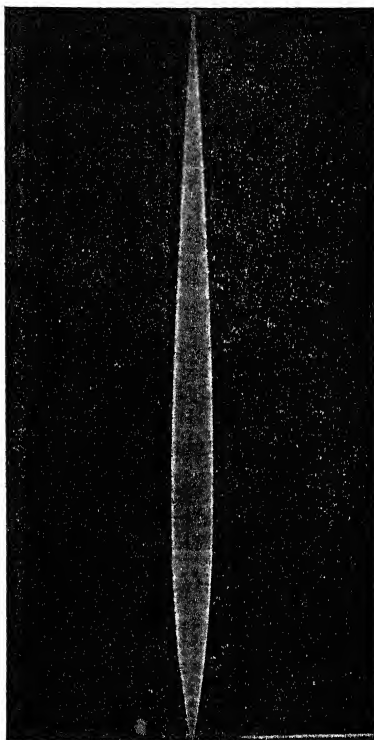
[1] Under these circumstances, the motion of the prong may be resolved into two components, one perpendicular, and the other parallel to the string. The first maintains an oscillation having the same frequency as that of the fork, and the secomd maintains an oscillation having half that frequency. The two oscillations occur, or can be made to occur in perpendicular planes.

[2] A parabolic arc is one of the Lissajons figures for the 1:2 composition ratio.

published (Fig. 2) gives only a very feeble idea of the effect. It can be seen from the shading that the paths of points on the string are curved arcs, and that the curvatures are in opposite directions in the two halves of the loop (which appear unequal in the photograph as the string pointed towards the lens of the camera).

7. For the study of the small motion at the nodes of the oscillation excited by transverse obligatory motion, it is necessary that the tension of the string be adjusted so that maintenance is as vigorous as possible. When this is done, it is noticed that points on the string (except near the nodes) describe 8 curves. The curve at the node, *i. e.*, the path compounded of the small motion at the node, and the large motion of half the frequency perpendicular to it, in neither an 8 curve not a straight line, but is a flat parabolic arc. From this, the phase-difference is again seen to be $\pi/2$. The direction of the curvature of the arc, in other words, the sign of the phase-difference, was found to agree with theory.

8. The above relates to the small motion at *any* node. One particular case is of importance, as it admits of independent experimental verification. For a string to be maintained in vigorous vibration by the imposition of an obligatory motion at one point, this point should itself lie at or near a node of the oscillation. It follows that under such circumstances, the imposed obligatory motion and the general oscillation of the string should differ in phase, the difference being equal to $\pi/2$ when the obligatory motion is imposed exactly at a node. This difference of phase between the motion of the prong and the general oscillation of the string, which may have been anticipated from the general principles of resonance, may be verified experimentally in two ways: (1) By stroboscopic observation and (2) by a tilting-mirror and Lissajous's figure arrangement. Before entering into experimental details regarding these, we may first discuss the mathematics of the questions dealt with above.

Mathematical Note.

9. The result of the investigation by Donkin of the problem of forced oscillations of stretched strings (Acoustics, 2d ed., pp. 121–124) is erroneous. Starting with the assumption that the obligatory

motion at the point $x = b$ is $p \sin nt + q \cos nt$ and taking dissipation of energy into account, the final approximate result obtained by him for the motion of points not near a node is

$$\sin \theta (p \sin nt + q \cos nt)/(\sin^2 \varphi + \delta_0^2 \cos^2 \varphi)^{\frac{1}{2}}.$$

From the original, it will be seen that $\sin \theta$ and $(\sin^2 \varphi + \delta_0^2 \cos^2 \varphi)$ do not involve the time t; the phase of the general motion of the string should therefore be identical with that of the obligatory motion at the point $x = b$. This result does not agree with that given in paragraph 8 above. The exact step in the mathematical work which introduces the error is putting $\tan \Phi = q/\rho$, where

$$\tan \Phi = (q\sigma_0 \sin \varphi + \rho\delta_0 \cos \varphi)/(\rho\sigma_0 \sin \varphi - q\delta_0 \cos \varphi).$$

To show that this step is erroneous, we may, without loss of generality, put $q = 0$. Then

$$\tan \Phi = \frac{\delta_0}{\sigma_0} \cot \varphi = \beta\varphi \cot \varphi = \frac{c\varphi}{2n} \cot \varphi.$$

Donkin's approximation is therefore equivalent to putting the damping factor $c = 0$. This is inadmissible, for the coefficient of the term, *i. e.*, $\cot \varphi$, is very large, and when $\Phi = i\pi$ at the exact stage of resonance, becomes infinite. At this stage $\tan \Phi = \infty$ and $\Phi = \pi/2$, whereas Donkin would have $\tan \Phi = 0$ and therefore $\Phi = 0$.

10. To compare the facts stated in paragraphs 1 to 8 above with the results of theory, we may make use of the notation and results given on pages 197–199 of Lord Rayleigh's Theory of Sound, Vol. I., second edition. The expression for the displacement at every point of the string maintained in vibration there given is

$$\gamma \frac{R_x}{R_b} \cos (pt + \Sigma_x - \Sigma_b),$$

where

$$R_x^2 = \sin^2 \alpha x + \frac{k^2 x^2}{4a^2} \cos^2 \alpha\lambda$$

and

$$\tan \Sigma_x = \frac{e^{\beta x} - e^{-\beta x}}{e^{\beta x} + e^{-\beta x}} \cot \alpha x.$$

corresponding to an obligatory motion $\gamma \cos pt$ at the point $x = b$. In this expression β is a small quantity, and therefore

$$\tan \Sigma_x = \beta x \cot \alpha x,$$

and

$$(\Sigma_x - \Sigma_b) = \tan^{-1}(\beta x \cot \alpha x) - \tan^{-1}(\beta b \cot \alpha b).$$

This value of $(\Sigma_x - \Sigma_b)$ is very small and may be put equal to zero except in the two cases, where $\cot \alpha x$ or $\cot \alpha b$ is very large, *i. e.*, αx or αb is nearly equal to any integral multiple of π. In other words, the motion at any point is in the same phase as the obligatory motion unless it happens that (1) the point of observation, or (2) the point at which the obligatory motion is imposed, or (3) both points either coincide with, or are situated near nodes of the forced oscillation. In short, it may be said that there is a localized change of phase at the nodes, the difference between the phase at the node and at some point a considerable distance away from it being $\pi/2$.

11. The expression for the displacement can be written in the form

$$\frac{\gamma}{R_b}\left(\sin \alpha x \cos pT + \frac{kx}{2a}\cos \alpha x \sin pT\right),$$

which is seen to be of the type given in paragraph 4 above.

12. We now return to the experimental details referred to at the conclusion of paragraph 8 above.

Method (1).—A short length of the vibrating string is brightly illuminated, and an image of it is focused in the field of view of a stroboscopic disc. A slit held parallel to the string at some distance from it is illuminated, and the light issuing therefrom suffers reflection at a small mirror attached to the prong of the electrically-maintained tuning-fork which keeps the string in vibration, and is then focused by a second lens into a linear image. The two images are adjusted so as to be in juxtaposition. On starting the tuning-fork and the stroboscopic disc, it can be seen that the two linear images are in different phases of motion, and the gradual change of the difference with the alteration of the tension of the string can be studied.

Method (*2*).—The motion of the prong and that of any point on the string are parallel to each other. To apply the method of Lissajous's figures for the observation of the phase-difference, a tilting-mirror arrangement was adopted. The tuning-fork actuates a light pivoted mirror by means of a thread which is kept taut by a spring pressing against the mirror. The plane of oscillation of the tilting-mirror is perpendicular to that of the vibration of the string, and a point on the latter brightly illuminated throughout its path with the aid of a cylindrical lens, and viewed by reflection, first at a fixed mirror and then at the tilting-mirror, is seen to describe a Lissajous's figure (circle, ellipse, or straight-line). From this figure the phase-difference can be inferred at once. It was found that the phase-difference was not quite independent of the amplitude of oscillation of the prongs: the explanation of this effect probably being that a large amplitude of oscillation involves a departure from constancy of the tension of the string, the average tension being greater than the normal value.

NOTE ON CROVA'S METHOD OF HETERO-
CHROMATIC PHOTOMETRY.

By Herbert E. Ives.

CROVA[1] suggested as a method of comparing the intensities of different colored lights, a comparison of their intensities at a single wave-length. By this means the actual photometry is reduced to the photometry of lights of the same color. For its utility and accuracy this method depends on the validity of a certain assumption, namely that there can be found a wave-length at which the relative energy emission in different illuminants is proportional to their total light. Such a wave-length may be looked for near the brightest part of the spectrum, since, owing to the very pronounced maximum of visual sensibility near .545μ and its rapid drop in value on either side, this point of maximum brightness is not greatly different for all the common incandescent illuminants. Obviously the method can only be applicable to light sources possessing continuous spectra, such as the black body, and sources whose energy distribution is not greatly different from the black body.

Crova found experimentally that he could measure the relative illumination from sunlight and from a Carcel lamp by a determination of the relative intensity at wave-length .582μ. It has been commonly assumed that this wave-length would also serve for the comparison of any illuminants lying between these two in color, either with one of these or with each other.

Lord Rayleigh[2] made a somewhat similar suggestion in a proposal to measure incandescent lamps with a "monochromatic telescope" using spectral yellow light. Rasch[3] and Lucas[4] have discussed the close relationhsip between the total light and the radiation

[1] Comptes Rendus, 93, p. 512.
[2] Phil. Mag., June, 1885.
[3] Ann. d. Physik, 14, pp. 193–203, 1904.
[4] Phys. Zeit., 6, 19–20, Jan. 1. 1906.

at a single wave-length, and the latter has deduced from Lummer and Kurlbaum's data on the luminous intensity of a black body at various temperatures the wave-length .542μ as the wave-length equivalent to total light in its behavior. Nernst[1] has used the Wien equation for the energy at one wave-length to represent total light in extrapolating from photometric observations on a black body. He obtained the values of his constants from observations between 1500° and 2300° and extrapolated to 2700°. Fery and Cheneveau[2] have derived equations connection the power expended and the candlepower of incandescent lamps, by assuming the candle-power to be proportional to monochromatic radiation following the Wien equation

$$I = Ae-\frac{B}{T}.$$

All of these are applications of the original idea of Crova.

While this method of heterochromatic photometry has not been very widely used, partly because of its limitation to continuous spectra and partly because of practical difficulties, it is nevertheless of considerable experimental and theoretic interest. Experimentally it offers a means of making photometric readings where the color of the measured illuminant is changed through a wide range under conditions which do not permit of many readings or the use of calibrated compensating colored glasses. Such for instance would be measurements on the relation between power consumption and candlepower in incandescent lamps as the measurements are carried near the burning out temperature. Obviously, also, color-blind observers can make heterochromatic comparisons with the same accuracy as can observers of normal vision, provided their observations are made at the proper wave-length as determined from the average of normal individuals. On the theoretical side, use of Crova's method makes possible the calculation of light-power relations which would be prohibitively complicated were it necessary to include the complete visibility function connecting light with radiation.

In connection with a problem on the efficiency of incandescent

[1] Phys. Zeit., 7, pp. 380–383, June 1, 1906.
[2] Bull. de la Societe Internationale des Electriciens, 1909, p. 655.

solid radiation, in which incandescent lamps are to be employed, the writer has found it convenient to make use of Crova's method for practically the reasons outlined above. It seemed advisable before so doing to investigate the range within which the method would be valid; and from this investigation it appears that certain of the assumptions commonly made in its use are not entirely correct.

One assumption either stated or tacitly made is that the equivalent wavelength is that corresponding to the maximum of visual sensibility, *i. e.*, the wave-length of maximum luminosity for an equal energy spectrum. Another assumption—or rather, another practice— is to use the equivalent wave-length idea over a large range of illuminant colors (such as black body temperatures) without raising any question as to the limits within which the method holds, or making any determination of the order of magnitude of the errors within that range. This latter is involved in the belief that the wavelength which Crova derived from comparison of sunlight and the Carcel would hold for all intermediate illuminants.
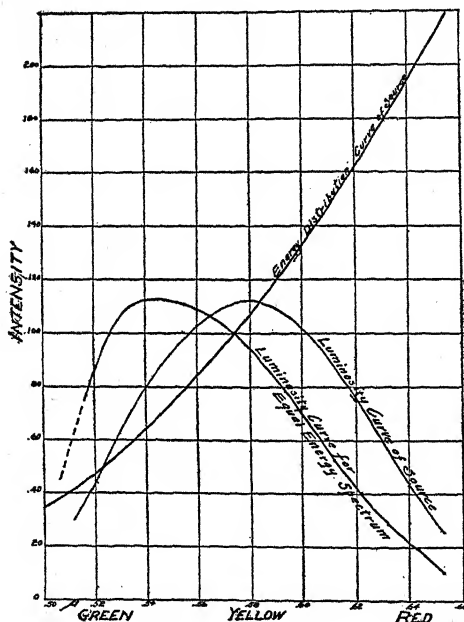
Fig. 1.

The error in the first assumption is most clearly indicated diagrammatically, as in Fig. 1. Here is given the actual luminosity curve of a source (tungsten lamp through prism, etc.) and on the same drawing the luminosity curve for a spectrum of equal energy at all wave-lengths.[1] The latter is the product of the luminosity values for the source by the reciprocals of the energy values. The

[1] Data taken from work shortly to be published.

curves are drawn to be approximately of the same area, and, on the assumption (to be discussed below) that these areas are proportional to total light, it is at once evident that the wave-length of maximum sensibility, .545μ, does not give the relative light values, the error being about twenty per cent.   The relative areas of the curves are actually given by the energy values at about wave-length .565μ. In other words the equivalent wave-length lies somewhere between the maxima of luminosity of the sources under comparison, and the wave-length of maximum sensibility does not enter in.
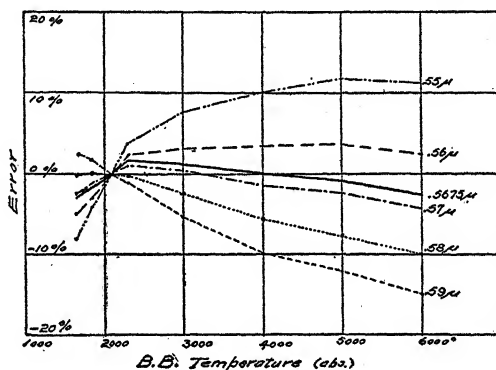
Fig. 2.

With regard to the second assumption, Fig. 2 gives in graphical 'orm some calculations on its validity in the case of the black body. The method of obtaining these data is as follows: Black body energy emission curves were calculated from the Wien equation; these were then each multiplied by the luminosity values corresponding to the spectral wave-lengths. These values were taken from a recently determined normal luminosity curve, from observations by five observers at a moderately high illumination.   The areas of the resultant curves (the type of which is exhibited in Fig. 1) are taken as proportional to the total light, as measured on a photometer. A word as to the assumption involved here.   Let the energy per unit of spectrum length at λ be W_λ, then the total energy in the spectrum is

$$ W = \int_0^\infty W_\lambda d\lambda. $$

Now the light values, or relative luminosities of the different wave-lengths, are different, as shown by the luminosity curve. To each wave-length we may ascribe a factor K_λ, its "light va'ue," which is zero outside the visible spectrum; then the assumption involved in the above process is, concisely stated, that the total light L is equal to

$$\int_0^\infty K_\lambda W_\lambda d\lambda.$$

The validity of this assumption has never been seriously questioned. Experimentally it has received verification by Abney and by Tufts, and has been used by others without introducing any observed errors. While recent work by the writer would indicate the necessity of a more searching inquiry as to its validity under all conditions there can be practically no doubt that this process of integration represents the truth to a fair degree of approximation.

Having in this manner determined the relative light values, the relative values for the energy at various wave-lengths were read off the energy distribution curves. A table was then made out, taking the light and energy values at 2100° abs. arbitrarily as the unit, and the changes in total light with temperature were compared with the changes in the emission at various wave-lengths in the spectrum. Fig. 2 gives the errors introduced by limiting the photometry to light of one wave-length, where the standard of intensity is the black body at 2100°. When the measured light is the same in color as the standard, any wave-length may be employed, hence the error at this point is zero.

Several facts are here brought out. First, no one wave-length can be used as an equivalent wave-length for more than a certain definite range of temperature, for any given allowable error. Wave-length $.58\mu$ for instance would be selected to compare our standard with a black body at any temperature between 1600° and 2300° with an error of not more than .1 per cent., but if a black body at a higher temperature is measured an increasing error is introduced, until at 6000° it amounts to fifteen per cent. The horizontal portion of each curve shows the temperature range through which the corresponding wave-length may be used as an equivalent to total light. Between 1600° and 2300°, $.58\mu$ is indicated; between

3000° and 4000°, .56μ; between 4500° and 5500°, .55μ. A wave-length which would serve for comparing black bodies at 2000° and 5000° would introduce errors of several per cent. if intermediate points were compared with either extreme.

In the second place it appears that the wave-length of maximum visual sensibility—in this case .545μ—is not the equivalent one for any black body comparisons except between 5000° and 6000°; that is, far above the temperatures represented by any incandescent lamp, or other artificial illuminant of the incandescent solid type.

Wave-length .5675μ has been picked out by trial as the one which could be employed for an equivalent wave-length over the longest range with the least error. The error with this wave-length is less than three per cent. in either direction. If, however, the method is to be used only within the limits of temperature represented by the ordinary incandescent lamps—namely 1600° to 2400° abs.—wave-length .58μ should be chosen.

In order to obtain a check on these calculations measurements were made on a tantalum lamp over a wide range of voltages, both of the total light and of the monochromatic light of various colors. For this purpose a simple Martens equality photometer was used, from which the eye-piece could be removed and a spectrometer substituted. Over the slit of the spectrometer was placed a lens to form an image of the photometer field in a convenient place in the optical system of the instrument. Observation was made through an eye-slit at the point where the spectrum is formed. Under these conditions the photometer field is seen in monochromatic light of the wave-length incident upon the eye. The tantalum lamp was operated at various voltages between 65 and 115, its normal voltage being 100. The highest voltage makes the color about that of a normally operated tungsten lamp while the lowest is about like the Hefner. A "4-watt" carbon lamp was used as comparison standard, the tantalum lamp matching it in color at 78 volts. Photometric observations for total light were made by two observers who possess very similar color vision, both of their maxima of sensibility for high illumination falling at .54μ. Their agreement in reading was excellent, the average difference being

0.2 per cent. and showing no systematic drift with increased color difference. The error of measurement with monochromatic light was larger than with the total because of the smaller amount of light available, the error amounting probably to as much as one
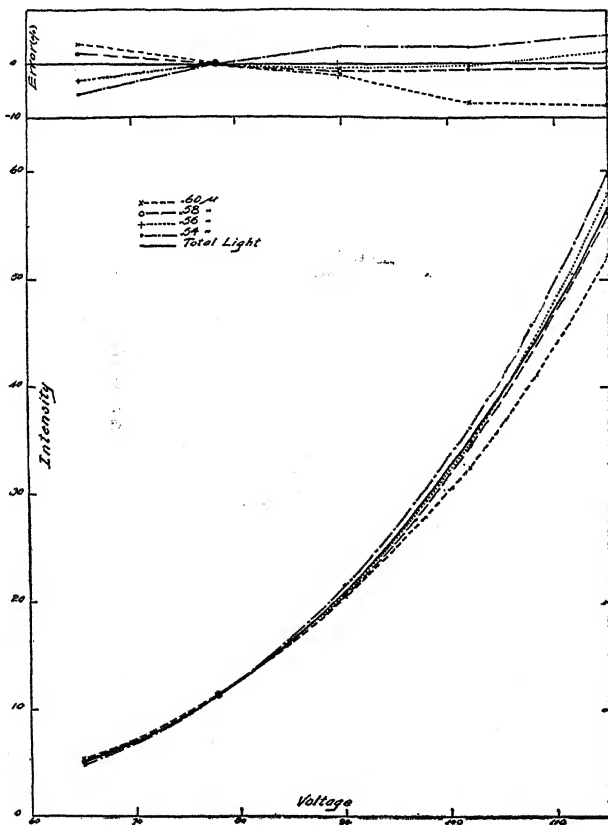

Fig. 3.

or two per cent. A larger number of measurements would have reduced the uncertainty, but those obtained (average of five settings) were so decisive as to make further experiment superfluous.

In Fig. 3 are plotted the experimental results. The full curve is the total light curve. The various dotted and dashed lines exhibit the monochromatic light curves, as designated in the key. At the top are plotted the percentage errors. At the voltage of

color match any wave-length may be chosen, but it is at once evident that as the color of the two lamps (standard and test lamp) become different, the relative candle powers are not followed equally well by all monochromatic radiations. The equivalent wave-length lies between .56μ and .58μ. The data of Fig. 2 indicate .58μ for this range of colors, but the two observers in this experiment have their maxima of sensibility further toward the blue than the mean of the five taken as normal in Fig. 2. It is seen that wave-length .54μ, the maximum of visual sensibility of the present observers, does not represent the total light change. The total error from end to end amounts in fact with this wave-length to about twelve per cent. In short, the results derived from calculation are very satisfactorily confirmed.

The result of the investigation is therefore to show that the oft-repeated statement that the equivalent wave-length is the wave-length of maximum sensibility is in error: and that any equivalent wave-length is such to a given degree of accuracy for but a limited range of black body temperatures. The equivalent wave-lengths for various black-body temperatures, and the one holding over the longest range with the least error for normal eyes, have been indicated.

The writer takes pleasure in acknowledging his indebtedness to Mr. M. Luckiesh for assistance in making the observations here given and for the preparation of the drawings.

A NEW METHOD OF PRODUCING RIPPLES.
OPTICAL ANALOGIES.

By A. H. Pfund.

THE use of ripples in illustrating optical phenomena is too well established to require justification. Since the publication of Vincent's[1] work on mercury ripples, little has been or can be added to the discussion of the application of these phenomena to experiments in optics. In view of their recognized pedagogical value, it seems strange that but little use is made of ripple experiments in lecture-room demonstration. The reason for this becomes apparent, however, when the complexity of the apparatus, necessary for producing ripples and making them visible, is taken into con-
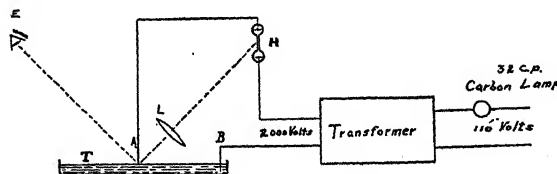
Fig. 1.

sideration. Recently I observed a phenomenon which was used as the essential feature in the construction of a ripple apparatus of great simplicity. The following diagram (Fig. 1) shows the complete apparatus.

The current from the secondary of a small transformer (2,000 volts, 60 cycles per second) passes through a helium vacuum tube *H*, then through the wire *A* (0.5 mm. diameter) into distilled water contained in the hard rubber tray *T* and finally back through the wire *B* to the transformer. While the wire *B* dips deeply into the water, the wire *A* just touches the surface—and it is at this point that the ripples are produced. The fact that the phenomenon is

[1] J. H. Vincent, Phil. Mag., 1897, 43, p. 417; 1898, 45, p. 191. H. M. Reese, Ap. Jnl., 1906, 24, p. 47.

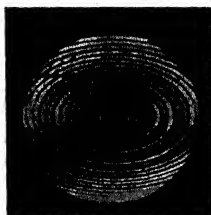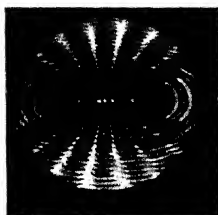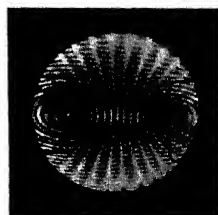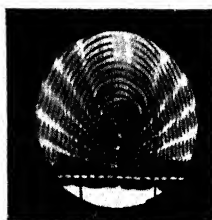Fig. 3.   Fig. 4.   Fig. 5.

Fig. 6.   Fig. 7.

A. H. PFUND.

observed only when such poorly conducting liquids as distilled water and alcohol are used, indicates that the ripples are produced by the effect of the large potential drop at the point of contact. No mechanical vibration of the wire A is present.

Each pulsation of the current produces a ripple which moves out from its source with so great a velocity that, in continuous illumination, the surface of the water appears undisturbed. However, by making the illumination intermittent, *i. e.*, synchronous with the rate of production of ripples (which is accomplished by the use of the vacuum tube H), the ripples apparently stand still and may be observed at leisure. For observing or photographing ripple-phenomena, the lens L is placed as indicated in the diagram
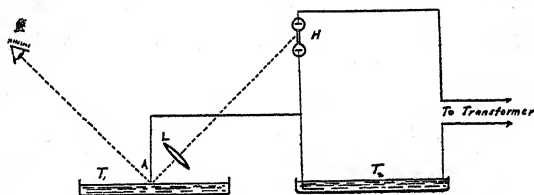

Fig. 2.

where the helium tube and the eye or camera lens are at conjugate foci. So as to obtain light from a "point" source, all light coming from the helium tube, with the exception of that coming from 1 mm. of the capillary, was screened off.

That the ripples are produced by electrostatic effects is further proved by the effects observed with the apparatus arranged according to diagram shown in Fig. 2. Here the current passes through the vacuum tube H and then through distilled water in tray T_2 (this water merely serves as a high resistance). The end of the wire A does not touch the surface of the water in tray T_1, but is brought as near it as possible without making contact. The eye placed at E sees beautiful ripples having A as their center.

The proper contact, necessary for giving the phenomenon at its best must be determined by trial. If the lower end of the wire A (Fig. 1) is too high above the general surface of the water, the ripples are too violent; if, on the other hand, the wire dips down into the water too deeply, the ripples are too feeble.

In order to avoid annoying reflections at the sides of the containing vessel, it is necessary to remove all traces of grease so that the water may be drawn up slightly where it meets the vessel. If, however, reflections are desired, a piece of glass or sheet-metal, covered with grease is placed into the water.

In explanation of the photographs it might be stated that, as a consequence of viewing the water-surface at an angle of about 45°, the reflected image of the wire A is always visible and the otherwise circular ripples appear elliptical. Whenever any object is placed in the water, the planarity of the surface is destroyed and, in consequence, a dark region of considerable extent is produced— making it difficult to determine the character of the object from the photograph. Whenever necessary, I have indicated with dotted lines the position of the object in question. The photographs here presented show only a few of the results which may be obtained with this particular arrangement of apparatus. Many modifications suggest themselves for showing more complicated phenomena, but, as these advantages are gained at the expense of simplicity, I have decided not to discuss them at present.

Fig. 3. This illustrates the character of ripples from a single point source.

Fig. 4. This illustrates Young's interference experiment. Two point sources, vibrating in phase, give rise to interference hands whose loci in space are confocal hyperbolæ of revolution (the sources lying at the foci). The existence of hyperbolæ is well shown in the photographs. To obtain these results, the current is led into the water by both wires.

Fig. 5. This shows the same phenomenon with wide separation of sources. The narrowness of the interference bands, as contrasted with the preceding case, is to be noted.

Fig. 6. The analogy of Lloyd's mirror is here shown. A greased piece of brass, at a distance of 1 cm. from the source, acts as a plane mirror—the incident and reflected waves interfering to show well-marked interference bands. As is well known, the waves leaving the reflector seem to come from a source situated as far behind the mirror as the real source is in front of it. As a result, interference bands of the same type as those shown in Figs. 4 and 5 are

produced. The reflector simply rests in the water and is not electrically connected.

Fig. 7. Here the ripples are produced by a point source in front of an obstacle and the phenomenon within the gemoetrical shadow is photographed. In the illustration it is first to be observed that the ripples rapidly enter the region of the geometrical shadow (diffraction). These diffracted waves, coming from the two edges of the obstacle, produce an interference pattern which could be duplicated by two real sources—one at each corner of the obstacle. This becomes evident from the hyperbolical form of the fringes. The straight central fringe (x) represents the analogy of Poisson's famous experiment.

Of the many other experiments which I have carried out with this apparatus, the one illustrating refraction is worthy of particular notice. This phenomenon was observed by floating a drop of machine oil on the surface of the water. The drop slowly expands into a circular disc on whose surface ripples are observed. Since the surface-tension of oil is much less than that of water, the velocity of the ripples is less on the oil surface, hence the waves are refracted. The phenomenon, though readily observable, was not photographed for the reason that during the necessary time of exposure (three minutes) the drop had expanded to such an extent that it covered the entire surface of the water.

In conclusion I wish to repeat that the only novelty claimed for these experiments lies in the simplicity of the method of producing ripples and making them visible. If this discussion will help in bringing about a more widespread use of ripples in the elucidation of optical principles, I shall consider my purpose in writing this note well accomplished.

JOHNS HOPKINS UNIVERSITY,
    October, 1910.

EFFECT ON THE CATHODE FALL IN GASES PRODUCED BY THE EVOLUTION OF GAS FROM THE CATHODE.

By L. A. Jones.

IT has been shown by Skinner[1] that a fresh metal electrode when used as cathode in a glow current in helium and argon gives off hydrogen at the rate required by Faraday's law for electrolytes. This effect was found to continue only for a few minutes after starting the current and then only when fresh metals were used. In an atmosphere of hydrogen no increase in pressure was observed, indicating either that no gas was given off by the cathode or that the anode absorbed gas at the same rate as the cathode evolved it. Experiments performed shortly afterward[2] with a nitrogen atmosphere gave similar results.

Holman first[3] and Chrisler,[4] more extensively, investigated the effect of absorbed hydrogen on the photo-electric activity of metals. Chrisler showed conclusively that absorbed hydrogen is an important factor in the photo-electric current. He found that use as cathode (which reduces the supply of hydrogen in the metal) always decreases the photo-electric effect; while use as anode in hydrogen or simply standing in hydrogen (which charges the metal with the gas) increases the photo-electric effect enormously.

Later Chrisler[5] proved by direct measurements the absorption of gas by the anode. He found that silver, mercury and the alkali metals when used as anode absorb hydrogen at the rate required by Faraday's law for electrolytes. These metals, with the exception of mercury, were also found to absorb nitrogen at the same rate. In some exceptional cases helium also was absorbed.

[1] C. A. Skinner, Phys. Rev., vol. 21, p. 1; Phil. Mag., 6, vol. 12, p. 481, 1906.
[2] C. A. Skinner, Phys. Rev., vol. 21, p. 169.
[3] W. F. Holman, Phys. Rev., vol. 25, p. 81.
[4] V. L. Chrisler, Phys. Rev., vol. 27, p. 267.
[5] V. L. Chrisler, Phys. Rev., vol. 24, p. 461.

Whiddington[1] investigated the effect on the emitted cathode rays arising from a treatment of the metal similar to that given by Chrisler in the photo-electric effect. He found that an electrode after having been used for some time as cathode emitted more homogeneous and slower moving cathode rays than were obtained when the electrode was fresh. On the other hand just after use as anode a flash of intense radiation was observed. Further, with a glow current through air using an aluminium cathode, he found the cathode fall increased rapidly to a maximum value in about three minutes after starting the current, then fell gradually to a final constant value which was lower than the initial.

The object of the present investigation was to make a more extended study of the effect which the evolution of gas by the cathode produces on the cathode fall.

The investigation was divided into two parts; the first dealing with the effect on the cathode fall under a constant current density; the second, with the effect on the "normal" cathode fall.

I. The Effect with Constant Current Density.

The plan followed was to use a discharge tube so constructed that after a cathode had been used for some definite time a fresh one of the same metal could be substituted for it immediately without changing any other conditions. This scheme eliminated any effect arising from a change in the gas, and gave, by comparing the final value of the old cathode with the initial value of the new one, a reliable indication of the effect of use on the metal alone.

Apparatus.

A vertical section of the discharge tube is shown in Fig. 1. The discharge chamber M (diam. 3 cm., length 20 cm.) terminates below in a large cylindrical one, the axis of which is parallel to the axis of the tube M. The lower chamber is about 12 cm. in diameter and 3 cm. deep. The entire discharge tube is of glass; all permanent joints, that could not be fused, being cemented with water glass and covered on the outside with a coating of de Khotinsky's laboratory cement. Inside the lower chamber is fitted a circular glass

[1] R. Whiddington, Proc. Camb. Phil. Soc., vol. 15, p. 183.

plate, slightly less in diameter than the chamber, capable of carrying ten cathodes, E, attached to the plate by means of hard rubber nuts, K. This circular plate can be rotated about its axis by means of a key introduced through the ground joint, J, thus allowing
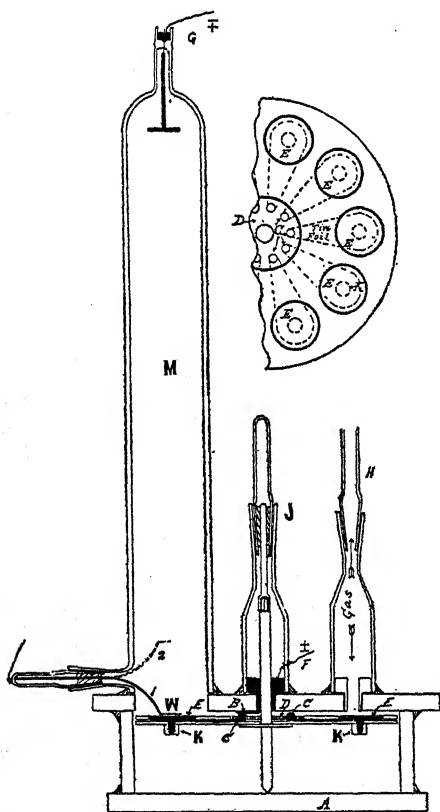

Fig. 1.

any cathode to be brought into position at the lower end of the discharge chamber. Tin-foil strips lead from the cathodes to small brass rivets, C, coming up through a hard rubber disk, D, fixed to the circular plate. These rivets make contact with a brass brush, B, connected to a brass collar which in turn is connected to a platinum wire, F, sealed through the wall of the tube. Thus all of the cathodes except the one directly beneath the discharge chamber are insulated from the circuit, but any one of them can be brought into position by simply turning the ground joint, J. A glass plate properly cut covered the tin-foil strips and fitted up to the rims of the cathodes. As a means of measuring the cathode fall a wire, W, was introduced through the ground joint, N. This probe is of fine aluminium wire sheathed with glass to within about 5 mm. of its inner end. It is bent into a form such that by simply turning the ground joint the distance between the probe and the cathode can be varied from three tenths to two centimeters. In order to change or repolish the cathodes it was necessary to take off the base plate, A, which allowed the circular plate, carrying the

cathodes, to be removed. The base plate was therefore sealed on with de Khotinsky's laboratory cement which was applied to the outside only. An aluminium anode of very small area (to prevent appreciable absorption of gas) was introduced at G. Connection to the evacuating and drying system was made through the ground joint, H.

The cathode fall with various metals was measured in both hydrogen and helium. The hydrogen was obtained from aluminium in a potassium hydrate solution, dried carefully with P_2O_5, and then stored in a glass bulb connected to the evacuating system by a ground joint.

The helium used was obtained in sealed tubes from Messrs. Thomas Tyrer and Co., London, and transferred to a glass bulb that had been carefully dried and evacuated for several days. This bulb was also connected to the system by a ground joint.

All ground joints and stop-cocks included in the evacuating, drying and gas-supply system were lubricated with a mixture free from volatile hydrocarbons.

Gas-pressure measurements were made by means of a McLeod gauge which magnified the pressure one hundred times. Evacuation was produced by a Barr and Stround mercury pump.

The cathode fall was measured by a Kelvin quadrant-electrometer, a definite potential being placed between the quadrants, and the cathode fall between the needle and one pair of quadrants. The instrument in this form was calibrated by comparison with a Weston voltmeter.

The electric current was furnished by a battery of six hundred small storage cells, was measured by a Weston milliammeter, and regulated by means of a resistance of cadmium iodide in amyl alcohol. The electrometer was protected by a similar resistance tube introduced in the line leading to the needle.

The cathodes used were 1.5 cm. in diameter and about 1 mm. thick. They were polished to a mirror surface with infusorial earth mixed with tallow, and finally with clean new felt and new cotton cloth.

EXPERIMENTAL RESULTS.

Platinum, zinc, aluminium and magnesium cathodes were tested in both gases.

The method followed was to evacuate the system (approximate volume 580 c.c.) and then after flushing with the gas to be used, to admit sufficient gas to bring the pressure to the desired value. The working limits of gas pressure were determined by making preliminary tests. The pressures used were such that the negative glow entirely covered the cathode; the Crooke's dark space varying in thickness from 1 mm. to 5 mm.

Gas having been admitted to the desired pressure the current was started, quickly adjusted to a definite value (one milliampere in all cases) and readings of gas-pressure and cathode-fall taken at regular time intervals (every 1.5 minutes in some cases, every two minutes in others).

The probe was placed in the negative glow so that a movement of 2 mm. or more, depending on the gas pressure, toward or away from the cathode caused no change in the electrometer reading. This adjustment showed that the fall of potential observed was the desired "cathode fall."

After taking readings for some time (10 to 40 minutes) a new cathode of the same metal was brought into position and its fall immediately measured. As already stated this mode of procedure was followed to eliminate variations arising from a change in the pressure or composition of the gas. Any variation in the cathode fall arising from a change in the cathode itself, was thus indicated by a difference in the electrometer reading the moment a fresh cathode was brought in.

The following tables give the results of tests with the four different metals (aluminium, zinc, platinum and magnesium) in both hydrogen and helium.

Table I. gives in detail the results obtained from an aluminium cathode in helium, observations of both gas pressure and cathode fall having been made at intervals of 1.5 minutes. It shows a gradual increase in gas pressure (2.74 mm. to 3.11 mm.) accompanied by a decrease in cathode fall (199.5 to 191 volts). When the fresh cathode was introduced however its fall was found only one volt less than the old (191 to 190 volts), the gas pressure having risen while measuring the fall of the new cathode from 3.11 mm. to 3.14 mm. This difference of one volt is too great to be attributed to

Table I.

Aluminium in Helium. Current Density: .56 (ma./cm.).

Duration of Current (min.)	Gas Pressure (mm.).	Cathode Fall (Volts).	Duration of Current (min.)	Gas Pressure (mm.).	Cathode Fall (Volts).
0	2.74	199.5	13.5	3.00	193.0
1.5	2.79	197.0	15.0	3.02	192.5
3.0	2.84	196.5	16.5	3.04	192.0
4.5	2.87	195.5	18.0	3.05	192.0
6.0	2.90	195.0	17.5	3.06	191.5
7.5	2.93	194.5	21.0	3.08	191.5
9.0	2.94	194.5	22.5	3.10	191.5
10.5	2.96	193.0	24.0	3.11	191.0
12.0	2.98	193.0			
			Fresh Cathode.		
				3.14	190.0

Table II.

Cathode Fall of Depleted and of Fresh Metals in Helium. Current Density: .56 ma./cm².

Cathode Metal.	Sample Tested for Duration.			Fresh Sample in same Gas.	
	Duration of Curr. (min.).	Gas Press. (mm.).	Cathode Fall (Volts).	Gas Press.	Cathode Fall.
	0	.89	255	—	—
	20	1.16	238	1.19	235
Aluminium.	0	2.74	196	—	—
	9	2.99	194	3.03	193
	0	2.04	214	—	—
	39	2.58	198	2.60	199
	0	.83	294	—	—
	20	1.11	270	1.13	273
Magnesium.	0	2.20	214	—	—
	16.5	2.63	207	2.69	204
	0	1.94	291	—	—
	24	2.31	286	2.34	285
	0	1.02	377	—	—
	25	1.33	355	1.33	356
Zinc.	0	2.83	262	—	—
	33	3.28	255	3.31	253
	0	.82	408	—	—
	20	.89	403	.89	402
Platinum.	0	1.85	214	—	—
	12	1.87	214	1.88	214

the change in gas pressure between the final observation on the old
and that on the new cathode, but it is less than the difference in
fall obtained under identical conditions with different fresh cathodes.
On the other hand the difference between the initial and final
observations with the first cathode (8.5 volts) is large in comparison
with the change produced by the introduction of the fresh cathode,
hence it is to be concluded that the observed variation in cathode
fall with use was caused not by any change in the cathode itself,
but rather by the change in condition of the gas content of the
discharge tube.

In Table II. are given condensed results of similar tests with
other metals in which the readings with the first cathode are fol-
lowed by the observation taken immediately after the introduction
of the second cathode of the same metal. In each case after the

<div align="center">

TABLE III.

Cathode Fall of Depleted and of Fresh Metals in Hydrogen.   Current density: .56 ma./cm².

</div>

Cathode Metal.	Sample Tested for Duration.			Fresh Sample in Same Gas.	
	Duration of Curr. (min.).	Gas Press. (mm.).	Cathode Fall (Volts).	Gas Press.	Cathode Fall.
Aluminium.	0	.82	408	—	—
	20	.89	403	.89	402
	0	1.91	241	—	—
	15	2.10	236	2.11	236
	0	.96	305	—	—
	20	1.28	285	1.28	288
Magnesium.	0	1.76	244	—	—
	15	2.06	234	2.11	231
	0	2.11	231	—	—
	12	2.35	227	2.38	228
Zinc.	0	.90	437	—	—
	22	1.29	406	1.32	404
	0	1.40	388	—	—
	15	1.40	386	1.40	385
Platinum.	0	.96	540	—	—
	20	.94	545	.94	550
	0	1.68	310	—	—
	21	1.83	314	1.85	314

fresh cathode was introduced the fall was measured as quickly as possible, the time required for the deflection of the electrometer to become steady being about one minute.

From the table the maximum change in cathode fall, occurring when the fresh cathode was introduced, was 3 volts, the minimum 0, and the mean 1.6 volts. This difference between the old and the fresh metal was in most cases accompanied by a slight increase in gas pressure, but could not be ascribed to that since the change in fall was sometimes an increase, sometimes a decrease. Table III. gives similar results obtained with an atmosphere of hydrogen.

While therefore the results reveal a considerable decrease in cathode fall with use (the only exception occurring with platinum in helium) accompanied in general by an increase of gas pressure, yet they appear to prove conclusively that with a constant current density there is only a relatively small change in the cathode fall in either hydrogen or helium arising from a change taking place in the metal. The change in fall taking place when the fresh cathode was introduced, sometimes positive and sometimes negative, was always much smaller than the difference between the initial and final values obtained with the original cathode. The fact that in general a relatively large decrease in cathode fall occurred when there was also a relatively large increase in gas pressure suggests that the change might very reasonably be ascribed to the increase in pressure simply, though no tests were made to verify this assumption.

II. The Effect with Normal Current Density.

The foregoing tests with a constant current density (greater than the "normal") having shown no change in the cathode fall caused by the condition of the cathode itself, it remained to investigate the effect with the "normal" current density.

The plan adopted was to use a long cylindrical cathode and a gas pressure such that the negative glow did not entirely cover it, thus making sure that the normal current density was obtained.

The apparatus in this case was not so constructed as to permit the introduction of a fresh cathode without changing the gas.

Apparatus.

The apparatus with the exception of the discharge tube was the same as that used in the previous work. A diagram of this is given in Fig. 2. It was about 3 cm. in diameter and 20 cm. long. A
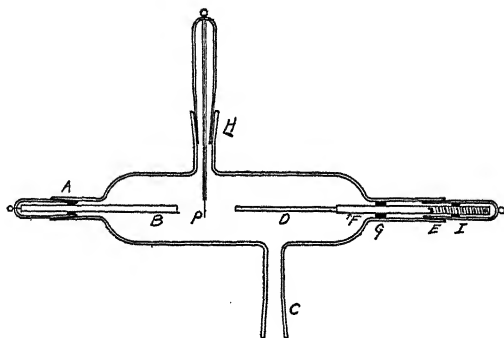

Fig. 2.

square brass rod F, threaded at one end, passing through the brass guide collar G, and a threaded nut I, carried the cathode D. By means of this arrangement the distance between the fixed probe, P, and the cathode could be readily varied. The probe was made of fine aluminium wire sheathed with glass to within 5 mm. of the end, and introduced through the ground joint, H. The anode, in this case, of rather large area, was introduced through the ground joint, A. Both anode and cathode were thus coincident with the axis of the tube, the probe crossing it at right angles. The ground joint, C, served to connect the tube with the rest of the system.

The cathodes used were of wire about 8 cm. long and of different diameters (varying from .5 mm. to 2 mm.). They were cleaned and polished in the same manner as before.

Experimental Results.

When the current was started the negative glow appeared at the end of the cathode, resembling in shape a small test-tube slipped over the end of the wire. The length of the cathode covered depended upon the kind of metal used, the kind of gas in the tube and the gas pressure. Conditions were always chosen so that the glow did not entirely cover the cathode, thus insuring normal current density.

In measuring the cathode fall, care was taken to see that the probe was in the negative glow, and so placed that the exposed portion was entirely in the region of zero potential gradient.

Observations on the fall were made at regular intervals (2 minutes). The pressure was observed at the beginning and end of the run, and at irregular intervals between.

The following tables give results of the tests, the first (Table IV.) in detail, the others (Tables V. and VI.) only the initial and final readings.

The same kinds of metal and gas were used as in the first part of the investigation. The magnitude of the current was one milliampere throughout.

TABLE IV.

Aluminium in Helium. Normal Current Density.

Duration of Current (min.)	Gas Pressure (mm.).	Cathode Fall (Volts).	Duration of Current (min.)	Gas Pressure (mm.).	Cathode Fall (Volts).
0	1.62	—	16	1.65	170
2	—	157	18	—	171
4	—	160	20	—	172
6	1.64	161	22	—	173
8	—	166	24	1.64	173
10	—	167	26	—	174
12	—	168	28	—	174
14	—	169	30	1.64	175

TABLE V.

Effect of Duration of Current on the Normal Cathode Fall in Helium.

Cathode Metal.	Duration of Current (min.).	Gas Pressure (mm.).	Cathode Fall (Volts).
Magnesium.	0	1.90	173
	34	1.93	192
Zinc.	0	1.69	170
	34	1.73	185
Platinum.	0	1.90	245
	36	1.90	249

The observations given in Table IV. (aluminium in helium) show that *the normal cathode fall increased* to a marked extent (157 volts to

175 volts) with duration of current while the gas pressure remained almost constant (1.62 to 1.64 mm.). The change here was therefore just the reverse of that noted in the first part.

Similar results were obtained in all other tests (Tables V. and VI.). The effect of duration of current was the same therefore, notwithstanding the fact that in two out of three tests made with helium the gas pressure increased slightly with duration of current, while with hydrogen (with but one exception) the pressure decreased.

TABLE VI.

Effect of Duration of Current on the Normal Cathode-Fall in Hydrogen.

Cathode Metal.	Duration of Current (min.).	Gas Pressure (mm.).	Cathode Fall (Volts).
Aluminium.	0	.85	190
	31	1.16	217
	0	3.12	199
	30	3.01	211
Magnesium.	0	3.23	205
	26	3.03	217
Zinc.	0	3.24	220
	32	3.14	234
	0	1.97	227
	32	1.83	250
Platinum.	0	3.48	315
	20	3.16	331
	0	3.23	310
	30	3.23	363

In order to make certain that the observed change in fall did not arise from the slight change in gas pressure which took place, observations were made on one cathode (with very short duration of current in each case) with pressures varying through a wide range. From the results of these tests (given in Tables VII. and VIII.) it may be seen that while the normal fall, with but one exception, decreased slightly with a decreasing gas pressure a relatively large change in pressure caused only a small lowering of the fall as compared with that produced by duration of current.

The results given in Tables VII. and VIII. are interesting aside

TABLE VII.

Effect on Normal Cathode Fall in Helium Caused by Changing the Gas Pressure.

Cathode Metal.	With Same Gas at Decreasing Pressures.		With Fresh Gas.	
	Gas Pressure (mm.).	Cathode Fall (Volts).	Gas Pressure (mm.).	Cathode Fall (Volts).
Aluminium.	4.00	175		
	1.85	170		
	.87	168		
	.56	165	3.97	176
Magnesium.	4.33	191		
	2.97	190		
	1.96	188		
	1.04	186	4.32	196

TABLE VIII.

Effect on Normal Cathode Fall in Hydrogen Caused by Changing the Gas Pressure.

Cathode Metal.	With Same Gas at Decreasing Pressures.		With Fresh Gas.	
	Gas Pressure (mm.).	Cathode Fall (Volts).	Gas Pressure (mm.).	Cathode Fall (Volts).
Aluminium.	3.72	213		
	2.03	211		
	1.13	210	3.90	213
Zinc.	6.80	256		
	3.65	253		
	1.45	250	6.35	256
Platinum.	6.55	363		
	3.85	360		
	2.35	354	6.46	364

from the fact that they prove that the observed change in the normal cathode fall with duration of current is not caused by the change in gas pressure. For, they appear to contradict the generally accepted conclusion that the normal cathode fall is independent of the gas pressure.[1] The variation here evidently cannot arise from a change in the cathode with use, for with fresh gas at the original pressure the fall returned to its original value. However

[1] Warburg, Wied. Ann., 31, p. 545, 1887; 40, p. 1, 1890.

it may, as shown in the first part of this paper, arise from a change in the condition of the gas, since the sequence of observations were made by reducing the gas pressure and not by introducing fresh gas.

Summing up the results: (*a*) It has been found that use of a metal as cathode with a constant current density greater than the "normal" *does not change the metal* in a way such as to appreciably change its cathode fall, but it does in general *lower* the cathode fall by producing some *change in the gas*. (*b*) On the other hand, use of a metal as cathode with "normal" current density *increases* its cathode fall, which in light of (*a*) appears reasonably explicable only by assuming that in the first case the cathode fall and current density were large enough to keep the surface of the metal clean by "disintegration," but not in the second.

The writer wishes here to express his thanks to Dr. C. A. Skinner, of the University of Nebraska, for help and encouragement given so freely during the pursuit of this investigation.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

TERMINAL VELOCITY OF FALL OF SMALL SPHERES IN AIR AT REDUCED PRESSURES.[1]

BY L. W. McKEEHAN.

IN previous work by John Zeleny and the author[2] it was found that Stokes' law for the terminal velocity of fall of a small sphere in a viscous fluid is upheld in air at atmospheric pressure, for spheres differing in size and composition.

E. A. Cunningham has given a theory[3] in which the molecular structure of the gaseous medium is considered, and finds that the velocity calculated by Stokes' law should be multiplied by a factor,

$$\left[1 + 1.63 \frac{l}{a} \left(\frac{1}{2 - f} \right) \right],$$

where l is the mean free path of the gas molecule, a is the radius of the sphere, and f is the fraction of the total number of molecular impacts on the surface of the sphere, which are regularly reflected. The second term in this expression becomes important only for low pressures, or for very minute spheres at atmospheric pressure. An examination of the derivation of this factor leads to the conclusion that it should be written

$$\left[1 + .940 \frac{l}{a} \left(\frac{4}{5 - f} \right) \right].$$

Neither of these factors agrees numerically with experiments at pressures down to a fraction of a millimeter of mercury, for spheres of wax ranging

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[1] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 27–30, 1910.

[2] PHYS. REV., Vol. XXX., p. 535, May, 1910.

[3] Proc. Roy. Soc., Ser. A, Vol. 83, p. 357, 1910.

from $a = .003$ cm., to $a = .0002$ cm.   An empirical factor,

$$\left[1 + k \frac{l}{a} \right],$$

is found, however, to represent with accuracy the results obtained.

UNIVERSITY OF MINNESOTA.

ON THE RESOLUTION OF THE SPECTRAL LINES OF MERCURY.[1]

BY J. C. McLENNAN AND E. N. MACALLUM.

WITH this communication a series of slides were exhibited which illustrated the resolving power of a high grade echelon spectroscope recently made by The Adam Hilger Co. for the Physical Laboratory at Toronto.   With this instrument the green line of mercury 5461 ÅU was shown to consist of a central doublet accompanied by three satellites of greater and by three of smaller wave-length, and the blue line 4359 ÅU to consist of a central strong line accompanied by three satellites of greater wave-length and by two of shorter wave-length.   These results are in good agreement with the components of the same lines recently obtained by Gale and Lemon and others with a seven-inch Michelson plane grating.

Slides were also shown which illustrated the magnetic resolution of the satellite of shortest wave-length of the line 5461 ÅU.   Under magnetic fields of 2,000 Gauss this line was resolved into a quartet, the inner doublet of which corresponded to vibrations parallel to the magnetic field, and the outer to vibrations perpendicular to the magnetic field.   Measurements made on the displacements of the lines constituting the quartet showed that the magnetic separation of the two inner doublets corresponded approximately to three fourths and that of the outer components to three halves of a normal triplet.

In taking the photographs of the magnetic resolution of this satellite it was necessary to use a source of light of strong intensity, and even with such a source to give long exposures.   The arrangement adopted is shown in Fig. 1.

A magnet was provided with pole pieces each of which was divided into two pieces that could be bolted together.   The pole pieces were provided with a groove, and in this groove an ordinary Cooper Hewitt lamp was fastened as shown in the figure with its axes parallel to the direction of the field. With this tube light of strong intensity was provided and there was no danger of the tube being melted by the discharge for the effect of the field was to concentrate the discharge along the axis of the tube.   With such an arrangement exposures of any duration could be used, and magnetic fields as high as 3,500 gauss could be obtained readily.

[1] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 28–31, 1910.

With this paper diagrams were also shown illustrating the dependence of the constitution of the mercury line 5461 ÅU upon the source of light.

With the Cooper Hewitt lamp no resolution of the main doublet could be observed but when a Hæreus quartz mercury lamp was used this line was clearly seen to be a doublet. A further peculiarity of the resolution of this line obtained with the quartz lamp was that after the lamp was in action a few minutes each of the lines of this main doublet and those corresponding to each of the satellites became somewhat diffuse and approximately of the same intensity and configuration. The general appearance of the resolved line with this source was a series of eight bands of approximately equal intensity and light distri-

Fig. 1.

bution. This peculiar appearance came on gradually after the lamp was lighted and while it suggested an inversion of the main line it also suggested an equi-partition of the energy of the line into its eight constituents. Janicki[1] who also observed this phenomenon with a Hæreus lamp, states that he found that the line 5461 ÅU finally broke up into five equidistant bands of equal intensity.
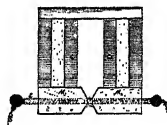
PHYSICAL LABORATORY,
    UNIVERSITY OF TORONTO,
        December 1, 1910.

ON A VARIATION IN THE INTENSITY OF THE PENETRATING RADIATION AT THE EARTH'S SURFACE OBSERVED DURING THE PASSAGE OF HALLEY'S COMET.[2]

BY ARTHUR THOMSON, M.A.

SINCE the discovery in 1902 of the penetrating radiation which exists at the surface of the earth, a number of investigators have attempted to ascertain whether the radiation had its origin in the rocks and soil of the earth or in the atmosphere of the latter, or whether it had its origin in one or more of the extra terrestrial bodies such as the sun or the moon.

A number of investigators including Borgmann,[3] Mach and Rimmer,[4] Wood and Campbell,[5] McKeon,[6] Strong,[7] Pacini[8] and Wulf[9] found in their

[1] Ann. der Physik, 19, p. 36, 1906.

[2] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 28–31, 1910.

[3] Borgmann, Sc. Abstracts, 1905, no. 1580.

[4] Mach and Rimmer, Phys. Zeit., 7, p. 617, Sept. 15, 1906.

[5] Wood and Campbell, Phil Mag., February, 1907.

[6] McKeon, PHYS. REV., 1907.

[7] Strong, PHYS. REV., July, 1908.

[8] Pacini, Rend. All. Lincei, 18, 123–129, 1909.

[9] Wulf, Phys. Zeit., 10, 1909, pp. 152–157.

various localities daily variations in the intensity of this radiation, and were able from their results to point to regular daily periods of maximum and minimum intensity.

Further, an examination of the results of these investigators shows that the variation in the intensity of the radiation covers an exceedingly wide range.

In some cases variations of ten per cent. from the normal intensity were observed, but in the majority of cases variations very much greater than this amount were commonly noted.

From a closer examination of the various results recorded, however, it does not appear that it is possible to establish any connection between the times of maxima and minima intensities obtained in different localities and as a consequence one is forced to the conclusion that the variations observed by them were due to local causes and not to any influences arising from the proximity of the sun or any of the other celestial bodies.

The results obtained by Professor McLennan and his students at Toronto when investigating this penetrating radiation have been quite different from those obtained elsewhere, in that they failed to show the marked variations observed by others. In particular C. S. Wright who made a number of observations at Toronto was not able to find any appreciable regular variation in the intensity of the penetrating radiation, and the variations which he did observe did not exceed three per cent. of the observed normal intensity of the radiation.

These observations were confirmed later by G. A. Cline and the conclusion was drawn by both these investigators that the soil contributed by far the greater proportion of the penetrating radiation present at the earth's surface at Toronto, and that any portion which might be contributed by the earth's atmosphere or by the sun was negligible in comparison with that arising from the soil.

The view that the soil was the chief source of the radiation was corroborated later by the discovery of Professor McLennan and C. S. Wright that the intensity of the radiation was very much less over the surface of Lake Ontario than over the surface of the land adjoining the waters of the lake.

This diminution in the intensity of the radiation over lake waters was also observed later by Gockel at Vierwaldstättersee in Switzerland.

From these experiments it seems then that there is no doubt about the soil and rocks of the earth being the chief source of the radiation. It appears too that the atmosphere of the earth contributes but a small proportion, if any, of the radiation. There is, moreover, no evidence to show that any part of it has its origin in either the sun or the moon.

Although it is clear from the evidence adduced above that it is not possible to ascribe an appreciable proportion of the penetrating radiation as ordinarily observed to a celestial origin, it seemed possible that the intensity of the

radiation might be increased by the passage near to the earth of a body possessing the constitution and characteristics of Halley's Comet.

And at the suggestion of Professor McLennan the writer undertook to make a series of observations on the intensity of the radiation extending over a number of days during the period when the comet and its tail was predicted to be in closest proximity to the earth.

To make the observations an ionization vessel, of about 30,000 c.c. capacity, similar to that used by C. S. Wright, and later by G. A. Cline, was installed on the roof of the physical laboratory, and readings on the conductivity of the air in the vessel were taken with a sensitive quadrant electrometer situated in a room directly below the point where the ionization vessel was installed.

In setting up the apparatus the most extreme care was taken to see that all insulating parts were clean and dry, that the receiver itself was hermetically sealed and that all electrical connections were effectively and thoroughly screened.

The observations were begun on the morning of May 18, and were continued without interruption until noon on May 21. As a result of these observations it was found that while the normal ionization was about 30 ions per c.c. per second two rather remarkable increases were noted in the observed ionizations. One of these occurred on May 19, beginning at about one o'clock P.M.[1] and lasting until about four o'clock P.M., and the other on May 21, commencing at about half-past nine in the morning and continuing for a period of about one hour and a half. On these two occasions it was found that the ionization rose to about 100 and 130 ions per c.c. per second respectively.

Whether these unusual increases in the ionization were due to the presence of the comet or not, it appears to the writer impossible to decide. A. Wigand,[2] in a paper on observations made at Halle from May 17 to May 20, records decided diminutions in the earth's horizontal magnetic field during the early hours of May 19 and again from 7.36 to 11.30 A.M. of the same day. He also records a marked increase in the electrical conductivity of the atmospheric air between the hours of five and nine o'clock on the morning of May 19 and at the same time an anomalous depression in the potential gradient. Wigand concludes from his observations that these magnetic and electrical disturbances were in all probability connected with the passage of the comet.

The disturbances noted by Wigand it will be seen took place on May 19 at 11 A.M., about twelve hours before the one at Toronto occurred. The phenomena observed by him were the electrical "dissipation" and the potential gradient, while that investigated by the writer was the ionization

[1] Throughout this paper the times refer to 75th meridian mean time.

[2] Wigand, Deutsch. Phys. Ges. Verh., 12, 13, pp. 511–530, July 15, 1910.

in the gas within an hermetically closed metallic vessel. It is possible to conceive that three such phenomena could be related but in the present case it is difficult to see any connection between the two sets of disturbances noted and still more difficult to establish a connection between them and the passage of the comet.

To the writer there appear to be four possible explanations of the disturbance noted by him. It could in the first place be caused by bringing radioactive bodies into the neighborhood of the ionizing chamber. It might also be produced by temporary faulty instrumental conditions or adjustments. Then again the disturbance might be the result of some intense temporary solar activity or finally it might be due to the passage of the comet. The first explanation can be easily dismissed for no radioactive bodies were brought near at any time while the observations were being taken. In regard to the second explanation it may be stated that extreme care was taken to perfect the conditions of measurement. Even after the measurements were finished the whole instrumental outfit was again closely scrutinized, but a rigid examination failed to reveal any faulty adjustment.

The writer is therefore inclined to the view that the disturbance was not due to faulty conditions of experiment or to faulty adjustments.

With the object of corroborating this view observations were again made for periods of about twelve hours on May 23, 24, 25 and 26. These observations failed to show anything abnormal. The ionization varied between narrow limits and the observations were practically of the same character as those made by Wright and Cline on previous occasions.

If the disturbances noted were due to faulty adjustments or to some cause associated with the sun's activity one would expect a repetition of the disturbance. None such occurred, however, during any of the four later periods of observation, and this absence of a repetition would go to confirm the view that the disturbance was not due to any defect in the measuring apparatus.

The absence of a repetition on the four days mentioned, however, would not entirely exclude solar activity as a cause of the disturbance. To establish this point it would be necessary to make a more extended series of observations.

All the readings recorded in the present paper were taken visually, but the strain of making measurements in this manner over long-continued periods of time is too great for visual observation to be practicable, and the writer therefore hopes to continue, at an early date, the observations with self-recording apparatus with a view to seeing whether solar activity is ever really accompanied by such terrestrial disturbances as those noted or not.

This information would be useful in deciding between solar activity and the comet's influence as the cause of the disturbance.

In conclusion the writer wishes to acknowledge his indebtedness to Professor McLennan for suggestions during the investigation, and for his kindness

in scrutinizing the installation of the apparatus, and also to Mr. W. T. Kennedy for his kind assistance in taking some of the readings.

PHYSICAL LABORATORY,
    UNIVERSITY OF TORONTO.
        Dec. 1, 1910.

AMERICAN PHYSICAL SOCIETY.

REPORT OF TREASURER, DECEMBER 1, 1910.

Receipts.

Dues for 1909...$	18.00
Dues for 1910...	3,339.00
Payments for Section "B," Science Abstracts..................	71.00
Entrance fees...	45.00
Interest on deposit and exchange...........................	25.91
Payments on old accounts..................................	94.00
Deposit in bank, December 1, 1909.........................	451.27
	$4,044.18

Disbursements.

Mileage of president.......................................$	90.36
Office of secretary: allowance..............................	100.00
Stationery and printing................................	140.45
Incidentals..	6.00
Mileage..	79.84
Office of treasurer: allowance..............................	50.00
Stationery and printing................................	63.00
Incidentals..	1.25
Subscriptions to PHYSICAL REVIEW..........................	1,744.50
Subscriptions to Science Abstracts "A"......................	863.25
Subscriptions to Science Abstracts "B"......................	72.00
Deposit in bank, December 1,1910..........................	833.53
	$4,044.18

J. S. AMES,
Treasurer.

I certify that I have audited these accounts and that I have found them correct.

WM. J. A. BLISS.

    December 12, 1910.

NEW BOOKS.

Die Verwertung des Luftstickstoffs mit Hilfe des elektrischen Flammenbogens. By J. ZENNECK. Leipzig : S. Hirzel, 1911. Pp. 1 + 29. Price, M. 1.60.

A History of the Cavendish Laboratory 1871–1910. New York: Longmans, Green and Co., 1910. Pp. x + 342.

Lehrbuch der Praktischen Physik. By FRIEDRICH KOHLRAUSCH. Leipzig: Teubner, 1910. Pp. xxxii + 736. Price, M. 11.

Die Mechanik, Eine Einführung mit Einem Metaphysischen Nachwort. By LUDWIG TESAR. Leipzig: Teubner, 1909. Pp. xii + 220.

Die Wissenschaft, Sammlung naturwissenschaftlicher und mathematischer Monographien. Grundlagen der Praktischen Metronomie. By KARL SCHEEL. Braunschweig: Vieweg and Son, 1911. Pp. xii + 168. Price, M. 5.20.

Principles of Electrical Engineering. By HAROLD PENDER. New York: McGraw-Hill Book Company, 1910. Pp. xi + 340. Price, $4.00 net.

Die Luftshiffahrt, Ihre wissenschaftlichen Grundlagen und Technische Entwicklung. By RAIMUND NIMFÜHR. Leipzig: Teubner, 1910. Pp. viii + 152.

An Elementary Text-Book of Physics. Part II. Sound. By R. WALLACE STEWART. London: Charles Griffin Company; Philadelphia: J. B. Lippincott Co., 1909. Pp. 1 + 141. Price, $1.25.

An Elementary Text-Book of Physics. Part III. Light. By R. WALLACE STEWART. London: Charles Griffin and Company; Philadelphia: J. B. Lippincott Co., 1909. Pp. vi + 219. Price, $1.50.

An Elementary Text-Book of Physics. Part IV. Heat. By R. WALLACE STEWART. London: Charles Griffin and Company; Philadelphia: J. B. Lippincott Co., 1910. Pp. 1 + 246. Price, $1.50.

A Text-Book of General Physics for Colleges. Mechanics and Heat. By J. A. CULLER. Philadelphia: J. B. Lippincott Company, 1909. Pp. ix + 311. Price, $1.80.

Traité de Radioactivité. By MADAME P. CURIE. Tome I. Paris: Gauthier-Villars, Imprimeur-Libraire, 1910. Pp. xiii + 426.

THE

PHYSICAL REVIEW.

THE ISOLATION OF AN ION, A PRECISION MEASURE-MENT OF ITS CHARGE, AND THE CORRECTION OF STOKES'S LAW.[1]

By R. A. Millikan.

§ 1. Introduction.

IN a preceding paper[2] a method of measuring the elementary electrical charge was presented which differed essentially from methods which had been used by earlier observers only in that all of the measurements from which the charge was deduced were made upon one individual charged carrier. This modification eliminated the chief sources of uncertainty which inhered in preceding deter-minations by similar methods such as those made by Sir Joseph Thomson,[3] H. A. Wilson,[4] Ehrenhaft[5] and Broglie,[6] all of whom had deduced the elementary charge from the average behavior in electrical and gravitational fields of swarms of charged particles.

The method used in the former work consisted essentially in catching ions by C. T. R. Wilson's method on droplets of water or alcohol, in then isolating by a suitable arrangement a single one of these droplets, and measuring its speed first in a vertical electrical

[1] A preliminary account of this work was read on April 23 before the American Physical Society and was published in Science, Vol. 32, p. 436, September, 1910.

[2] Millikan, Phys. Rev., December, 1909, and Phil. Mag., 19, p. 209.

[3] Thomson, Phil. Mag., 46, p. 528, 1898; 48, p. 547, 1899; 5, p. 346, 1903.

[4] H. A. Wilson, Phil. Mag., 5, p. 429, 1903.

[5] Ehrenhaft, Phys. Zeit., Mai, 1909.

[6] Broglie, Le Radium, Juillet, 1909.

and gravitational field combined, then in a gravitational field alone.[1]

The sources of error or uncertainty which still inhered in the method arose from: (1) the lack of complete stagnancy in the air through which the drop moved; (2) the lack of perfect uniformity in the electrical field used; (3) the gradual evaporation of the drops, rendering it impossible to hold a given drop under observation for more than a minute, or to time the drop as it fell under gravity alone through a period of more than five or six seconds; (4) the assumption of the exact validity of Stokes's law for the drops used. The present modification of the method is not only entirely free from all of these limitations, but it constitutes an entirely new way of studying ionization and one which seems to be capable of yielding important results in a considerable number of directions.

With its aid it has already been found possible:

1. To catch upon a minute droplet of oil and to hold under observation for an indefinite length of time one single atmospheric ion or any desired number of such ions between 1 and 150.

2. To present direct and tangible demonstration, through the study of the behavior in electrical and gravitational fields of this oil drop, carrying its captured ions, of the correctness of the view advanced many years ago and supported by evidence from many sources that all electrical charges, however produced, are exact multiples of one definite, elementary, electrical charge, or in other words, that an electrical charge instead of being spread uniformly over the charged surface has a definite granular structure, consisting, in fact, of an exact number of specks, or atoms of electricity, all precisely alike, peppered over the surface of the charged body.

3. To make an exact determination of the value of the elementary electrical charge which is free from all questionable theoretical assumptions and is limited in accuracy only by that attainable in the measurement of the coefficient of viscosity of air.

4. To observe directly the order of magnitude of the kinetic energy of agitation of a molecule, and thus to bring forward new

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[1] In work reported since this paper was first presented, Ehrenhaft (Phys. Zeit., July, 1910) has adopted this vertical field arrangement so that he also now finds it possible to make all his measurements upon individual charged particles.

direct and most convincing evidence of the correctness of the kinetic theory of matter.

5. To demonstrate that the great majority, if not all, of the ions of ionized air, of both positive and negative sign, carry the elementary electrical charge.

6. To show that Stokes's law for the motion of a small sphere through a resisting medium, breaks down as the diameter of the sphere becomes comparable with the mean free path of the molecules of the medium, and to determine the exact way in which it breaks down.

§2. The Method.

The only essential modification in the method consists in replacing the droplet of water or alcohol by one of oil, mercury or some other non-volatile substance and in introducing it into the observing space in a new way.

Fig. 1 shows the apparatus used in the following experiments. By means of a commercial "atomizer"[1] A a cloud of fine droplets of oil is blown with the aid of dust-free air into the dust-free chamber C. One or more of the droplets of this cloud is allowed to fall through a pin-hole p into the space between the plates M, N of a horizontal air condenser and the pin-hole is then closed by means of an electromagnetically operated cover not shown in the diagram. If the pin-hole is left open air currents are likely to pass through it and produce irregularities. The plates M, N are heavy, circular, ribbed brass castings 22 cm. in diameter having surfaces which are ground so nearly to true planes that the error is nowhere more than .02 mm. These planes are held exactly 16 mm. apart by means of three small ebonite posts a held firmly in place by ebonite screws. A strip of thin sheet ebonite C passes entirely around the plates, thus forming a completely enclosed air space. Three glass windows, 1.5 cm. square, are placed in this ebonite strip at the

[1] The atomizer method of producing very minute but accurately spherical drops for the purpose of studying their behavior in fluid media, was first conceived and successfully carried out in January, 1908, at the Ryerson Laboratory, by Mr. J. Y. Lee, while he was engaged in a quantitative investigation of Brownian movements. His spheres were blown from Wood's metal, wax and other like substances which solidify at ordinary temperatures. Since then the method has been almost continuously in use here, upon this and a number of other problems, and elsewhere upon similar problems.

angular positions 0°, 165° and 180°. A narrow parallel beam of light from an arc lamp enters the condenser through the first window and emerges through the last. The other window serves for observing, with the aid of a short focus telescope placed about 2 feet distant, the illuminated oil droplet as it floats in the air between the plates. The appearance of this drop is that of a brilliant star on a black background. It falls, of course, under the
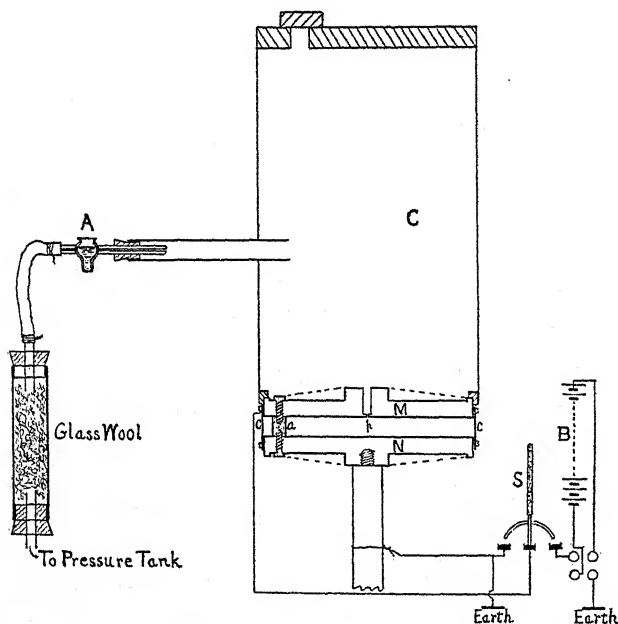
Fig. 1.

action of gravity, toward the lower plate; but before it reaches it, an electrical field of strength between 3,000 volts and 8,000 volts per centimeter is created between the plates by means of the battery *B*, and, if the droplet had received a frictional charge of the proper sign and strength as it was blown out through the atomizer, it is pulled up by this field against gravity, toward the upper plate. Before it strikes it the plates are short-circuited by means of the switch *S* and the time required by the drop to fall under gravity the distance corresponding to the space between the cross hairs of the observing telescope is accurately determined. Then the rate

at which the droplet moves up under the influence of the field is measured by timing it through the same distance when the field is on. This operation is repeated and the speeds checked an indefinite number of times, or until the droplet catches an ion from among those which exist normally in air, or which have been produced in the space between the plates by any of the usual ionizing agents like radium or X-rays. The fact that an ion has been caught, and the exact instant at which the event happened is signalled to the observer by the change in the speed of the droplet under the influence of the field. From the sign and magnitude of this change in speed, taken in connection with the constant speed under gravity, the sign and the exact value of the charge carried by the captured ion are determined. The error in a single observation need not exceed one third of one per cent. It is from the values of the speeds observed that all of the conclusions above mentioned are directly and simply deduced.

The experiment is particularly striking when, as often happens, the droplet carries but one elementary charge and then by the capture of an ion of opposite sign is completely neutralized so that its speed is altogether unaffected by the field. In this case the computed charge is itself the charge on the captured ion.

The measurement of the distance between the cross hairs, correct to about .01 mm., is made by means of a standard scale placed vertically at exactly the same distance from the telescope as the pin-hole p.

§ 3. The Deduction of the Relative Values of the Charges Carried by a Given Droplet.

The relations between the apparent mass[1] m of a drop, the charge e_n, which it carries, its speed, v_1 under gravity, and its speed v_2 under the influence of an electrical field of strength $\mathscr{F}$, are given by the simple equation

$$\frac{v_1}{v_2} = \frac{mg}{\mathscr{F}e_n - mg} \quad \text{or} \quad e_n = \frac{mg}{\mathscr{F}}\left(\frac{v_1 + v_2}{v_1}\right). \tag{1}$$

[1] The term "apparent mass" is used to denote the difference between the actual mass and the buoyancy of the air.

This equation involves no assumption whatever save that the speed of the drop is proportional to the force acting upon it, an assumption which is fully and accurately tested experimentally in the following work. Furthermore, equation (1) is sufficient not only for the correct determination of the relative values of all of the charges which a given drop may have through the capture of a larger or smaller number of ions, but it is also sufficient for the establishment of all of the assertions made above, except 3, 4 and 6. However, for the sake of obtaining a provisional estimate of the value of m in equation (1), and therefore of making at once a provisional determination of the absolute values of the charges carried by the drop, Stokes's law will for the present be assumed to be correct, but it is to be distinctly borne in mind that the conclusions just now under consideration are not at all dependent upon the validity of this assumption.

This law in its simplest form states that if μ is the coefficient of viscosity of a medium, x the force acting upon a spherical drop of radius a in that medium, and v the velocity with which the drop moves under the influence of the force, then

$$x = 6\pi\mu a v. \tag{2}$$

The substitution in this equation of the resulting gravitational force acting on a spherical drop of density σ in a medium of density ρ gives the usual expression for the rate of fall, according to Stokes, of a drop under gravity, viz.,

$$v_1 = \frac{2}{9}\frac{ga^2}{\mu}(\sigma - \rho). \tag{3}$$

The elimination of m from (1) by means of (3), and the further relation $m = \frac{4}{3}\pi a^3(\sigma - \rho)$ gives the charge e_n in the form

$$e_n = \frac{4}{3}\pi \left(\frac{9\mu}{2}\right)^{\frac{3}{2}} \left(\frac{1}{g(\sigma-\rho)}\right)^{\frac{1}{2}} \frac{(v_1 + v_2)v_1^{\frac{1}{2}}}{\mathscr{F}}. \tag{4}$$

It is from this equation that the values of e_n in tables I.–XII. are obtained.

§ 4. Preliminary Observations upon the Catching of Ions by Oil-Drops.

Table I. presents the record of the observations taken upon a drop which was watched through a period of four and one half hours as it was alternately moved up and down between the crosshairs of the observing telescope under the influence of the field F and gravity G. How completely the errors arising from evaporation, convection currents or any sort of disturbances in the air are eliminated is shown by the constancy during all this time in the value of the velocity under gravity. This constancy was not attained without a considerable amount of experimenting which will be described in section 11. It is sufficient here to state that the heating effects of the illuminating arc were eliminated, first by filtering the light through about two feet of water, and second, by shutting off the light from the arc altogether except at occasional instants, when the shutter was opened to see that the star was in place, or to make an observation of the instant of its transit across a cross-hair. Further evidence of the complete stagnancy of the air is furnished by the fact that for an hour or more at a time the drop would not drift more than two or three millimeters to one side or the other of the point at which it entered the field.

The observations in Table I. are far less accurate than many of those which follow, the timing being done in this case with a stopwatch, while many of the later timings were taken with a chronograph. Nevertheless this series is presented because of the unusual length of time over which the drop was observed, and because of the rather unusual variety of phenomena which it presents.

The column headed G shows the successive times in seconds taken by the droplet to fall, under gravity, the distance between the crosshairs. It will be seen that, in the course of the four and one half hours, the value of this time increases very slightly, thereby showing that the drop is very slowly evaporating. Furthermore, there are rather marked fluctuations recorded in the first ten observations which are probably due to the fact that, in this part of the observation, the shutter was open so much as to produce very slight convection currents.

The column headed F is the time of ascent of the drop between

TABLE I.

Negative Drop.

Distance between cross-hairs = 1.010 *cm.*
Distance between plates = 1.600 *cm.*
Temperature = 24.6° C.
Density of oil at 25° C. = .8960.
Viscosity of air at 25.2° C. = .0001836.

	G sec.	F sec.	n	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
	22.8	29.0	7	34.47	4.923
	22.0	21.8	8	39.45	4.931
	22.3	17.2 ⎫			
G = 22.28	22.4	— ⎪	9	44.42	4.936
V = 7950	22.0	17.3 ⎬			
	22.0	17.3 ⎭			
	22.0	14.2	10	49.41	4.941
	22.7	21.5	8	39.45	
	22.9	11.0	12	59.12	4.927
	22.4	17.4	9	44.42	
	22.8	14.3	10	49.41	
V = 7920	22.8	12.2 ⎫			
G = 22.80	22.8	12.3 ⎬	11	53.92	4.902
	23.0	—			
	22.8	14.2 ⎫			
F = 14.17	—	— ⎬	10	49.41	4.941
	22.8	14.0 ⎭			
	22.8	17.0 ⎫			
F = 17.13	—	17.2 ⎬	9	44.42	4.936
	22.9	17.2 ⎭			
	22.8	10.9 ⎫			
F = 10.73	22.8	10.9 ⎬	12	59.12	4.927
	22.8	10.6 ⎭			
	22.8	12.2	11	53.92	4.902
V = 7900	22.8	8.7	14	68.65	4.904
G = 22.82	22.7	6.8 ⎫	17	83.22	4.894
F = 6.7	22.9	6.6 ⎭			
	22.8	7.2 ⎫			
	—	7.2 ⎪			
	—	7.3 ⎪			
F = 7.25	—	7.2 ⎬	16	78.34	4.897
	23.0	7.4 ⎪			
	—	7.3 ⎪			
	—	7.2 ⎭			
F = 8.65	22.8	8.6 ⎫	14	68.65	4.904
	23.1	8.7 ⎭			
	23.2	9.8 ⎫	13	63.68	4.900
	—	9.8 ⎭			
F = 10.63	23.5	10.7 ⎫	12	59.12	4.927
	23.4	10.6 ⎭			

TABLE I.—*Continued.*

	G sec.	F sec.	n	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
	23.2	9.6			
	23.0	9.6			
	23.0	9.6			
	23.2	9.5			
$V = 7820$	23.0	9.6	13	63.68	4.900
$G = \ \ 23.14$	—	9.4			
$F = \ \ \ 9.57$	22.9	9.6			
	—	9.6			
	22.9	9.6			
	—	10.6	12	59.12	4.927
$F = \ \ \ 8.65$	—	8.7	14	68.65	4.904
	23.4	8.6			
	23.0	12.3			
$F = \ \ 12.25$	23.3	12.2	11	53.92	4.902
	—	12.1			
	23.2	12.4			

Change forced with radium.

	G sec.	F sec.	n	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
	23.4	72.4			
	22.9	72.4			
$F = \ \ 72.10$	23.2	72.2	5	24.60	4.920
	23.5	71.8			
	23.0	71.7			
	23.0	39.2	6		
$V = 7800$	23.2	39.2			
$G = \ \ 23.22$	—	27.4	7	34.47	
	—	20.7	8	39.38	4.922
	—	26.9	7	34.47	4.923
	—	27.2			
	23.3	39.5			
	23.3	39.2			
$F = \ \ 39.20$	23.4	39.0	6	29.62	4.937
	23.3	39.1			
	23.2	71.8	5	24.60	4.920
	23.4	382.5	4		
	23.2	374.0			
	23.4	71.0	5	24.60	4.920
	23.8	70.6			
$V = 7760$	23.4	38.5	6		
$G = \ \ 23.43$	23.1	39.2			
	23.5	70.3			
	23.4	70.5			
	23.6	71.2			
	23.4	71.4	5	24.60	4.920
	23.6	71.0			
	23.4	71.4			

TABLE I.—*Continued.*

	G sec.	F sec.	n	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
	23.5	380.6			
	23.4	384.6			
	23.2	380.0			
$F = 379.6$	23.4	375.4	4	19.66	4.915
	23.6	380.4			
	23.3	374.0			
	23.4	383.6			
	——	39.2			
$F = 39.18$	23.5	39.2			
$V = 7730$	23.5	39.0	6	29.62	4.937
$G = 23.46$	23.4	39.6			
	——	70.8			
$F = 70.65$	——	70.4	5	24.60	4.920
	——	70.6			
	23.6	378.0	4	19.66	

Saw it, here, at end of 305. sec., pick up two negatives.

	G sec.	F sec.	n	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
	23.6	39.4	6	29.62	4.937
	23.6	70.8	5	24.60	4.920

Mean of all e_1s $= 4.917$

Differences.

$$24.60 - 19.66 = 4.94$$
$$29.62 - 24.60 = 5.02$$
$$34.47 - 29.62 = 4.85$$
$$39.38 - 34.47 = \underline{4.91}$$
$$\text{Mean dif.} = \underline{4.93}$$

the cross-hairs under the action of the field. The column headed e_n is the value of the charge carried by the drop as computed from (4). The column headed n gives the number by which the values of the preceding column must be divided to obtain the numbers in the last column. The numbers in the e_n column are in general averages of all the observations of the table which are designated by the same numeral in the n column. If a given observation is not included in the average in the e_n column, a blank appears opposite that observation in the last two columns. On account of the slow change in the value of G, the observations are arranged in groups and the average value of G for each group is placed opposite that group in the first column. The reading of the voltmeter, taken at the mean time corresponding to each group, is labelled V

and placed just below or just above the mean G corresponding to that group. The volts were in this case read with a ten thousand volt Braun electrometer which had been previously calibrated, but which may in these readings be in error by as much as one per cent., though the error in the relative values of the volts will be exceedingly slight. The *PD* was applied by means of a storage battery. It will be seen from the readings that the potential fell somewhat during the time of observation, the rate of fall being more rapid at first than it was later on.

§ 5. Multiple Relations shown by the Charges on a Given Drop.

Since the original drop in this case was negative, it is evident that a sudden increase in the speed due to the field, that is, a decrease in the time given in column F, means that the drop has caught a negative ion from the air, while a decrease in the speed means that it has caught a positive ion.

If attention be directed, first, to the latter part of the table, where the observations are most accurate, it will be seen that, beginning with the group for which $G = 23.43$, the time of the drop in the field changed suddenly from 71 sec. to 380 sec., then back to 71, then down to 39, then up again to 71, and then up again to 380. These numbers show conclusively that the positive ion caught in the first change, *i. e.*, from 71 to 380, carried exactly the same charge as the negative ion caught in the change from 380 to 71. Or again, that the negative ion caught in the change from 71 to 39, had exactly the same charge as the positive ion caught in the change from 39 to 71.

Furthermore, the exact value of the charge caught in each of the above cases is obtained in terms of mg from the difference in the values of e_n, given by equation (1), and if it be assumed that the value of m is approximately known through Stokes's law, then the approximately correct value of the charge on the captured ion is given by the difference between the values of e_n obtained through equation (4). The mean value of this difference obtained from all the changes in the latter half of Table I. (see Differences), is 4.93×10^{-10}.

Now it will be seen from the first observation given in the table that the charge which was originally upon this drop and which was obtained, not from the ions in the air, but from the frictional process involved in blowing the spray, was 34.47×10^{-10}. This number comes within one seventh of one per cent. of being exactly seven times the charge on the positive, or on the negative, ion caught in the observations under consideration. In the interval between December, 1909, and May, 1910, Mr. Harvey Fletcher and myself took observations in this way upon hundreds of drops which had initial charges varying between the limits 1 and 150, and which were upon as diverse substances as oil, mercury and glycerine and found in every case the original charge on the drop an exact multiple of the smallest charge which we found that the drop caught from the air. The total number of changes which we have observed would be between one and two thousand, and *in not one single instance has there been any change which did not represent the advent upon the drop of one definite invariable quantity of electricity, or a very small multiple of that quantity.* These observations are the justification for assertions 1 and 2 of the introduction.

For the sake of exhibiting in another way the multiple relationship shown by the charges on a given drop the data of Table I. have been rearranged in the form shown in Table II.

TABLE II.

n	$4.917 \times n$	Observed Charge.	n	$4.917 \times n$	Observed Charge.
1	4.917	——	10	49.17	49.41
2	9.834	——	11	54.09	53.92
3	14.75	——	12	59.00	59.12
4	19.66	19.66	13	63.92	63.68
5	24.59	24.60	14	68.84	68.65
6	29.50	29.62	15	73.75	——
7	34.42	34.47	16	78.67	78.34
8	39.34	39.38	17	83.59	83.22
9	44.25	44.42	18	88.51	——

No more exact or more consistent multiple relationship is found in the data which the chemists have amassed on combining powers, and upon which the atomic theory of matter rests, than is found in tables I. to XIII.

§ 6. Direct Observation of the Energy of Agitation of a Molecule.

Before discussing assertion 4 it is desirable to direct attention to three additional conclusions which can be drawn from Table I.:

1. Since the time of the drop in the field varied in these observations from 380 sec. to 6.7 sec., it will be seen that the resultant moving force acting upon the drop was varied in the ratio 1 to 55, without bringing to light the slightest indication of a dependence of e_1 upon the velocity. Independently of theory, therefore, we can assert that the velocity of this drop was strictly proportional to the moving force. The certainty with which this conclusion can be drawn may be seen from a consideration of the following numerical data. Although we had upon our drop all possible multiples of the unit 4.917×10^{-10} between 4 and 17, save only 15, there is not a single value of e_1 given in the table which differs by as much as .5 per cent. from the final mean e_1. It is true that the observational error in a few of the smaller times is as much as 1 or 2 per cent., but the observational error in the last half of the table should nowhere exceed .5 per cent. In no case is there here found a divergence from the final value of e_1 of more than .4 per cent.

2. Since the charge on the drop was multiplied more than four times without changing at all the value of G, or the apparent value of e_1, the observations prove conclusively that in the case of drops like this, the drag which the air exerts upon the drop is independent of whether the drop is charged or uncharged. In other words, the apparent viscosity of the air is not affected by the charge in the case of drops of the sort used in these experiments.

3. It will be seen from the table that in general a drop catches an ion only when the field is off. Were this not the case there would be many erratic readings in the column under F, while in all the four and one half hours during which these experiments lasted, there is but one such, and the significance of this one will presently be discussed. A moment's consideration will show why this is. When the field is on, the ions are driven with enormous speed to the plates as soon as they are formed, their velocities in the fields here used being not less than 10,000 cm. per sec. Hence an ion cannot be caught when the field is on unless the molecule which is

broken up into ions happens to be on the line of force running from
the plates through the drop. With minute drops and relatively
small ionization this condition is very unlikely to occur. When the
field is off however, the ions are retained in the space between the
plates, and sooner or later, one or more of them, by virtue of its
energy of agitation, makes impact upon the drop and sticks to it.

These considerations lead up to assertion 4 in the introduction.
It will be seen from the readings in the first half of the table that
even when the drop had a negative charge of from 12 to 17 units
it was not only able to catch more negative ions, but it apparently
had an even larger tendency to catch the negatives than the posi-
tives. Whence then does a negative ion obtain an amount of
energy which enables it to push itself up against the existing electro-
static repulsion and to attach itself to a drop already strongly
negatively charged? It cannot obtain it from the field, since the
phenomenon occurs when the field is not on. It cannot obtain it
from any explosive process which frees the ion from the molecule
at the instant of ionization, since again in this case, too, ions would
be caught as well, or nearly as well, when the field is on as when it
is off. *Here then is an absolutely direct proof that the ion must be
endowed with a kinetic energy of agitation, which is sufficient to push it
up to the surface of the drop against the electrostatic repulsion of the
charge on the drop.*

This energy may easily be computed as follows: As will appear
later the radius of the drop was in this case .000197 cm. Further-
more, the value of the elementary electrical charge obtained as a
mean of all of our observations, is 4.891×10^{-10}. Hence the energy
required to drive an ion carrying a unit charge up to the surface
of a charged sphere of radius r, carrying 16 elementary charges, is

$$\frac{16e^2}{r} = \frac{16 \times (4.891 \times 10^{-10})^2}{.000197} = 1.95 \times 10^{-14} \text{ ergs.}$$

Now the kinetic energy of agitation of a molecule as deduced
from the value of e herewith obtained, and the kinetic theory equa-
tion, $p = \frac{1}{3}nmu^2$, is 5.75×10^{-14} ergs. According to the Max-
well-Boltzmann Law, which certainly holds in gases, this should
also be the kinetic energy of agitation of an ion. It will be seen

that the value of this energy is approximately three times that required to push a single ion up to the surface of the drop in question. If, then, it were possible to load up a drop with negative electricity until the potential energy of its charge were about three times as great as that computed above for this drop, then the phenomenon here observed, of the catching of new negative ions by such a negatively charged drop, should not take place, save in the exceptional case in which an ion might acquire an energy of agitation considerably larger than the mean value. Now, as a matter of fact, it was regularly observed that the heavily charged drops had a very much smaller tendency to pick up new negative ions than the more lightly charged drops, and, in one instance, *we watched for four hours another negatively charged drop of radius .000658 cm., which carried charges varying from 126 to 150 elementary units, and which therefore had a potential energy of charge (computed as above on the assumption of uniform distribution) varying from 4.6×10^{-14} to 5.47×10^{-14}, and in all that time this drop picked up but one single negative ion, and that despite the fact that the ionization was several times more intense than in the case of the drop of Table I.* This is direct proof independent of all theory that the order of magnitude of the kinetic energy of agitation of a molecule is 5×10^{-14}, as the kinetic theory demands.

§ 7. THE QUESTION OF VALENCY IN GASEOUS IONIZATION.

The correctness of assertion 5 in the case of the ionization existing in the observing chamber at the time at which the data in Table I. were taken is directly proved by the readings shown in that table, since the great majority of the changes recorded in column 4 correspond to the addition or subtraction of one single elementary charge. There are, however, some changes which correspond to the addition or subtraction of two or three times this amount and which therefore seem at first sight to indicate the existence of multiply charged ions. The conclusion, however, that valency is exhibited in gaseous ionization is not to be so easily drawn. During the observations recorded in the first half of the table, a closed tube of radium, containing 500 mg. of radium bromide of activity 3,000, stood about five feet away from the testing chamber, so that its

γ rays and a portion also of its β rays could enter this chamber. At the end of the observations in the group in which $G = 23.14$, this radium was brought up to within a few inches of the testing chamber, and six elementary charges were forced upon the drop in a manner which will be explained in section 8. The radium was then taken entirely out of the room, so that the changes recorded in the last half of the table are entirely due to such ionization as exists in air under normal atmospheric conditions.

Now, so long as changes take place only when the field is off there is no way of telling whether an observed change of two units is due to the addition to the drop of a double ion or to the successive additions of two single ions. It might be possible to account, therefore, for all the multiple changes which occurred when the field was off on the theory of successive single changes. There is, however, one single change recorded in the last part of Table I. which is not to be so easily accounted for upon this hypothesis. It will be seen that the drop made one particular trip up in 378 sec., then one down (recorded in the same horizontal line) in 23.6 sec. Immediately thereafter it was being pulled back again under the influence of the field at the 380 sec. rate—a rate so slow that it could scarcely be seen to be moving at all if observed for a short time. After the lapse of 305 seconds, during which time the shutter had been opened every 30 seconds or so to see that the star was still in view it changed instantly *while I was looking at it, the field being on*, from the 380 sec. to the 39 sec. speed skipping entirely the 71 sec. speed.

This sort of a multiple change, when the field was on, has been observed a dozen or more times when the ionization was so weak that it seemed very improbable that two or three different molecules could have been simultaneously ionized in the minute tube of force having for its diameter the diameter of the drop. In fact at the time at which the preliminary report upon this work was made it was thought that these changes constituted pretty good evidence that the ionization produced by radium does not always consist in the detachment of one single elementary charge from a neutral molecule but consists in occasional instances, in the separation of two or three such charges from a single molecule. The method

of studying ionization herewith presented is capable of furnishing a definite answer to the question here raised in the case of any particular ionizing agent. Recent work which will be reported in detail in another paper has shown that if either radium radiations or X rays of the intensities thus far used ever produce multiply-valent ions in air, the number of such ions formed cannot exceed one or two per cent. of the number of univalent ions formed. At the present time therefore it seems probable that, despite the contrary evidence presented by Townsend[1] and Franck and Westphal,[2] *the process of gaseous ionization by both radium and X rays always consists in the detachment from a neutral molecule of one single elementary electrical charge.*

§8. Mechanism of the Change of Charge of a Drop.

It has been tacitly assumed thus far that the only way in which a drop can change its charge is by the capture of ions of one sign or the other from the air. When a negative charge increases there seems to be no other conceivable way by which the change can be produced. But when it decreases there is no *a priori* reason for thinking that the change may not be due as well to the direct loss of a portion of the charge as to the neutralization of this same amount by the capture of a charge of opposite sign. Table I. shows conclusively, however, that if direct losses occur at all they take place with exceeding infrequency as compared with the frequency with which ions are captured from the air even when there is no external source of ionization whatever. For if there were two comparable processes tending to diminish the charge (viz., direct loss and capture of opposite ions) and only one tending to increase it (viz., capture of ions of the same sign) and that one of approximately the same efficiency as one of the first two, the drop, instead of maintaining as it did in these experiments for three and one half hours after the radium was removed from the room, essentially the same mean charge despite its repeated changes, would have quickly lost its charge and gone to the lower plate. The fact that it did not do this furnishes perhaps the most convincing evidence

[1] J. Townsend, Proc. Roy. Soc., 80, p. 207, 1908.
[2] J. Frank u. Westphal, Verh. d. D. Phys. Ges., 11, pp. 146 and 276, 1909.

which has yet been brought forward that the process of evaporation, which must have been going on continuously at the surface of the drop (see § 13) does not have the power of removing at all an electrical charge which resides upon an evaporating surface.[1]

There is but one more comment to be made upon Table I. At a point indicated in the table by the remark "change forced with radium," it will be noticed that the charge was suddenly changed from eleven negative units to five negative units; *i. e.*, that six positive units were forced upon the drop. This sort of a change was one which, after the phenomenon had once been got under control, we could make at will in either direction; *i. e.*, we could force charges of either sign or in any desired number, within limits, upon a given drop. We did this as follows: when it was desired to load the drop up negatively, for example, we held it with the aid of the field fairly close to the positive plate, and placed the radium so that it would produce uniform ionization throughout the chamber. Under these conditions if the positive and negative ions were alike in both number and mobility the chance that the drop would catch a negative ion would be as many times its chance of catching a positive ion as the distance from the drop to the negative plate was times the distance from the drop to the positive plate. Similarly, if we wished to load the drop positively it was held by the field close to the negative plate. On account of the slightly greater mobility of the negative ion, and also on account of the somewhat greater numbers in which they occur, we found, in general, a greater tendency of the drops to take up negative than positive charges. In view, therefore, of the greater ease with which negative drops could be held for long intervals without being lost to the plates most of the drops studied have been of negative sign.

§ 9. The Failure of Stokes's Law.

When the values of e_1 were computed as above for different drops, although each individual drop showed the same sort of consistency which was exhibited by the drop of Table I., the

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[1] This question has been considerably discussed in the past and the experiments of Henderson (Phil. Mag., 50, p. 489, 1900) and at Schwalbe (Ann. de Phys., 1, p. 295, 1900) strongly support the conclusions here reached, despite the opposite evidence brought forward by Pellat (Jour. de Physique, 8, p. 225, 1899).

values of e_1 at first came out differently, even for drops showing the same value of the velocity under gravity. This last irregularity was practically eliminated by blowing the drops into air which was strictly dust free, but even then drops of different sizes, as determined by v_1, always gave consistently different values of e_1. This is illustrated by the observations shown in tables III., IV., V., VI., VII. and VIII.

TABLE III.

Negative Drop No. 5.

Distance between cross hairs $=1.303$ *cm.*
Temperature $=24.6°$ *C.*
Density of oil at 25.0° C. $=.9041$

	G sec.	F sec.	n	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
	120.8	26.2	2	10.98	5.490
$F=$ 11.9	121.0	11.9	4	21.98	5.495
	121.2	16.5	3	16.41	5.470
	120.1	16.3			
$F=$ 26.40	120.2	26.4	2		
	119.8	67.4	1	5.495	5.495
$G=120.07$	120.1	26.6	2	10.98	
$V=$ 91.50	—	16.6			
$F=$ 16.50	120.2	16.6	3	16.41	
	—	16.5			
$F=$ 67.73	120.2	68.0	1	5.495	
	119.9	67.8			
		26.4		10.98	

$v_1 = .01085$ Mean e_1 (weighted) $=5.490$

TABLE IV.

Negative Drop No. 8.

Distance between cross hairs $=1.033$ *cm.*
Temperature $=20°$ *C.*

	G sec.	F sec.	n	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
	88.0	—			
$V=3512$	88.8	95.3	2	10.98	5.490
$G=$ 87.85	87.8	31.0	4	21.93	5.482
$F=$ 30.9	87.4	30.8			
	87.8	47.0	3	16.41	5.470
	87.3	—			

$v_1 = .01176$ Mean e_1 (weighted) $=5.482$

TABLE V.

Negative Drop No. 2.

Distance between cross hairs $=1.005$ *cm.*

Temperature $\qquad =24.3°$ *C.*

	G sec.	F sec.	n	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
$F=\ 49.15$	53.8	49.2	4	21.46	5.365
	53.7	49.1			
$G=\ 53.80$	54.0	95.2			
$V=3990$	——	95.5	3	16.00	5.333
	53.7	96.6			
$F=\ 95.78$	53.7	95.8			
$v_1=.01868$				Mean $e_1=5.349$	

TABLE VI.

Positive Drop No. 15.

Distance between cross hairs $=1.033$ *cm.*

Temperature $\qquad =20°$ *C.*

	G sec.	F sec.	n	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
	30.4	12.8	10	52.06	5.206
	30.5	17.9	8	41.61	5.200
	30.6	43.8	5	26.08	5.216
	30.2	85.9			
$G=\ 30.48$	30.5	85.9			
$V=9010$	30.7	86.4			
	30.5	85.6	4	20.84	5.210
	30.7	86.2			
$F=\ 86.09$	30.5	86.2			
	——	86.4			
	30.2	2520.0	3	15.55	5.183
$v_1=.04265$				Mean e_1 (weighted) $=5.208$	

The drops shown in tables III. and IV. were of almost the same size, as is seen from the closeness of the values of the two velocities under gravity, and although the field strength was in one case double that in the other the values of e_1 obtained are almost identical. Similarly Tables VII. and VIII. are inserted to show the consistency which could be attained in determining the values of e_1 so long as the drops used were of the same size. On the other hand, the series of tables III., V., VI. and VII., or IV., V., VI. and VIII. show conclusively that the value of e_1 obtained in this way

TABLE VII.

Positive Drop No. 16.
Distance between cross hairs =1.317 *cm.*
Temperature =27.6° C.

	G sec.	*F* sec.	*n*	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
	24.61 [1]	151.9			
	24.4	152.9			
F = 152.9	24.63	152.4	5	25.75	5.150
	24.6	153.5			
	24.4	153.9			
V =9075	24.7	39.4	7	36.03	5.147
G = 24.57	24.8	29.2			
	24.6	28.6			
F = 28.92	24.50	28.9	8	41.07	5.134
	24.59	29.0			
	24.54	16.0			
F = 15.93	24.53	16.0	11	56.25	5.114
	———	15.8			
$v_1 = .05360$				Mean e_1 (weighted) =*5.143*	

TABLE VIII.

Negative Drop No. 17.
Distance between cross hairs =1.305 *cm.*
Temperature =26.8° C.

	G sec.	*F* sec.	*n*	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
	23.8	31.5			
F = 31.33	23.6	31.3	8	41.10	5.139
	23.4	31.2			
	23.7	43.8			
G = 23.58	23.7	43.6			
V =8975	23.8	43.7	7	36.09	5.156
F = 43.72	23.5	43.4			
	23.2	43.4			
F = 24.2	23.5	24.2	9	46.29	5.144
$v_1 = .05534$				Mean e_1 (weighted) =*5.145*	

diminishes as the velocity of the drop increases. This means of course that Stokes's law does not hold for these drops.

In order to find in just what way this law breaks down we made an extended series of observations upon the drops the velocities

[1] The reading carried to hundredths of a second were taken with a chronograph, the others with a stop watch: The mean *G* from the chronograph readings is 24.567, that of the stop watch readings 24.583.

of which varied in the extreme cases 360 fold. These velocities lay between the limits .0013 cm. per sec. and .47 cm. per sec. Complete records of a few of these observations are given in tables IX., X., XI. and XII.

On account of the obvious importance of obtaining accurate readings on the larger drops, for which Stokes's law should most nearly hold, the times of fall of such drops under gravity were taken with a chronograph with as great care as possible. Also, wherever it was possible the same drop was timed by both Mr. Fletcher and myself in order to eliminate the personal equation. The degree of precision which we attained can be judged from the readings recorded in the columns headed G in the tables IX., X., XI. and XII. The letter F before a reading means that it was taken by Fletcher, the letter M that it was taken by Millikan. It will be seen that we very seldom made a reading of the time interval involved in the passage of our star between the cross hairs which differed from the mean time interval by more than one twenty-fifth of a second. Furthermore, F.'s and M.'s mean times on a given drop in no case differ by as much as one fiftieth of a second.

All of the times recorded under F in these tables were taken with a stop watch for the reason that in view of the way in which v_1 and v_2 enter into formula 4, and also in view of the fact that F was in all these observations very much larger than G, no increase

TABLE IX.

Negative Drop No. 20.
Distance between cross hairs $= 1.314$ *cm.*
Temperature　　　　　　 $= 23.4°$ *C.*

	G Sec.	F Sec.	n	$c^n \times 10^{10}$	$e_1 \times 10^{11}$
$V = 8431$ $F = 114.9$	M 14.87 " 14.88 " 14.87	114.7 114.8 115.3	11	56.14	5.104
$G = 14.857$ $V = 8428$ $F = 64.35$	" 14.90 " 14.85 " 14.82 " 14.84	64.2 64.8 64.2 64.2	12	61.20	5.100
$V = 8423$ $F = 117.0$	" 14.84 " 14.84	117.0 117.0	11	56.12	5.102
$v_1 = .08843$					Mean $e_1 = 5.102$

TABLE X.

Negative Drop No. 27.
Distance between cross hairs =1.317 cm.
Temperature 　　　　　=25.2° C.

	G sec.	F sec.	n	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
$V=8793$	F　8.03	48.6	28	114.78	5.063
$F=$ 99.35	"　8.03	98.9 ⎱			
$V=8792$	"　8.08	99.8 ⎰	26	131.58	5.061
$F=$ 67.05	"　8.06	67.2 ⎱			
$V=8790$	"　7.96	66.9 ⎰	27	136.34	5.050
	"　7.98	32.7 ⎱			
	M 7.96	32.6 ⎰	30	151.69	
	"　8.04	27.6	31		
$F=$ 32.66	——	32.6 ⎱			
$V=8788$	"　7.92	32.7 ⎰			
$G=$ 8.013	——	32.7	30	151.69	5.056
	"　8.02	32.7 ⎰			
$F=$ 24.67	——	24.7 ⎱			
$V=8786$	——	24.6	32	161.41	5.044
	"　8.06	24.7 ⎰			
	Forced change with radium.				
$V=8785$	"　8.03	50.5	28	141.20	5.043
$F=$ 68.3	——	68.2 ⎱			
$V=8784$	"　8.01	68.4 ⎰	27	136.17	5.043
$F=$ 107.15	——	107.2 ⎱			
$V=8782$	"　8.01	107.4 ⎰	26	131.05	5.040
$v_1 = .16436$					Mean $e_1 = 5.050$

F's mean $G=8.023$.　M's mean $G=8.007$

Differences.

	e_n	n	e_1	Prob. Error. Per Cent.
141.78	— 131.58 = 10.20	÷ 2 =	5.10	1
136.34	— 131.58 = 4.76	÷ 1 =	4.76	2
151.69	— 136.34 = 15.35	÷ 3 =	5.12	2
161.41	— 141.20 = 20.20	÷ 4 =	5.05	1
141.20	— 136.17 = 5.03	÷ 1 =	5.03	2

Weighted mean difference = 5.03

in the accuracy of e_1 could be obtained by the use of a chronograph in the observations on v_2.

The volts were read just before and just after the observations on a given drop by dividing the bank of storage cells into 11 parts and reading the *PD* of each part by means of a 900 volt Kelvin and White electrostatic voltmeter which we calibrated with an

TABLE XI.

Negative Drop No. 29.

Distance between cross hairs $= 1.007$ *cm.*

Temperature $\qquad = 21.8°$ *C.*

	G sec.	F sec.	n	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
$V = 8845$	—	16.8	46	232.07	
$F = 15.07$	—	15.0			
$V = 8845$	—	14.8	47	238.43	
	—	15.4			
$F = 18.60$	—	18.5			
$V = 8844$	—	18.7	45	227.21	
	—	18.6			
	—	20.6	44	222.67	
	F 4.66	27.5			
	" 4.69	27.5			
	" 4.57	27.8			
	" 4.61	27.9			
	—	27.9			
$F = 27.73$	" 4.66	27.7	42	212.70	5.064
$V = 8843$	" 4.58	27.6			
	" 4.60	27.7			
	" 4.65	27.6			
	—	27.7			
	M 4.60				
	" 4.62	28.0			
	" 4.61	27.9			
	" 4.60	33.6			
	" 4.68	33.8			
$F = 33.75$	" 4.61	33.8	41	207.33	5.057
$V = 8841$	—	33.7			
	" 4.64	33.7			
	" 4.62	33.9			
$F = 42.55$	" 4.61	42.5	40	202.28	5.057
$V = 8840$	" 4.61	42.6			
	" 4.64	33.8			
$F = 34.05$	—	34.2	41	207.30	5.055
$V = 8839$		34.2			
	" 4.66	34.0			
	" 4.67	34.8			
	—	34.4	41		
$G = \quad 4.630$	" 4.68	34.8			
	" 4.61	28.8	42		
$F = 34.67$	" 4.66	34.5			
$V = 8837$	—	34.8	41	206.86	5.045
	" 4.62	34.7			
		Forced change with radium.			
$F = 59.50$	F 4.58	59.4	39	196.75	5.045
$V = 8836$	" 4.63	59.6			

Table XI.—*Continued.*

	G sec.	F sec.	n	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
	" 4.64	60.0			
$F=$ 44.1	—	44.1 ⎫			
$V=8835$	" 4.64	44.0 ⎬	40	201.69	5.041
	" 4.63	44.2 ⎭			
		Forced change with radium.			
$F=$ 219.3	F 4.66	216.7 ⎫	37	186.39	5.038
$V=8834$	—	222.0 ⎭			
		Forced change with radium.			
	" 4.64	35.0 ⎫			
$F=$ 35.2	" 4.60	35.2 ⎬			
$V=8833$	" 4.65	35.4 ⎬	41	206.59	5.039
	" 4.65	35.2 ⎭			
	" 4.67	44.8 ⎫			
$F=$ 45.66	—	45.2 ⎬			
$V=8831$	" 4.60	45.4 ⎬	40	201.30	5.033
	—	45.4 ⎬			
	—	45.5 ⎭			
	—	35.6 ⎭	41		
		Forced change with radium.			
	—	19.1 ⎫			
	—	19.6			
	—	19.2			
	—	19.6			
	—	19.5			
$F=$ 19.42	—	19.4			
$V=8829$	—	19.3 ⎬	45	226.21	
	—	19.2			
	—	19.7			
	—	19.6			
	—	19.3			
	—	19.2			
	—	19.7			
	—	19.5 ⎭			
		Forced change with radium.			
	—	64.0 ⎫			
$F=$ 63.45	—	63.4 ⎬	39	196.12	
$V=8827$	—	63.0 ⎬			
	—	63.4 ⎭			
$F=$ 100.2	—	100.0 ⎫	38	191.11	
$V=8826$	—	100.3 ⎭			
$v_1=.2175$				Mean $e_1=5.046$	

F's mean $G=4.629$.   M's mean $G=4.632$.

Differences.

	c_n	n	c_1	Prob. Error. Per Cent.
$196.12 - 191.11 =$	5.01	$\div 1 =$	5.01	1
$226.21 - 196.12 =$	30.09	$\div 6 =$	5.08	1
$226.21 - 201.30 =$	24.11	$\div 5 =$	4.98	2
$206.59 - 186.39 =$	20.20	$\div 4 =$	5.04	1
$201.69 - 186.39 =$	15.30	$\div 3 =$	5.10	1

Mean difference (weighted) $= 5.035$

TABLE XII.

Negative Drop No. 32.

Distance between cross hairs $= 1.003$ *cm.*

Temperature $= 23.2°$ *C.*

	G sec.	F sec.	n	$c_n \times 10^{10}$	$c_1 \times 10^{10}$
$F = \quad 8.5$ $V = 8577$	— — —	8.7 ⎫ 8.3 ⎬ 8.5 ⎭	123	622.40	
		Changed without radium.			
$F = \quad 28.70$ $V = 8573$	M 2.44 — —— " 2.46 " 2.54 " 2.46 " 2.45 " 2.43	28.4 ⎫ 28.7 28.7 28.4 ⎬ 29.0 29.0 28.8 28.6 ⎭	104	524.25	5.040
		Change forced with radium.			
$G = \quad 2.462$ $F = \quad 15.72$ $V = 8568$	" 2.44 " 2.48 — — —	15.7 ⎫ 15.7 15.7 ⎬ 15.7 15.8 ⎭	111	558.78	5.034
		Change forced with radium.			
$F = \quad 59.1$ $V = 8565$	— " 2.50	59.1 ⎫ 59.1 ⎬	100	503.42	5.034
$F = \quad 60.0$ $V = 8563$	$F = 2.45$	59.8 ⎫ 60.2 ⎬	100	503.23	5.032
		Change forced with radium.			
$F = \quad 81.5$ $V = 8561$	— —	81.0 ⎫ 82.1 ⎬	99	498.12	5.031
		Change forced with radium.			
$F = \quad 20.0$ $V = 8555$	" 2.44 " 2.50 " 2.42	19.9 ⎫ 20.1 ⎬ 20.0 ⎭	108	543.41	5.032
$v = .4074$		Mean e_1 (weighted) $= 5.033$			

F's mean $G = 2.452$.   M's mean $G = 2.467$.

Differences.

	e_n	n	e_1	Prob. Error. Per Cent.
543.41 − 498.12 =	45.29 ÷	9 =	5.032	.5
503.23 − 498.12 =	5.11 ÷	1 =	5.11	3.0
558.78 − 503.42 =	55.36 ÷	11 =	5.033	.5
558.78 − 524.25 =	34.53 ÷	7 =	4.94	3.0

Mean difference (weighted) = 5.031

accuracy of .1 per cent. by comparing it with a Weston voltmeter which had been standardized at the Bureau of Standards. Furthermore, care was taken to use the cells under such conditions as would render the diminution in potential small and as uniform as possible.

It will be seen from the tables that even in the case of the largest drops used, which were charged with as many as 130 elementary units, the values of n are in every case unmistakably determined by the differences summarized at the bottoms of the tables. In fact, in general, even with the largest drops the relative value[1] of e_1 can be determined with an accuracy of .5 per cent. from the differences alone. The accuracy is, of course, increased by dividing the values of e_n by n as soon as n has been found with certainty from the differences.

The readings shown in these tables are merely samples of the sort of observations which we took on between 100 and 200 drops between December, 1909, and May, 1910. The sort of consistency which we attained after we had learned how to control the evaporation of the drops and after we had eliminated dust from the air may be seen from Table XIII. which contains the final results of our observations upon all of the drops except three which were studied throughout a period of 47 consecutive days. The three drops which have been excluded all yielded values of e_1 from two to four per cent. too low to fall upon a smooth $e_1 v_1$ curve like that shown in Fig. 2 which is the graph of the results contained in Table XIII. It is probable that these three drops corresponded not to single drops but to two drops stuck together. Since we have never in all our study observed a drop which gave a value of e_1 appreciably above the curve of Fig. 2, the hypothesis of binary

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[1] Since the same value of G is used in computing all of the e_ns the relative values of e_n are practically independent of the error in G.

drops to account for an occasional low value of e_1 is at least natural. Before we eliminated dust we found many drops showing these low values of e_1, but after we had eliminated it we found not more than one drop in ten which was irregular. The drop shown in Table I. is perhaps the best illustration of the case under consideration which we have observed. It yields a value of e_1 which is four per cent. too low to fall on the curve of Fig. 2. This is as large a departure from this curve as we have thus far obtained.
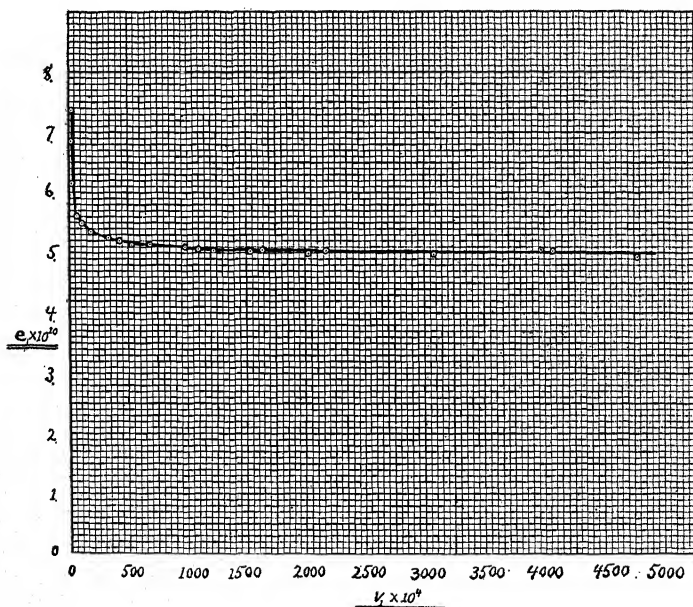
Fig. 2.

§ 10. THE CORRECTION OF STOKES'S LAW.

The simple form of Stokes's law which has been used in obtaining the values of e_1 involves the assumption that there is no slip at the bounding surface between the medium and the drop, or that the coefficient of external friction between oil and air is infinite. From the standpoint of the kinetic theory this surface slip, though in general very small, is, strictly speaking, never zero, and to take it into account a term must be introduced into the equation of motion which is proportional to the ratio between the mean free path of the

TABLE XIII.

No.	Velocity cm./sec.	Radius cm.	$e \times 10^{10}$	Per Cent. Prob. Error.
1	.001315	.0000313	7.384	6.
2	.001673	358	6.864	4.
3	.001927	386	6.142	2.5
4	.006813	755	5.605	1.5
5	.01085	967	5.490	.5
6	.01107	979	5.496	.7
7	.01164	.0001004	5.483	.4
8	.01176	1006	5.482	.4
9	.01193	1016	5.458	.8
10	.01339	1084	5.448	.5
11	.01415	1109	5.448	.4
12	.01868	1281	5.349	.5
13	.02613	1521	5.293	.5
14	.03337	1730	5.257	.5
15	.04265	1954	5.208	.5
16	.05360	2205	5.148	.4
17	.05534	2234	5.145	.5
18	.06800	2481	5.143	.7
19	.07270	2562	5.139	.5
20	.08843	2815	5.102	.3
21	.09822	2985	5.107	.4
22	.1102	3166	5.065	.4
23	.1219	3344	5.042	.5
24	.1224	3329	5.096	.5
25	.1267	3393	5.061	.5
26	.15145	3712	5.027	.5
27	.1644	3876	5.050	.3
28	.2027	4297	4.989	.7
29	.2175	4447	5.046	.4
30	.3089	5315	4.980	1.
31	.3969	6047	5.060	1.
32	.4074	6104	5.033	1.
33	.4735	6581	4.911	1.5

gas molecule and the radius of the drop.[1] Since it is conceivable however that there is some other cause for slip than that assigned by the kinetic theory, it will be well to make this discussion as independent as possible of all theoretical considerations.

From whatever point of view, then, the phenomenon of external

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[1] See O. E. Meyer, Kinetische Theorie der Gase, p. 211, for the correction of Poiseuille's law for slip, and Cunningham, Proc. Roy. Soc., 83, p. 357, 1910, for the corresponding correction of Stokes's law.

slip be regarded it is clear that the very existence of any surface effect of this sort between the medium and the drop must tend to produce an actual velocity higher than that computed from the simple form of Stokes's law, *i. e.*, it must tend to produce departures from Stokes's law of the kind actually shown in the experiments herewith recorded. Furthermore, it will be evident from the analysis underlying Stokes's law (see § 11) that any surface effect whatever between oil and air which might modify the velocity given by Stokes's law must be more and more effective in so modifying it the more the radius of the drop is diminished, and that when the radius is taken sufficiently large the term which represents this surface effect must become negligible. We could then write a corrected form of Stokes's law which would take into account any kind of surface phenomenon which might alter the speed, in the general form

$$X = 6\pi\mu a v \left\{ 1 + f\left(\frac{l}{a}\right) \right\}^{-1} \tag{5}$$

in which l is a constant of the medium and a the radius of the drop. If we were in complete ignorance of the form of the function f we could express it in terms of the undetermined constants, A B, C, etc., thus

$$f\left(\frac{l}{a}\right) = 1 + A\frac{l}{a} + B\frac{l^2}{a^2} + C\frac{l^3}{a^3} \text{etc.} \tag{6}$$

and so long as the departures from the simple form of Stokes's law were small we could neglect the second order terms in l/a and have therefore

$$X = 6\pi\mu a v \left\{ 1 + A\frac{l}{a} \right\}^{-1} \tag{7}$$

or

$$v_1 = \frac{2}{9}\frac{ga^2(\sigma - \rho)}{\mu}\left\{ 1 + A\frac{l}{a} \right\}. \tag{8}$$

Using this form of equation to combine with (1) and denoting now by e the absolute value of the elementary charge and by e_1 as heretofore the value of the charge obtained from the use of (4) there results at once

$$e\left(1 + A\frac{l}{a} \right)^{\frac{2}{3}} = e_1 \quad \text{or} \quad e^{\frac{2}{3}}\left(1 + A\frac{l}{a} \right) = e_1^{\frac{2}{3}}. \tag{9}$$

If Al were known a could be determined directly from (8) and then e could be determined from (9). In fact Al is not known but the departures from Stokes's law shown in the experimental curve of Fig. I are not large except for the very small values of e_1. Leaving these for the present out of consideration, and remembering that a appears in the second power in (3), it will be evident that we can obtain very nearly correct values of a from the assumption of (3). We can then find the approximate value of Al by plotting
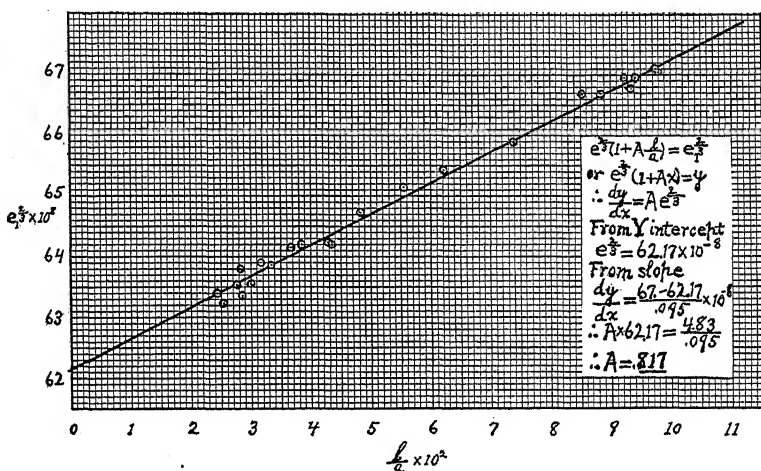
Fig. 3.

the observed values of $e_1^{\frac{2}{3}}$ as ordinates and the observed values of $1/a$ as abscissæ, and obtaining the slope of the resulting straight line (see Fig. 3), providing a linear relation is found between these quantities. If no such linear relation is found then an equation of the form (7) is not sufficient for the representation of the phenomena. As a matter of fact, a very satisfactory linear relationship was found between $e_1^{\frac{2}{3}}$ and $1/a$ as is shown by Fig. 3. In this way we at first determined the approximately correct value of Al and then went back and recomputed a from the corrected formula (8) which is a simple quadratic containing no unknown except a. We then corrected the value of Al by plotting a new curve between $e^{\frac{2}{3}}$ and the corrected values of $1/a$. *When this was done it was found that the constant Al so determined agreed closely with the*

*value of Al given by the analysis of Cunningham based upon kinetic
theory considerations, provided the value of f in his formula[1] was made
equal to zero.* Thus, if the constant of the medium l was taken
as the mean free path of the molecules of air, then the value which
we found it necessary to assign to A was .817 (see Fig. 3). Further-
more, A could easily be determined from our curve with an accuracy
of two or three per cent. as will be seen from the figure.

The value of A which results from placing $f = 0$ in Cunningham's
equation is .815. The agreement however is not as close as at first
appears. The empirical equation (7) was set up without any reference
to Cunningham's theoretical work, which in fact appeared while we
were in the process of finding empirically the correction to Stokes's
law, and the values of l used in obtaining the values of l/a plotted
in Fig. 3 and tabulated in Table XIV. were computed from the
Boltzmann formula $\mu = .3502 \rho \bar{c} l$,[2] in which $\bar{c}$ is the average molecular
velocity. In order to obtain from Cunningham's theoretical work
the value $A = .815$ it is necessary not only to put $f = 0$ but to com-
pute l from the formula $\mu = \frac{1}{3}\rho u l$ in which u is not the average
molecular velocity but the square root of the mean square velocity.
If l is computed, as above, from Boltzmann's formula Cunning-
ham's theory gives when $f = 0$ $A = .788$ instead of $A = .815$, so
that the above empirical value of A is actually 3.6 per cent. higher
than that given by Cunningham's theory[3] when $f = 0$.

In the above computation, as in all of this, the value of μ at 15°
C. is taken as .00017856, and for temperatures within a range of say
12° on either side of 15° C. μ_t is computed from the formula

$$\mu_t = \mu_{15}(1 + .00276)(t - 15).$$

This equation gives precisely the rate of change of μ with tempera-
ture, within this region, which is obtained independently by all
three of the observers Breiterbach,[4] Schultze[5] and Fisher[6] in their

[1] Cf. p. 361, *l. c.*

[2] Boltzmann, Gastheorie, 1, p. 81.

[3] If l is computed from $\mu = \frac{1}{3}\rho \bar{c} l$ in which $\bar{c}$ is the *average* molecular velocity
Cunningham's corrective term, when the substitutions are correctly carried out, is
$(1 + 1.5 \cdot l/a)$ instead of $(1 + 1.63 \cdot l/a)$.

[4] P. Breiterbach, Ann. der Phys., 5, p. 168, 1901.

[5] H. Schultze, Ann. der Phys., 5, p. 157, 1901.

[6] W. J. Fischer, Phys. Rev., 28, p. 104, 1909.

exceedingly careful work upon this subject. The reliability of the absolute value of μ_{15} will be discussed in section 12.

It is most interesting that the agreement between Cunningham's rational formula and the above empirical results is as good as it is, but it is to be particularly emphasized that the correctness of the final value of the elementary electrical charge is completely independent of the correctness of any theory whatever as to the cause of the failure of Stokes's law for small drops. *The corrective constant A is a purely empirical one.* It is entirely possible that a series of experiments of this kind upon substances other than oil might lead to other values of A but the value of e should in no way be effected thereby. It is of immense interest to know whether varying the mean free path by varying the pressure and the nature of the gas will affect the value of e_1 in the way in which it ought according to Cunningham's theory and enough data have already been obtained to indicate that that theory holds, approximately at least, for wide ranges of pressure. This work, however, will be reported in a later article.

§ 11. COEFFICIENTS OF EXTERNAL FRICTION AND OF SLIP.

The modified form of Stokes's law which takes into account surface slip[1] is

$$X = 6\pi\mu a v \left\{ \frac{1 + 2\gamma}{1 + 3\gamma} \right\}$$

or

$$v_1 = \frac{2}{9} \frac{ga^2}{\mu} (\sigma - \rho) \left\{ \frac{1 + 3\gamma}{1 + 2\gamma} \right\} \qquad (10)$$

in which γ is defined by the equation

$$\gamma = \frac{\mu}{\beta a},$$

β being the coefficient of external friction. If the last factor is expanded in powers of γ the resulting series

$$1 + \gamma + 2\gamma^2 + 4\gamma^3 \cdots (-1)^n 2^{n-2}\gamma^{n-1} + \cdots$$

is convergent for $\gamma < 1$. If we neglect powers of γ higher than the

[1] See Basset's Hydrodynamics, Vol. II., p. 271.

first (10) becomes

$$v_1 = \frac{2}{9} \frac{ga^2(\sigma - \rho)}{\mu} \{1 + \gamma\} \qquad (11)$$

which is identical with the empirical equation (8) when

$$\gamma = Al/a.$$

But since

$$\gamma = \frac{\mu}{\beta a}$$

and the coefficient of slip ζ is defined by

$$\zeta = \frac{\mu}{\beta}^1$$

we obtain, by inserting the value of Al given by the above experiments viz., .0000077,

$$\zeta = .0000077 \quad \text{and} \quad \beta = 23.7.$$

This direct determination of these constants for oil and air at atmospheric pressure agrees well with the result ($\zeta = .0000076$) computed from Warburg's[2] observations on the flow of gases at low pressures though glass capillaries.

§ 12. The Absolute Value of e.

Taking the value of A as .817 the value of e was determined from (9) and the values of e_1, a, and l obtained as explained above. The next to the last column of Table XIV. gives the results of this computation of e for all of the observations recorded in Table XIII. except the first four and the last four. These are omitted not because their introduction would change the final value of e, which as a matter of fact is not appreciably affected thereby, but solely because of the experimental uncertainties involved in work upon either exceedingly slow or exceedingly fast drops. When the velocities are very small residual convection currents and Brownian movements introduce errors, and when they are very large the time determination becomes unreliable, so that it is scarcely legiti-

[1] Meyer, Kin. The. d. Gases, p. 207.
[2] Warburg, Pogg. Ann., 1876, Vol. 159, p. 399.

mate to include such observations in the final mean.   However, for
the sake of showing how completely formula (9) fits the ex-
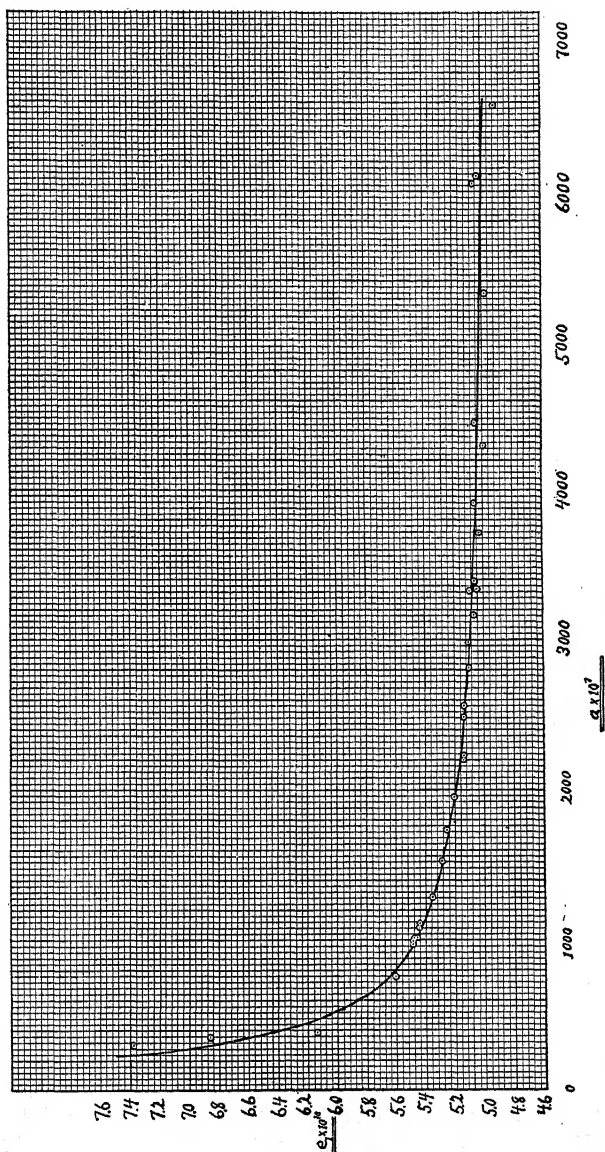
Fig. 4.

perimental results throughout the whole range of the observa-
tions of Table XIII., Fig. 4 has been introduced.   The smooth

TABLE XIV.

No.	Tem. °C.	Dew Point °C.	$l \times 10^8$ cm.	Velocity cm./sec.	$a' = $radius} cm.	l/a	$e_1 \times 10^{10}$	Max. Obs'l Error. $\%$	$e \times 10^{10}$	Dif. from Mean. $\%$
1	24.0	5.3	945	.001315	.0000313	.3020	7.384	6.		
2	26.0	10.8	954	.001673	358	.2172	6.864	4.		
3	23.8	9.3	944	.001927	386	.1993	6.142	2.5		
4	19.9	1.8	929	.006813	755	.1230	5.605	1.5		
5	24.6	3.7	948	.01085	967	.0980	5.490	.5	4.892	.20
6	26.4	6.0	955	.01107	979	.0975	5.496	.7	4.889	.26
7	24.0	0.0	945	.01164	.0001004	.0941	5.483	.4	4.903	.03
8	20.0	1.8	929	.01176	1006	.0923	5.482	.4	4.916	.28
9	24.8	0.0	949	.01193	1016	.0934	5.458	.8	4.891	.22
10	26.3	6.0	955	.01339	1084	.0883	5.448	.5	4.908	.10
11	23.6	3.7	943	.01415	1109	.0850	5.448	.4	4.921	.42
12	24.3	11.0	947	.01868	1281	.0739	5.349	.5	4.900	.03
13	24.0	0.0	945	.02613	1521	.0621	5.293	.5	4.910	.17
14	27.0	6.0	959	.03337	1730	.0554	5.257	.5	4.918	.34
15	23.2	−1.2	942	.04265	1954	.0483	5.208	.5	4.913	.21
16	27.6	12.2	959	.05360	2205	.0435	5.143	.4	4.884	.36
17	26.8	6.0	958	.05534	2234	.0429	5.145	.5	4.885	.34
18	25.2	4.0	951	.06800	2481	.0384	5.143	.7	4.912	.21
19	23.8	5.0	944	.07270	2562	.0369	5.139	.5	4.913	.01
20	23.2	13.5	942	.08843	2815	.0325	5.102	.3	4.901	.01
21	24.6	1.7	948	.09822	2985	.0318	5.107	.4	4.915	.27
22	25.0	9.2	950	.1102	3166	.0300	5.065	.4	4.884	.36
23	27.7	15.0	959	.1219	3344	.0287	5.042	.5	4.882	.40
24	22.6	1.6	939	.1224	3329	.0282	5.096	.5	4.923	.44
25	24.0	3.7	944	.1267	3393	.0278	5.061	.5	4.894	.15
26	23.8	5.0	944	.15145	3712	.0254	5.027	.5	4.880	.44
27	25.2	0.3	948	.1644	3876	.0245	5.050	.3	4.903	.03
28	22.3	−0.7	938	.2027	4297	.0218	4.989	.7	4.858	.85
29	21.8	−0.1	936	.2175	4447	.0211	5.046	.4	4.918	.36
30	22.3	4.2	938	.3089	5315	.0177	4.980	1.		
31	24.4	1.0	947	.3969	6047	.0157	5.060	1.		
32	22.8	1.0	940	.4074	6104	.0154	5.033	1.		
33	25.2	2.7	951	.4735	6581	.0144	4.911	1.5		

Mean $e = 4.901$

Six months after the original work on this table was done the laboratory obtained a very reliable Weston laboratory standard voltmeter which made it possible to obtain a more perfect calibration curve of the Kelvin and White electrostatic instrument than had been made at first. With the aid of this new calibration curve every value of e_1 in the above table was recomputed with the result that the final value of e was reduced .06 per cent. Furthermore in the computation of the above table the m of equation (1) was through oversight treated as the real mass instead of as the apparent mass. This necessitates a further reduction of e amounting to .14 per cent. so that the most reliable value obtainable from the work thus far done is

$$e = 4.891 \times 10^{-10}.$$

curve in this figure is computed from (7) under the assumption of $e = 4.891 \times 10^{-10}$ and the experimentally determined values of e_1 are plotted about this curve, every observation contained in Table XIII. being shown in the figure.

The probable error in the final mean value 4.891×10^{-10}, computed by least squares from the numbers in the last column, is four hundredths of one per cent. If there is an error of as much as 3 per cent. in the determination of A the final value of e would be affected thereby by only about .2 per cent. Since, however, the coefficient of viscosity of air is involved in the formula, the accuracy with which e is known is limited by that which has been attained in the measurement of this constant. There is no other factor involved in this work which has not been measured with an accuracy at least as great as .2 per cent.

The value of μ_{15} which has been used in the computation of all of the preceding tables, viz., .00017856, is in my judgment the most probable value which can be obtained from a study of all of the large mass of data which has been accumulated within the past forty years upon this constant. It represents not only the result of what seems to me to be the most reliable single determination of μ which has thus far been made, viz., that of Stokes and Tomlinson[1] who deduced it from the damping of oscillating cylinders and spheres, but it is exactly the mean of the three most recent and very concordant values obtained by the outflow method (Table XV.) and it is furthermore the mean of all of the most reliable determinations which have ever been made. These are summarized in Table XV.

In this summary I have discarded the early work of Maxwell[2] and O. E. Meyer[3] by the damping method, because it is admittedly inaccurate, the work of Puluj[4] by the transpiration method because he used but one capillary tube and obtained a value completely out of line with the results obtained by all others under similar conditions, and two recent results obtained by Zemplén[5] because he himself discards the first while the second has just been definitely

[1] Stokes, Math. and Phys. Papers, Vol. 5, p. 181.

[2] Phil. Trans., 156, p. 240, 1866.

[3] Pogg. Ann., 143, p. 14, 1871.

[4] Wien. Sitz. Ber., 69, p. 287; 70, p. 243, 1874.

[5] Ann. Phys., 19, p. 442, 1906; 29, p. 869, 1909.

Table XV.

Date.	Observer.	Method.	Reference.	$\frac{\mu_{15}}{\times 10^7}$
1874	Schumann.	Damping method.	Wed. Ann., 23, p. 374, 1884.	1752
1875	Obermeyer.	Transpiration method.	Wiener. Sitz. Ber., 71, p. 281; 73, p. 433.	1748
1885	Schneebeli.[1]	Transpiration method.	Arch. des sc. phys. et nat. Geneve, 14, p. 197.	1780
1886	Tomlinson.	Damping method.	Phil. Trans., 177, p. 767.	1785
1899	Breiterbach.	Transpiration method.	Ann. der. Phys., 5, p. 168.	1807
1901	Schultze.	Transpiration method.	Ann. der. Phys., 5, p. 157.	1811
1906	Tanzler.	Transpiration method.	Verh. d. D. Phys. Ges., 8, p. 222.	1812
1908	Grindly and Gibson.	Flow through long pipe.	Proc. Roy. Soc., 80, p. 114.	1788
1909	Fisher.	Transpiration method.	Phys. Rev., 28, p. 104.	1782
1910	Rankine.	Transpiration method.	Proc. Roy. Soc., ser. A, pp. 516–525.	1788

Mean = 1785

shown by work now in progress in the Ryerson Laboratory to be 6 per cent. in error. O. E. Meyer's[2] early transpiration experiments are also excluded for the reason that, though his mean is in close agreement with the above result the individual observations show great divergence.

Despite the agreement shown in the results obtained by the "outflow"[3] and "damping"[4] methods I am inclined to rate both of them as inferior in reliability and precision to the "constant deflection" method (with concentric cylinders) used by Gurney at the Ryerson laboratory and also by other observers, in determining the coefficient of viscosity of liquids. Mr. Lachlan Gilchrist

[1] Obermeyer and Schneebeli are in perfect agreement when they treat their observations in the same way. One considers the air saturated, the other dry. It is probably in an intermediate condition.

[2] Pogg. Ann., 148, p. 37, and 203, 1873.

[3] The consistently high values found by Briterbach, Schultze and Tanzler may possibly be due to the fact that they all use high values of $(p_1 - p_2)$. Barus' results (Ann. der Phys., 36, p. 358, 1889) seem to indicate an apparent slight increase in μ with increasing $(p_1 - p_2)$.

[4] Schumann's results by the damping method are not at all comparable in accuracy with Tomlinson's.

is now making in this laboratory a very careful determination of μ for air by this method and it is already certain that his result will not differ from Tomlinson's by more than a fraction of a per cent. It seems therefore impossible that the value of μ which has been used in these computations can be in error by more than .5 per cent. and it is probable that its error is even less than this. Tomlinson estimates it at not more than .2 per cent. and the agreement to within less than .1 per cent. which he obtained both in the use of two wholly distinct damping methods and in the use of different suspensions by the same method seems to justify his estimate. It may be confidently expected that within a very few years at most the uncertainty in the absolute value of the coefficient of viscosity of air will be not more than one or two tenths of one per cent.

Tomlinson's work, like the above, was done with ordinary rather than with dry air, his mean humidity being apparently much the same as that shown in column 3, Table XIV. He estimated however that the presence of the aqueous vapor in the air used in his experiment could scarcely affect his results by .1 per cent. Mr. Gilchrist has experimentally demonstrated the correctness of this estimate in the case of the present experiments and has further shown that the drag which the air exerts upon an oiled cylinder is exactly the same as the drag which it exerts upon a brass cylinder, thereby removing all question as to the applicability of the above constant to the conditions of these experiments.

We have devised two modifications of this method of determining e which do not involve the value of μ. It is scarcely likely however that the necessary experimental error in these methods can be reduced below the error in μ. It is probable therefore that any increased accuracy in our knowledge of e is to be looked for in increased accuracy in the determination of μ.

§ 13. Experiments upon Substances other than Oil.

All of the preceding experiments except those recorded in Table I. were made with the use of a specially cleaned gas engine oil of density .9041 at 25° C. Those in Table I. were made with the use of a similar, though more volatile, mineral oil (machine oil) of density .8960. The reason that we worked so continuously upon

a single substance was that it was found that in order to maintain a drop of constant size it was necessary, even with these very non-volatile substances to have the drop in equilibrium with its saturated vapor. This is shown by the following observations. The inner surfaces of the condenser plates had been covered with a very thin coat of machine oil in order that they might catch dust particles. Drops blown from a considerable number of non-volatile substances were introduced between the plates and were found in the main to evaporate too rapidly to make accurate observing possible. This was true even of so non-volatile substances as glycerine and castor oil as the following observations show:

Glycerine,	Den. 1.25		Castor Oil,	Den. .975
G	F		G	F
28.3	11.5		73.8	18.0
32.5	9.8	.	75.8	12.9
38.7			77.3	18.0
45.6	8.4		78.7	102.2
59.2			79.6	17.8
			84.8	30.2
			87.7	12.7
			90.7	18.1

In order to get rid of this continuous increase in G, the drops were next blown from the least volatile liquid at hand, viz., gas engine oil, and the behavior of a given drop showed immediately that it was growing in size instead of evaporating. This can be seen from the following readings:

Gas Engine Oil.	
G	F
17.6	6.1
17.4	76.2
17.2	82.0
16.9	87.2
16.8	92.4
17.1	97.8
16.7	104.6
16.4	112.0

This behavior was shown consistently by all the drops experimented upon (six or eight in number) throughout a period of two days. Imagining that the vapor from the more volatile machine oil upon the plates was condensing into the less volatile but similar

oil of the drop I took down the apparatus, cleaned the plates carefully, and oiled them again, this time with the gas engine oil. Every gas engine oil drop tried thereafter showed the sort of constancy which is seen in tables III. to XII. Series of observations similar to that made upon gas engine oil and tabulated in tables XIII. and XIV. will ultimately be made upon other substances. Thus far the aim has been to take enough observations upon other substances to make sure that the results obtained from these substances are substantially in agreement with those obtained from gas engine oil and to concentrate attention upon an accurate series of observations upon one substance. As a matter of fact we have a fairly complete series upon machine oil and a number of observations upon watch oil, castor oil, and glycerine, all of which are in agreement within the limits of observational error, in some cases as much as 2 or 3 per cent., with the observations upon gas engine oil.

The only observations which have been taken upon mercury are tabulated below.

TABLE XVI.

Negative Mercury Drop.

Distance between cross hairs = 1.033 cm.
Temperature   = 24.4° C.
Density of mercury at 25° C. = 13.52.
Viscosity of air at 25.2° C. = .0001837.

	G sec.	F sec.	n	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
$G=$ 6.747 $V=8383$ $F=$ 64.1	6.780 6.686 6.776	68.0 ⎫ 63.3 ⎬ 61.0 ⎭	6	32.77	5.448
$v_1 = .1531$					$e_1 = 5.448$

The drop of Table XVI. yields a value of e within 1 per cent. of the value obtained with the gas engine oil if A is assumed to be .817. This will be readily seen from the consideration that v_1 for an oil drop of the same size as the mercury drop would have been

$$.1531 \times \frac{.9041}{13.52} = .01025.$$

This is very close to the value of the velocity shown by the oil

TABLE XVII.

Positive Mercury Drop.

Distance between cross hairs = .980 cm.
Temperature   = 21° C.

	G sec.	*F* sec.	*n*	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
F = 4.8	3.0	4.8	27	137.1	5.077
	3.1	4.8			
	3.0	4.8			
	2.8	4.8			
F = 7.83	2.8	8.0	23	116.8	5.078
	3.0	7.5			
	2.8	8.0			
	3.1	6.7	24	122.2	5.092
	3.1	36.6			
	3.2	36.2			
G = 2.988	3.0	36.4			
V = 9070	3.0	36.1			
	—	36.2			
F = 36.34	2.8	36.1	18	91.45	5.081
	3.2	36.1			
	2.9	36.6			
	3.0	36.5			
	—	36.6			
v_1 = .3280				Mean e_1 = 5.082	

drop of Table III., and the values of e_1 obtained from tables III. and XVI. differ by not more than 1 per cent. Since *G* in Table XVI. is uncertain to 1 per cent., as is shown by the differences between individual readings (which were taken with the chronograph), the uncertainty in e_1 is about 1.5 per cent. (*cf.* equation 4) so that within the limits of observational error this drop is in good agreement with the oil.

The drop of Table XVII., while yielding readings which are most consistent among themselves, gives a value of *e* about 4.5 per cent. too low. However this drop was timed with a stop watch and *G* is therefore uncertain by about one tenth second, or 3 per cent., which means that e_1 contains an uncertainty of 4.5 per cent. The experiments upon mercury have not been pushed farther for the reason that its great density makes it unsuitable for precise measurements unless the drops are very small, and then Brownian movements produce enormous irregularities.

The conclusion to be drawn then from all of the work thus far done on substances other than oil is merely that there is nothing in it to cast a doubt upon the correctness of the value of *e* obtained from the much more extended and much more accurate work upon gas engine oil.

§ 14. Comparisons with Other Determinations.

The value of *e* herewith obtained is in perfect agreement with the result reached by Regener[1] in his remarkably careful and consistent work on the counting of the number of scintillations produced by the particles emitted by a known amount of polonium and measuring the total charge carried by these same particles. His final value of this charge is 9.58×10^{-10}, and upon the assumption that this is twice the elementary charge—an assumption which seems to be justified by Rutherford's experiments[2]—he finds for *e* 4.79×10^{-10}, with a probable error of 3 per cent. Since the difference between this value and 4.89×10^{-10} is but 2 per cent. the two results obviously agree within the limits of observational error.

On the other hand, the present value of *e* is 4 per cent. higher than the simple mean value which I previously obtained in work by a similar method upon drops of water and alcohol[3] and when the correction to Stokes's law is applied the difference becomes as high as 8 per cent. Although the observational error in these earlier experiments was enormously greater than that found in the present work it is not probable that more than 2 per cent. or 3 per cent. of the difference can be accounted for by mere timing errors in the preceding determination. The difference is due I think in the main to the instability of the conditions which prevail in any expansion chamber immediately after a sudden expansion. Since the temperature rises rapidly after such an expansion the convection currents produced by this rise must on the whole have an upward tendency and consequently the apparent rate of fall under gravity is lower than it should be. This always

[1] E. Regener, Sitz. Ber. d. k. Preuss. Acad. d. Wiss., XXXVII., p. 948, 1909.

[2] Rutherford, Phil. Mag., 17, p. 281, 1909.

[3] Millikan, Phil. Mag., 19, p. 209, 1909.

tends to make the value of e come out too small (cf. equation 4).
Furthermore the lack of perfect uniformity in the electrical field
between plates as small as those which have been used by all
experimenters who have worked with the cloud method tends to
push down the apparent value of e.

Turning next to Begeman's experiments[1] the observational errors
are here still larger, the last criticism applies with equal force, and
in addition Cunningham shows that the group velocity of a
falling cloud is less than the velocity of an individual drop and
finds that because of this fact alone Sir Joseph Thomson's calcu-
lation of e is 7 per cent. too low.[2]   The same sort of a correction
would apply to Begeman's experiment.   On the other hand, Bege-
man's result would be three or four per cent. too high because of
the fact that individual drops of the sizes which he uses actually
fall faster than they would if Stokes's law held.   The net result
of the consideration of all of these causes would be to raise
Begeman's value so that it would be well within the limits of ob-
servational error of the value 4.89×10^{-10}.

The only other investigations which appear to yield results which
are in any way adverse to the conclusions herein contained are re-
ported in very recent papers by Ehrenhaft[3] and Przibram[4] both of
which have appeared since this work was first presented to the
Physical Society on April 23, 1910.

Ehrenhaft, who in his preceding work, which appeared simul-
taneously with the completion of the work reported in October,
1909,[5] had deduced e from average rates of fall and average
velocities in a horizontal electric field, has in this later work used the
vertical electric field and therefore now makes all the observations
from which e is deduced, as I have done since the spring of 1909,
upon a single charged particle.   His present observations then
differ from these only in these important respects:

I. He observes through an ultra-microscope and therefore deter-

[1]Begeman, PHYS. REV., 31, p. 41, 1910.
[2]Loc. cit., p. 365.
[3]Ehrenhaft, Phys. Zeit., Juli 15, 1910.
[4]Przibram, Wien. Sitz. Ber., CXIX., p. 1, 30 June, 1910.
[5]Millikan, PHYS. REV., 1909.

mines rates of fall and rise through exceedingly minute distances, about .01 cm. in place of the 1.3 cm. which is here employed.[1]

2. He moves a single particle up and back but once, and holds it under observation at most a minute, as I did in the earlier work, instead of from four to five hours as I am now doing.

3. Instead of using oil drops he sucks into the observing chamber the metallic dust arising from the volatilization produced in a metallic arc.

4. He assumes Stokes's law instead of correcting it as is done above.

5. He computes simply the charges upon his dust particles, and in no case the charges upon ions captured from the air, *i. e.*, he makes no study whatever of the phenomena of *change* of charge.

His results are so irregular that he concludes that if there is any elementary charge it is much smaller than the value herein assigned. His irregularities are all easily and simply explained, however, by a consideration of the failure of Stokes's law shown above and the Brownian movements.[2] His particles have diameters which lie between those of our very smallest drops and values only one tenth as large. Now it was found in this work on oil drops that consistency could not be obtained in the readings upon the successive rates of fall under gravity of a given particle when that particle was smaller than the smallest shown in Table XIV., and that for the simple reason that the displacements of such particles due to

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[1] The apparatus of Fig. 1 was designed with especial reference to (1) uniformity of field, (2) freedom from convection currents and (3) the largest attainable accuracy in the measurement of rates of fall even with large drops.   The distance between the plates was therefore made as large as possible and the magnification used as small as possible (only about 4 diameters).   The accuracy and consistency of the results is attributable chiefly to the largeness of the distance of fall and the smallness of the magnification.

[2] Einstein's formula

$$\Delta \overline{X^2} = \frac{RT}{N} \frac{1}{3\pi a \mu} \tau$$

shows that the displacement in time τ is independent of the mass of the particle. Since however the kinetic energy of agitation of all particles is the same the velocity at any instant of a platinum particle is but about one fifth as much as that of an oil or water particle of the same radius.   This accounts presumably for the fact that Ehrenhaft did not notice the agitation of the particles when he was using dense substances but did observe it when he was using light substances.

their Brownian movements[1] became comparable with the displacements produced by gravity. Indeed, we have many series of observations upon such particles in which the successive values of G vary because of the Brownian movements from two to four fold. Table XVIII. shows observations upon one such. It will be seen

<div align="center">

TABLE XVIII.

</div>

Distance between plates   = 1.600 *cm.*
Distance between cross hairs = .0148.
Temperature   = 23° *C.*
Volts   = 326.

G	F	e_1	
10.3	10.6	5.25	
7.7	20.0	4.52	
6.8	13.2	7.50	
5.6	13.4	9.40	
8.3	8.8	7.15	Mean $e_1 = 7.8 \times 10^{-10}$
5.0	11.5	11.3	
7.8	16.5	5.87	
6.8	11.2	8.0	
5.7	7.2	11.6	
4.7	13.0	12.1	
7.3	13.2	6.95	
7.4	15.4	6.50	
8.6	17.0	5.24	
6.5	3.6	14.9	
6.0	2.6	19.8	
8.4	3.0	13.8	
5.6	3.0	18.8	
6.4	3.0	15.1	Mean $e_2 = 15.2 \times 10^{-10}$
6.0	3.6	15.9	
7.2	4.3	12.2	
6.5	3.3	15.8	
7.8	3.0	14.4	
8.2	3.4	12.7	
5.5	2.7	13.9	

<div align="center">

$a = .00004$ cm.

</div>

that while the drop carried one single elementary charge the value of this charge, if computed from single observations, would have appeared to oscillate between 5.2×10^{-10} and 11.6×10^{-10} and similarly that the value of the double charge would have appeared

[1] A more complete analysis will be given in a separate paper by Mr. Fletcher.

to oscillate between 12.7×10^{-10} and 19.8×10^{-10}. The Brownian movement theory can be made to account quantitatively as well as qualitatively for all of the irregularities of Table XVIII., as well as for those in Ehrenhaft's tables.[1]

The irregularities in Ehrenhaft's results furnish then no evidence whatever against any of the conclusions which have been drawn from the above data.

Przibram's observations, carried on between April and June, 1910, were made under conditions which were practically identical with those which I used in the work published in December, 1909, save that his drops had but one third as large diameters and were formed differently. He obtained more consistent results than did Ehrenhaft, because he used larger particles. However tables V. and VI. of his paper, in which he records the only observations made as much as twice upon the same particle show, as he himself points out, divergencies as high as 40 per cent. between the two successive timings and an average divergence of 12.4 per cent. in the computed values of e. The only way in which data of this sort can be treated, if results of any significance whatever are to be drawn from it, is to take averages of a large number of observations upon the same particle, or since these are not available in this case, averages of as many observations as possible upon different particles. The mean value of e which Przibram obtains by plotting the results of 1,000 observations on "Phosphornebel" of mean radius .000054 cm. is 6.0×10^{-10} which is seen from the curve of Fig. 3 to agree perfectly with the above results.

The only result in any of the Vienna work which is not to be predicted at once from the smallness of the particles used, taken in connection with the laws governing Brownian movements, is the fact that some of Ehrenhaft's irregularities fluctuate about smaller values of e_1 than any shown in the above tables. It is to be observed however that Przibram's observations made upon the same substance "Phosphornebel" fluctuate about the correct value. In view then of (1) the disagreement between Ehrenhaft's and Przibram's results on the same substance, (2) the uncertainty as

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[1] A more complete analysis will be given in a separate paper by Mr. Fletcher.

to the density and sphericity of ultramicroscopic particles, and (3) our complete ignorance to date of the law of motion through a resisting medium of such particles especially when charged,[1] it is obvious that there is nothing in either Ehrenhaft's or Przibram's work to raise a suspicion as to the validity of any of the conclusions herein drawn.

There is but one more experiment which needs to be mentioned in this connection, namely, the important work of Zeleny and McKeehan[2] on the direct measurement of the dimensions and rates of fall in air of spheres whose radii lie within the limits .00366 cm. and .00035 cm. This work constitutes direct verification within the rather large limits of experimental error of the constants of Stokes's law, though the individual observations are too scattering to throw any light upon the way in which the rate of fall varies with the radius of the drop—a gap which is filled in by the above observations. For spheres of the average radius which they use the correction term which is here applied to Stokes's law would influence their final mean by only 1 per cent. Since their probable error is very much larger than this there is evidently no discrepancy between their "verification of Stokes's law" and the proof herewith presented of its inadequacy. They originally used a high value of μ, but they now inform me that the value $\mu_{15} = .0001785$ yields better agreement between observed and calculated times of fall than does the value which they first employed.

In conclusion there is presented a summary of the most important of the molecular magnitudes, accurate values of which are made possible by an accurate determination of e. The Faraday constant is taken as $Ne = 9,655$ absolute electromagnetic units.

$e = 4.891 \times 10^{-10}$ E.S.U.   the smallest quantity of electricity capable of separate existence.

$N = 5.922 \times 10^{23}$ the number of molecules in one gram molecule of any substance.

$n = 2.644 \times 10^{19}$ the number of molecules in 1 c.c. of any gas at 0° C. and 76 cm.

$\alpha = 2.106 \times 10^{-16}$ ergs.   the constant of molecular energy. Molecular energy $\epsilon = \alpha T$.

[1] When the particles become sufficiently small the apparent viscosity of the medium should be a function of the charge.

[2] Phys. Rev., XXX., p. 525, May, 1910.

$\epsilon_0 = 5.750 \times 10^{-14}$ ergs.   the kinetic energy of agitation of a single molecule at
$\qquad\qquad\qquad\qquad$ 0° C. and 76 cm. $\epsilon_0 = 273a$.

$m = 1.702 \times 10^{-24}$ gms.   the weight of the hydrogen atom.

Weights and Diameters of Molecules.

Substance.	Molecular Wt. ($H=1$).	Absolute Wt., Grams.	Diameter,[1] cm.	Absolute Density, g.cm.
Hydrogen...........	2	3.40×10^{-24}	2.28×10^{-8}	.55
Helium.............	4	6.81×10^{-24}	2.00×10^{-8}	1.63
Carbon monoxide	27.8	$47.4^r \times 10^{-24}$	2.89×10^{-8}	3.76
Ethylene............	27.8	47.4×10^{-24}	3.40×10^{-8}	2.34
Nitrogen............	27.8	47.4×10^{-24}	3.06×10^{-8}	3.17
Air................	28.9	49.2×10^{-24}	2.99×10^{-8}	3.53
Nitric oxide	29.81	50.8×10^{-24}	2.69×10^{-8}	5.00
Oxygen.............	31.8	54.2×10^{-24}	2.89×10^{-8}	4.30
Argon	39.6	67.5×10^{-24}	2.78×10^{-8}	6.01
Carbon dioxide	43.7	74.4×10^{-24}	3.11×10^{-8}	4.73
Nitrous oxide........	43.7	74.4×10^{-24}	3.48×10^{-8}	3.39
Chlorine	70.4	119.8×10^{-24}	3.01×10^{-8}	3.90
Water vapor.........	17.9	30.5×10^{-24}	$3(?) \times 10^{-8}$	
Ethyl chloride	64.0	108.9×10^{-24}	$4(?) \times 10^{-8}$	

My thanks are due to Professors Crew, Carman and Guthe, for loaning to me tubes of radium when my own supply met with an accident.   I wish also to acknowledge my great indebtedness to Mr. Harvey Fletcher who has most ably assisted me throughout the whole of this investigation.

RYERSON LABORATORY,
$\qquad$ UNIVERSITY OF CHICAGO,
$\qquad\qquad$ November 28, 1910.

[1] These diameters have been obtained from the above value of n and the viscosity equation

$$\mu = \frac{350\rho\bar{c}}{\sqrt{2}\pi nD^2},$$

Sutherland's correction for cohesional force (Phil. Mag., 17, p. 320, 1909) and Jean's correction for persistence of velocities being added.   This procedure is thought to yield more reliable results than applying the above corrections to means of D obtained from viscosity, diffusion, heat conduction, and departures from Boyle's law, since computations based on the last three phenomena involve both theoretical and experimental uncertainties of large magnitude.

MAXIMUM CURRENT IN THE SECONDARY OF A TRANSFORMER.[1]

By John Stone Stone.

THE condition of maximum current in the secondary of an alternating current transformer is one of considerable importance, especially since the advent of wireless telegraphy.

In this paper the condition particularly considered is that in which the amplitude of the secondary current is at its greatest value. In practice this condition is in general attained by the adjustment of the capacities, inductances and mutual inductance of the primary and secondary circuits. Accordingly in this paper the periodicity of the current will be considered as constant and the reactances of the primary and secondary circuits as well as the mutual reactance will be treated as the independent variables.

In the case of a single isolated circuit, having inductance L in series with a condenser of capacity C, the reactance is

$$K = L\omega - 1/C\omega,$$

where $\omega = 2\pi/T$, T being the periodic time of the impressed electromotive force.

When the capacity of such a circuit varies from 0 to ∞, the reactance varies from $-\infty$ to $L\omega$, and when the inductance varies from 0 to ∞, the reactance varies from $-1/C\omega$ to ∞, so that by suitably varying the inductance and capacity of such a circuit, its reactance can be made to vary from $-\infty$ to $+\infty$ without discontinuities.
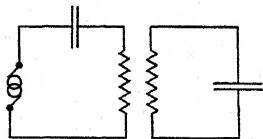
Fig. 1.

The impedance of such a cricuit is

$$E/I = \sqrt{R^2 + K^2},$$

where E is the amplitude of the impressed electromotive force, I is the amplitude of the current and R is the resistance of the circuit.

For variations of the reactance K, this impedance is evidently a minimum when

$$K = 0 \quad \text{or} \quad L\omega = 1/C\omega,$$

and this therefore is the adjustment for maximum current in the circuit in question.

In the case of the transformer shown in Fig. 1, let the impressed electromotive force be $E \sin \omega t$, the resistance, inductance and capacity of the primary be R_1, L_1 and C_1, respectively, the resistance, inductance and capacity of the secondary be R_2, L_2 and C_2 respectively and the mutual inductance between the circuits be M. The expression for the amplitude of the current in the secondary may be written in the form

$$I_2 = \frac{EK_{12}}{\{(R_1{}^2 + K_1{}^2)(R_2{}^2 + K_2{}^2) + K_{12}{}^4 + 2(R_1R_2 - K_1K_2)\}^{\frac{1}{2}}}, \quad (1)$$

where

$$K_1 = L_1\omega - 1/C_1\omega,$$

$$K_2 = L_2\omega - 1/C_2\omega$$

and

$$K_{12} = M\omega.$$

The function corresponding to the impedance of the single isolated circuit previously considered is here

$$\frac{E}{I_2} = \left\{ \frac{(R_1{}^2 + K_1{}^2)(R_2{}^2 + K_2{}^2)}{K_{12}{}^2} + K_{12}{}^2 + 2(R_1R_2 - K_1K_2) \right\}^{\frac{1}{2}}. \quad (2)$$

This function is a minimum when

$$K_1 = K_2 \frac{K_{12}{}^2}{R_2{}^2 + K_2{}^2}. \quad (A)$$

The expression (A) therefore gives the value of the primary reactance which makes the amplitude of the current in the secondary a maximum.

The value of K_2 which makes (2) a minimum is

$$K_2 = K_1 \frac{K_{12}^2}{R_1^2 + K_1^2}. \tag{B}$$

(B) is therefore the expression for the secondary reactance which makes the amplitude of the secondary current a maximum.

Finally (2) is a minimum when

$$K_{12}^2 = \{(R_1^2 + K_1^2)(R_2^2 + K_2^2)\}^{\frac{1}{2}}, \tag{C}$$

and (C) therefore gives the mutual reactance of the circuits that will make the secondary current amplitude a maximum.

Before proceeding further it is interesting to note that (A) is not only one of the conditions for a maximum amplitude of secondary current but that it is also the condition for unity power factor in the primary of the transformer, since the expression for the primary current may be written in the form

$$I_1 \sin (\omega t + \vartheta) = \frac{E \sin (\omega t + \vartheta)}{\left\{ \left(R_1 + R_2 \frac{K_{12}^2}{R_2^2 + K_2^2} \right)^2 + \left(K_1 - K_2 \frac{K_{12}^2}{R_2^2 + K_2^2} \right)^2 \right\}^{\frac{1}{2}}},$$

where

$$\vartheta = \tan^{-1} \frac{K_1 - K_2 \dfrac{K_{12}^2}{R_2^2 + K_2^2}}{R_1 + R_2 \dfrac{K_{12}^2}{R_2^2 + K_2^2}}.$$

Evidently when the condition of (A) is imposed, this reduces to

$$I_1 = \frac{E}{R_1 + R_2 \dfrac{K_{12}^2}{R_2^2 + K_2^2}}, \tag{3}$$

and $\vartheta = 0$.

The primary current is, under these circumstances, equal to the ratio of the impressed electromotive force to the apparent or effective resistance of the primary, the apparent reactance of the primary is zero and the current in the primary is in phase with the impressed electromotive force.

Similarly it is of interest to note that the condition (B) for a maximum amplitude of secondary current is also the condition

which would produce unity power factor in the secondary circuit if it were used as the primary or driving circuit.

When all these conditions (A), (B) and (C) are imposed upon the expression for the primary current it becomes

$$I_1 = E/2R_1, \tag{4}$$

and when all these conditions are imposed on the expression for the secondary current it becomes

$$I_2 = E/2\sqrt{R_1 R_2}. \tag{5}$$

This is the greatest maximum value of I_2 which can be attained by adjustment of the capacities, inductances and mutual inductance of the primary and secondary circuits.

Conditions (A) and (C) and conditions (B) and (C) both lead to the relation

$$K_1 R_2 = K_2 R_1, \tag{6}$$

so that condition (C) becomes

$$K_{12}{}^2 = K_1 K_2 + R_1 R_2.$$

The condition which some authors term the condition of resonance in the secondary of a transformer is

$$K_2 = 0 \quad \text{or} \quad L_2\omega - 1/C_2\omega. \tag{D}$$

This condition does not *per se* give the maximum of current in the secondary.

When condition (D) is imposed upon the expression for the primary current this becomes

$$I_1 \sin(\omega t + \vartheta) = \frac{E \sin(\omega t + \vartheta)}{\left\{ \left(R_1 + \dfrac{K_{12}{}^2}{R_2} \right)^2 + K_1{}^2 \right\}^{\frac{1}{2}}}, \tag{7}$$

where

$$\vartheta = \tan^{-1} \frac{K_1}{R_1 + \dfrac{K_{12}{}^2}{R_2}}.$$

Under condition (D) therefore the apparent resistance of the primary of the transformer is greater than the real resistance, but

the apparent reactance of the primary is the same as the real reactance of the primary.

When condition (D) is imposed upon the expression for the amplitude of the current in the secondary this reduces to

$$I_2 = \frac{EK_{12}}{\{R_2^2(R_1^2 + K_1^2) + K_{12}^4 + 2K_{12}^2R_1R_2\}^{\frac{1}{2}}}. \tag{8}$$

If, however, in addition to the condition (D) we also impose the condition

$$K_1 = 0 \quad \text{or} \quad L_1\omega = 1/C_1\omega, \tag{E}$$

the expression for the primary current becomes

$$I_1 \sin \omega t = \frac{E \sin \omega t}{R_1 + \dfrac{K_{12}^2}{R_2}}, \tag{9}$$

and $\vartheta = 0$. This is a condition of unity power factor in the primary.

Under conditions (D) and (E), the amplitude of the secondary current is

$$I_2 = \frac{EK_{12}}{R_1R_2 + K_{12}^2}. \tag{10}$$

For variations of K_{12}, this is a maximum when

$$K_{12} = \sqrt{R_1R_2}, \tag{F}$$

so that when conditions (D), (E) and (F) are imposed the primary current amplitude is given by (4), and the amplitude of the current in the secondary is given by (5).

This shows that conditions (D), (E) and (F) when simultaneously imposed are the equivalent of conditions (A), (B) and (C) simultaneously imposed. In fact, the set of conditions (D), (E) and (F) constitute one limiting case of the set of conditions (A), (B) and (C). The other limiting case being when

$$K_1 = K_{12}^2/2K_2, \tag{G}$$

$$K_2 = K_{12}^2/2K_1, \tag{H}$$

and

$$K_{12}^2 = 2K_1K_2. \tag{J}$$

From the expressions (A) and (B) it is apparent that there are in general two values of K_1 and two values of K_2 which will satisfy these conditions.   They are

$$K_1' = K_{12}^2/2K_2 + \sqrt{K_{12}^4/4K_2^2 - R_1^2}, \tag{A'}$$

$$K_1'' = K_{12}^2/2K_2 - \sqrt{K_{12}^4/4K_2^2 - R_1^2}, \tag{A''}$$

$$K_2' = K_{12}^2/2K_1 + \sqrt{K_{12}^4/4K_1^2 - R_2^2}, \tag{B'}$$

$$K_2'' = K_{12}/2K_1 - \sqrt{K_{12}^4/4K_1^2 - R_2^2}. \tag{B''}$$

With the conditions of (A') and (B') goes the corresponding condition

$$K_{12}'^2 = \{(R_1^2 + K_1'^2)(R_2^2 + K_2'^2)\}^{\frac{1}{2}}$$

or

$$K_{12}'^2 = K_1'K_2' + R_1R_2, \tag{C'}$$

while with the conditions (A'') and (B'') goes the condition

$$K_{12}''^2 = \{(R_1^2 + K_1''^2)(R_2^2 + K_2''^2)\}^{\frac{1}{2}},$$

or

$$K_{12}''^2 = K_1''K_2'' + R_1R_2. \tag{C''}$$

The set of conditions expressed by (A), (B) and (C) is seen therefore to consist really of the two sets of conditions expressed by (A'), (B'), (C') and by (A''), (B''), (C'') respectively.

The two sets of conditions expressed by (A'), (B'), (C') and by (A''), (B'') and (C'') only become the same in the limiting case given by (G), (H) and (J).

The conditions for the greatest maximum value of the current in the secondary circuit of a transformer given above will probably be most readily apprehended through an examination of the diagram of Fig. 2.

Since the reactances of both the primary and secondary circuits as well as the mutual reactance between the two circuits are variable, the only elements of the transformer circuits which remain constant are the resistances R_1 and R_2 of these circuits.   In constructing the diagram of Fig. 2 which represents graphically the combined conditions of (A), (B) and (C) therefore the two constants R_1 and R_2 will form the basis for the measurement of all the other quantities.

To construct the diagram of Fig. 2 proceed as follows:

From the point O as origin lay off the horizontal line OA, equal to R_2. On O, drop the vertical FO, and about O as a center describe the arc OB with OA as its radius. Project the horizontal line BS to the left from the point of intersection B of the arc OB with the vertical FO. About O as a center, with a radius OD equal to R_1, describe the second arc DE. Draw the right line OG through O and through the point of intersection E of this arc with the horizontal at line BS. Choose some convenient point P on the line OG, such that OP is equal to or greater than OE and from this point describe the semi-circle $OWHV$ with radius OP. From D draw the line DH parallel to OG and from the points of intersection H and W of this line with the arc $OWHV$ erect perpendiculars to
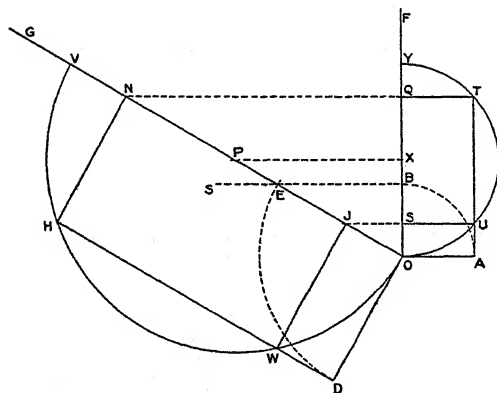
Fig. 2.

the line OG and from the points N and J where these perpendiculars meet the line OG, draw horizontal lines intersecting the vertical OF.

So far as defining the relation between the quantities R_1, R_2, K_1, K_2 and K_{12}, the construction need not be carried further, but the significance of the conditions of (A), (B) and (C) is best seen when the diagram is completed as follows:

From the point P draw the horizontal line PX to the vertical FO and taking the point X as a center describe the arc OUT with radius XO. Draw a perpendicular to the line OA at A. This will pass through the points T and U of intersection of the horizontal lines NQT and JSU with the arc OUT and will complete the diagram.

In the figure,

$$OA = R_2,$$
$$OD = R_1,$$
$$OS = K_2',$$
$$OQ = K_2'',$$
$$OJ = K_1',$$
$$ON = K_1'',$$
$$OY = K_{12}^2/K_1 = K_{12}'^2/K_1' = K_{12}''^2/K_1'',$$
$$OV = K_{12}^2/K_2 = K_{12}'/K_2' = K_{12}''^2/K_2''.$$

From the fact that P may be chosen at any point along the line OG beyond the point E, it follows that in the case of any particular transformer there are an infinite number of values of K_{12}^2/K_2, each greater than R_1 that are consistent with the conditions of (A), (B) and (C). Similarly there are an infinite number of values of K_{12}^2/K_1 each greater than R_2 that are consistent with this set of conditions.

The limiting case in which the conditions of (G), (H) and (J) obtain is represented by a diagram in which the points P and X coincide with E and B respectively, under which conditions evidently W coincides with H and T coincides with U, so that in this case

$$K_1 = R_1,$$
$$K_2 = R_2$$

and

$$K_{12}^2 = 2R_1R_2.$$

The condition of the greatest maximum value of the current in the secondary circuit indicated by (4) and (5) and defined by (A), (B) and (C) may therefore always be attained when

$$K_{12}^2 \geqq 2R_1R_2.$$

A METHOD OF MEASURING THE SUSCEPTIBILIY OF WEAKLY MAGNETIC SUBSTANCES AND A STUDY OF THE SUSCEPTIBILITY OF ALLOYS OF BISMUTH WITH TELLURIUM AND THALLIUM.

By C. E. MENDENHALL AND W. F. LENT.

Fig. 1.

IN the PHYSICAL REVIEW, Volume XXV.I., Number 6, Clifford describes a method for measuring susceptibility which gave excellent results but which involved the rather tedious determination of the magnetic field-intensity throughout a region in which the specimen under test was to beused, in order that the equation

$$f_x = \frac{dH}{dx} HkV,$$

might be used. In this equation, H is the magnetic field-intensity, dH/dx, its rate of change along x; k, the susceptibility and V the volume of the specimen, while f_x is the resulting ponderamotive force, along x, on the specimen. It occurred to one of us that considerable simplification, and possibly some increase in accuracy, would result if the arrangements were changed so as to give virtually a "zero" method. The present paper gives an account of the new method, together with the results of a study of the series of bismuth-tellurium and bismuth-thallium alloys, and some preliminary observations on the effects of heat treatment on the susceptibility of bismuth.

The principle of this method, which is applicable equally well to measurement of
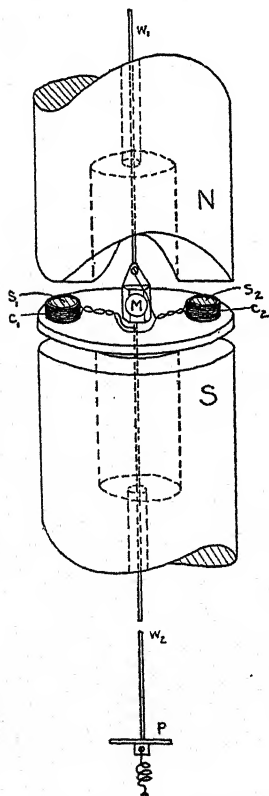
para- and dia-magnetic bodies, is, briefly, to balance the torque produced by the action of a non-uniform magnetic field upon two symmetrically-placed specimens of the substance under test, by the torque due to the action of the same field upon coils immediately surrounding the specimens, the current in which can be measured. The general arrangement of the apparatus will be clear from Fig. 1, where N, S are the poles of the magnet producing the testing field; S_1S_2, the small cylinders of the metal under test; c_1c_2, the cylindrical coils accurately surrounding the test specimens, through which the balancing current is passed and w_1w_3 the suspending wires which also serve to carry the current to and from the coils c.

In order to bring out the quantitative conditions on which the method rests, consider a section through the (circular) specimens by a plane perpendicular to the axis of suspension, Fig. 2, where r
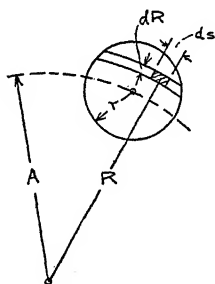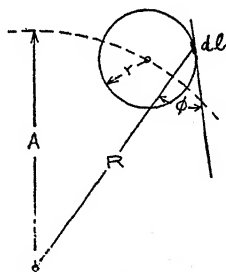
Fig. 2.   Fig. 3.

is the radius of the specimen, and also of the single layer coil surrounding it; R, the distance from the axis of suspension to an element of the circumference. The expression for the moment of the forces acting upon a thin section of the specimen, cut parallel to the plane of the paper, around an axis through O perpendicular to the plane of the paper is:

$$
\begin{aligned}
M_S &= dt \int_{A-r}^{A+r} R \int_{H_1}^{H_2} kH dR \frac{dH}{dS} dS \\
&= kdt \int_{A-r}^{A+r} R dR \frac{(H_2 + H_1)}{2}(H_2 - H_1),
\end{aligned}
$$

(I)

where dt is the thickness of the section and H_2, H_1 are the com-

ponents of the magnetic field perpendicular to the section, at the ends of the elementary strip of width, dR.

Similarly (Fig. 3) for the coil of radius r, n turns per centimeter of thickness (dt) and carrying a current i the moment around o is:

$$(2) \quad M_c = indt \int_{A-r}^{A+r} Rdl \cos \phi (H_2 - H_1) = indt \int_{A-r}^{A+r} RdR(H_2 - H_1).$$

If, now, $(H_2 + H_1)$ is independent of R we have for the balanced case when $M_s = M_c$,

$$(3) \qquad k \frac{H_2 + H_1}{2} = in.$$

It is evidently necessary, then, that the specimens should as nearly as possible fill the coils and that the mean field $\frac{H_2 + H_1}{2}$ should not vary with R. Furthermore, if $\frac{H_2 + H_1}{2}$ is to be determined, as is most convenient, by using the suspended coils as test coils in connection with a standardized fluxmeter, it is necessary that the lateral variations of H should be practically linear, in order that the mean value of H over the *area* of the specimen, as measured by the fluxmeter, should equal the mean value around the *boundary* of the specimen. Our specimens were turned to fit the coils to about 0.02 mm., so that any error introduced hereby is negligible. Since two symmetrical coils are used it is to be observed that the H and n occurring in (3) are the means for the two coils.

The mean value of H was determined by connecting the two coils in series with a Grassot fluxmeter and determining the deflection produced by reversing the magnetic field;[1] the fluxmeter having been calibrated in the usual way with a standard solenoid.

The expression for H is then:

$$H = \frac{1}{2} \frac{d}{A\,Na},$$

where: d = deflection observed on reversing H through coils;

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[1] This reversal method gives correct results in the present case because of the narrow hysteresis loop due to the large air-gap.

A = constant of fluxmeter;

$N = 2nh$ = total number of turns on coils 1 and 2 (h = height of coils);

a = area of one coil.

So that

$$k = \frac{A}{d} Na \cdot \frac{N}{2h} \cdot i,$$

$$= \frac{A N^2 a^2 i}{dV_c}.$$

(V_c = volume of both coils.

= $2ha$.)

The actual test pieces used were not simple cylinders as above assumed, but had small projections on top for convenience in handling; the magnetic torque produced by them would therefore be V_s/V_c times that produced by specimens just filling the coils, where V_s = actual volume of specimens, and V_c = volume of coils. The expression for k should therefore be multiplied by V_c/V_s giving, finally, $k = A N^2 a^2 i/dV_s$. V_s was determined from the loss of weight in water.

In working this out practically the general procedure was first to adjust the torsion head, with no field on, and no specimen in place, but with equal counter-weights in the pan, P, below, until the coils were in the proper part of the field, this zero position being located by a scale reading reflected from the mirror M of the suspended system. The counter-weights which were removed when the specimens were put in place were to avoid any error due to twisting of the suspension caused by the change in tension which would otherwise occur. The specimens were then put in place and the field put on, and the current determined which was necessary to bring the suspended system back to the zero position.[1] The suspended system was then clamped in the zero position, the coils (through the suspension) disconnected from the current circuit and connected to the fluxmeter and the field H determined by reversals. In determining H it was found necessary to make two observations—one with the

[1] To eliminate any slight asymmetry of specimens and coils readings were also taken with specimens interchanged.

coils in series with the fluxmeter, as described, and the other with only the upper and lower suspension in series with each other and the fluxmeter—*i. e.*, the coils themselves cut out of circuit by a very small switch on the suspended system. By this means allowance could be made for the change in flux (on reversal of H) in all parts of the circuit other than the coils themselves. It was also necessary to make "blank" tests of the magnetic deflection obtained with no specimens in position—in order to correct for the magnetic properties of the suspended system. Experience emphasized the necessity for placing the counter-weights some distance below the magnet in order to avoid the torque due to stray fields.

A tantalum ribbon served excellently as an upper suspension, with a coiled phosphor-bronze ribbon for the lower one. The coils c_1 and c_2 were made by winding no. 36 enamel insulated copper wire on a carefully turned brass rod, coating with plaster of Paris and removing. The difference between the mean radius of the coil and the outside radius of the specimen was only the radius of the insulated wire, 0.07 mm., plus 0.01 mm. to permit the easy removal of the specimens. The diameter of the coils was measured with a micrometer microscope. The dimensions of the moving system were:

Diameter of coils = 0.8131 cm.,
Height of coils = 3.5 mm. (approximately),
Distance from axis of suspension to center of coils = 2 cm.
Total number of turns per coil = 25.

In order to minimize the radial variation of H, and to produce an approximately linear transverse variation, the end of the upper hollow cylindrical pole-piece was given an approximately helicoidal shape, the air-gap varying with θ but not with R. The width of the annular face along R was about twice the diameter of the specimens. Errors in the results due to variations in H different from those assumed would in all probability be different for saturated and unsaturated pole-pieces; hence determinations of the susceptibility of bismuth were made with different magnetizing currents (saturated and unsaturated pole-pieces). As may be seen from the observations on a single specimen of bismuth grouped in

Table I., the relations between k and the exciting current (H) is:

<div style="text-align:center">

Current k.

5 amperes, 14.27
4 amperes, 14.30
3 amperes, 14.41

</div>

There is apparently a tendency to get larger values of k with smaller field strengths—the maximum variation of these means being 1 per cent. It is doubtful whether this variation is really due to a departure of the field from the linear variation assumed, for it will be noticed that the *accidental* variations are fully as large as this. All readings except those here given were made with four amperes field current; and it would seem a reasonable conclusion from these figures of Table I. that errors of somewhat less than 1 per cent. are as great as need be expected in the *means* of a number of observations.

<div style="text-align:center">

TABLE I.

Susceptibility of Bismuth.
Volume, 0.377 c.c.   A = 2.965×19⁻⁵.

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I	$-i_{c_1}$ $\times 10^2$	$-i_{s_1}$ $\times 10^2$	$-i_{s_2}$ $\times 10^2$	$-i_{c_2}$ $\times 10^2$	$-i \times 10^2$	d	$-k \times 10^6$
5.00	0.1182	0.7602	0.7552	0.1189	0.642	25.50	14.39
	0.1179	0.7510	0.7545	0.1179	0.635		14.21
	0.1657	0.8023	0.7997	0.1704	0.633		14.20
4.00	0.1714	0.7714	0.7700	0.1634	0.603	24.02	14.29
	0.1037	0.7083	0.7116	0.1078	0.604		14.30
	0.1966	0.8002	0.8018	0.1940	0.607		14.36
	0.2209	0.8235	0.8239	0.2216	0.603		14.28
	0.2096	0.8089	0.8115	0.2069	0.602		14.25
3.00	0.1594	0.6941	0.6913	0.1594	0.533	20.92	14.42
	0.1554	0.6893	0.6862	0.1585	0.531		14.40
	0.2222	0.7558	0.7524	0.2222	0.532		14.41

Mean $k = -(14.32 \pm 0.11) \times 10^6$.
Maximum deviation from mean, 0.77 per cent.

The Susceptibility of Bismuth.—As a first test of the method a number of observations were taken on the susceptibility of bismuth. The bismuth used was merely the purest obtainable from Kahlbaum, but great care was taken to avoid contaminating it with paramagnetic material, especially iron, while preparing it for use, it

being cast in glass tubes and turned with a carborundum crystal as tool.   The susceptibilities determined for a number of specimens, prepared by casting in a glass tube and allowing to cool freely in air, are as follows:

$$k = 14.32$$
$$14.38$$
$$14.25$$
$$\underline{14.35}$$
$$\text{Mean} = 14.32$$

We also attempted to determine whether the susceptibility of bismuth could be altered by the heat treatment used in preparation—*i. e.*, sudden chilling after casting, cooling in air (rather rapid) or long slow annealing.   Some difference might perhaps be expected due to the first two and the last methods of preparation.   To this we cannot as yet give a definite answer, though there is indication that such an effect exists, for the mean of the values of k for specimens cooled in air, and of those slowly cooled in the furnace from the molten state are:

air cooled, $k = 14.32$

annealed, $k = 14.76$ (weighted mean, 3 specimens, 7 measurements)

That this difference is not due to the presence or absence of blow-holes in one case or the other is shown by the fact that the average density of the air-cooled specimens is 9.82, of the slow-cooled, 9.83. This matter of the effect of heat treatment (crystal structure) will be followed further.   Our average value of k for air-cooled bismuth agrees well with the mean of previous observations, and the individual measurements are somewhat more consistent.

The Susceptibility of Bismuth-Tellurium and Bismuth-Thallium Alloys.—Aside from the work of Laws and Clifford, relatively little work has been done on the relation between diamagnetic susceptibility and composition of alloys.   We selected the above group of alloys for work for the reasons, first, that their chemical and (other) physical properties were more or less known, and, second, because they both indicate, by their melting points and other physical properties, the existence of definite compounds between the constituents.

The tellurium used was obtained from Professor Lenher, of the

Chemistry Department of the University of Wisconsin, and was prepared with great care under his direction by Dr. Charles Tibbals. The thallium was Schuckhardt's chemically pure; the bismuth, Kahlbaum's chemically pure (the same as that used in the measurements on the susceptibility of bismuth above). Since tellurium and thallium oxidize very rapidly when heated in air the alloys were fused in a reducing atmosphere of hydrogen in an electric furnace, the temperature in which was determined by a thermopile. Great care was exercised in the preparation of the alloys to introduce no impurities. The tellurium as obtained was in a powdered form which melts at a higher temperature than the solid material. In order, then, to reduce the losses by vaporization in the fusing process this powder was first melted together. After cooling, the solid mass could be conveniently broken into pieces, for the preparation of the alloys, by pounding in a mortar. The thallium was cut into chips with a knife and boiled in dilute HCl to remove abraded iron. Both tellurium and thallium alloy very well with bismuth and the compositions calculated from the weights of the materials originally put into the alloy are known to check very closely with chemical analyses (Monkemeyer and Chikashige).[1] The alloys were, therefore, not analyzed after the measurement of their susceptibilities. The alloys were turned up in a lathe to the size required by a tool formed from a carborundum crystal. This eliminates the effect, noted by Clifford,[2] of iron abraded from an ordinary tool.

The susceptibilities for the alloys with tellurium are given in Table II.; for thallium in Table III. The values given are the mean of two or more observations except in the case of nos. 21 to 25, inclusive, for which but one observation on each was made The measurements were made at temperatures varying from 20° to 25° C. Tables II. and III. are plotted in the curves of Figs. 4 and 5.

The susceptibility of bismuth is taken from our results given above (-14.32×10^{-6}). The susceptibility found for thallium (mean of two specimens and three observations $-2.46 \pm 0.06 \times 10^{-6}$) is considerably lower than that found by Königsberger[3]

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[1] Zeitschrift f. anorg. Chem., 46, 415, 1905, and 51, 328, 1906.

[2] *Loc. cit.*

[3] Königsberger, Wied. Ann., 66, 698, 1892.

TABLE II.

Bismuth Tellurium Alloys.

Specimen Number.	Per Cent. Bi. Weight.	Method of Preparation.	Ave.— $k \times 10^6$
14	00.00	Cast in glass tube.	2.26
33	10.00	From No. 16 by adding Te.	1.87
16	25.00	From No. 15 by adding Te.	2.14
15	52.14	Melting together.	2.89[2]
35	52.14	From No. 33 by adding Bi.	2.60
17	75.00	From No. 15 by adding Bi.	4.48
21	95.00	From No. 17 by adding Bi.	8.25
A_c	100.00	Fusing from Kahlbaum.	14.32

TABLE III.

Bismuth Thallium Alloys.

Specimen Number.	Per Cent. Bi. Weight.	Method of Preparation.	Ave. — $k \times 10^6$
18	00 00	Cut directly from stick.	2.47
25	00 00	Cut directly from stick.	2.45
36	10.90	From No. 23 by Adding Tl.	1.49
23	25.00	Melting together.	0.87
20	47.53	Melting together.	1.38
37	56.00	From No. 20 by adding Bi.	1.57
19	63.00	Melting together.	1.30
26	63.00	Melting together.	0.97
38	67.00	From No. 37 by adding Bi.	2.42
22	80.34	From No. 19 by adding Bi.	6.08
24	95.00	Melting together.	11.40
A_c	100.00	Melting from Kahlbaum.	14.32

$(- 4.6 \times 10^{-6})$, the only other experimenter who has made measurements on thallium, so far as we are aware. This may indicate that out thallium was less pure than Königsberger's though he makes no statement concerning the source or purity of his material.

The susceptibility found for tellurium (one specimen and mean of two observations $- 2.26 \pm 0.05 \times 10^{-6}$) is slightly higher than the result of Königsberger,[3] $(- 2.1 \times 10^{-6})$. Other experimenters have found for tellurium $- .006$,[4] $- 1.6$[5] and $- 1.9 \times 10^{-6}$). Our

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[1] The composition for numbers 15 and 35 was Bi_2Te_3.
[2] The composition for numbers 19 and 26 was Bi_5Tl_3.
[3] Königsberger, Wied. Ann., 66, 698, 1892.
[4] St. Meyer, Wied. Ann. 68, 375; 69, 236, 1899.
[5] Von Ettinghausen, Wied. Ann., 17, 272.
[6] Curie, Comptes Rendus, 115 and 116, 1892.

tellurium was very pure so that the value obtained is probably very good.

That the two determinations on the compound Bi_5Tl_3 (Fig. 4) do not check very closely probably indicates that the compound is on a steep maximum of the curve. This deviation may also be partially accounted for by the fact (as shown by Table II.) that
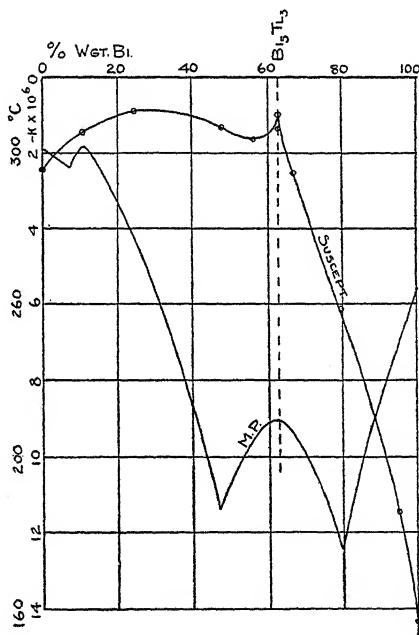
Fig. 4.

Bismuth thallium alloys.

one of the specimens was prepared after four successive fusions from other alloys, and here its assumed composition may have been somewhat in error. The points for the compound Bi_2Te_3 (Fig. 5) lie on a smooth curve through the other points. It is possible that the points should be on a small cusp similar to that obtained for Bi_5Tl_3. Its detection is, however, either beyond the limits of accuracy of the method or would require the determination of a great number of points in the neighborhood of the cusp.

The melting-point curves in Figs. 4 and 5 are taken from the

work of Monkemeyer and Chikashige;[1] those of thermo-electric force and electrical conductivity for tellurium from Haken.[2] No corresponding data on thallium are available at present.

It is interesting to note the difference in the behavior of the tellurium and thallium alloys at the point of chemical combination. The thallium shows a well-defined cusp, while the tellurium curve is quite smooth, in spite of the fact that other physical properties show marked discontinuities. That thallium is a metal and tel-
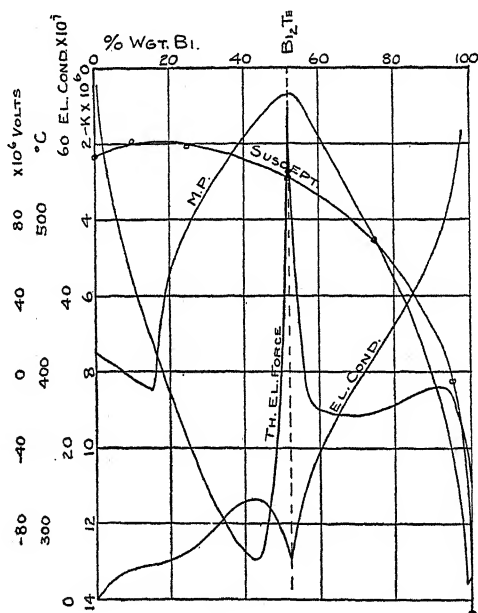
Fig. 5.

Bismuth tellurium alloys.

lurium on the border line between metals and non-metals may or may not be a factor in the explanation. Chemists generally accept maxima on the melting-point curves as good evidence of chemical combination. On this basis we have two chemical compounds in the bismuth-thallium alloys (Fig. 4). Strangely enough we have unique magnetic properties for Bi_5Tl_3 but not for the other compound, and not for Bi_2Te_3. Other groups of alloys will be studied with this point in mind.

[1] *Loc. cit.*

[2] Haken, Ber. d. Deutsch. Phys. Gesell., March 15, 1910.

SUMMARY.

1. A null method for measuring magnetic susceptibility is described and its accuracy discussed.

2. The mean value $k = 14.32$ is obtained for pure bismuth.

3. Preliminary results on the effect of heat-treatment (crystal structure) on the susceptibility of bismuth are given.

4. The curves connecting k and percentage composition for the series of Bi-Te alloys is given, showing no peculiar magnetic properties for the compound Bi_2Te_3.

5. The corresponding curve for Bi-Th alloys is also determined and does show peculiar magnetic properties for the compound Bi_5Tl_3.

UNIVERSITY OF WISCONSIN,
     DEPARTMENT OF PHYSICS,
          November, 1910.

THE SECOND POSTULATE OF RELATIVITY AND THE ELECTROMAGNETIC EMISSION THEORY OF LIGHT.

BY O. M. STEWART.

INASMUCH as it appears that the results of the relativity principle are to be accepted or rejected as we accept or reject the two postulates, these postulates should receive very careful attention.

Einstein has stated the first postulate in a general form: The laws governing natural phenomena are independent of the motion of the system of coördinates used in observations on these phenomena, provided there is no acceleration of this system. In other words, one can never tell from any observations he may make what the absolute velocity of his origin of coördinates may be. The observed operation of any laws would be the same whether one is absolutely at rest or moving with a uniform linear motion. Two observers, one *A*, the other *B*, have relative motion. It is a matter of indifference whether we say *A* is moving and *B* at rest, or *B* moving and *A* at rest, since all observations made by *A* can be duplicated by *B*. There thus arises an interesting symmetry which will be referred to later. This postulate has been accepted in mechanics since the days of Newton, and is confirmed in other branches of physics by all the experiments that man has been able to devise. Such for example are the experiments of Michelson and Morley,[1] and Trouton and Noble.[2]

The second postulate, as stated by Einstein, is that light is always propagated in vacuo with the same velocity, this velocity being independent of any motion of the source. This postulate is based in part on experiment and in part on an *assumption*. The experiments of Michelson and Morley and others show that

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[1] Am. Jour. Sci., 34, p. 333, 1887.
[2] Phil. Trans. Roy. Soc., (A), 202, p. 165, 1904.

the time required for light to travel to a mirror and return is independent of any translatory motion of the apparatus as a whole. In other words, all experiments that have been made on the velocity of light show an agreement with the first postulate. The assumption is that the velocity of light is independent of the velocity of the source. We may then say that the results of the relativity principle are due to a generalization of a law of mechanics (the first postulate) and to the assumption that the velocity of light is independent of the velocity of the source. This assumpton has been generally accepted on account of our concept of the ether as a fixed medium filling all space. But, as has been shown by Einstein and others, the first postulate of relativity leads to the rejection of this concept of the ether. In fact, in all experiments where it has been possible to test this conflict between the stationary ether theory and the first postulate, the results have been in favor of the latter. The experiments already referred to, those of Michelson and Morley, and Trouton and Noble, are illustrations of this. Thus we have the principle of relativity destroying a concept which is used in one of its postulates.

It has been pointed out in many places that if we deny this assumption and admit that the velocity of light is a function of the velocity of the source, the complete explanation of such experiments as those of Michelson and Morley follows at once, and at the same time the strange conclusions of the relativity principle are destroyed. Such an hypothesis makes it necessary for us to modify our conception of the ether, but this we must do in either case. If we retain the old idea that the velocity of light is independent of the velocity of the source, we apparently must accept the relativity principle with its rejection of a fixed medium filling all space together with an entire revision of our concepts of the fundamental and derived units. At present it seems less revolutionary to assume that the velocity of light is a function of the velocity of the source. It is the purpose of this paper to attempt to clear up certain objections that have been raised to this view.

Sir J. J. Thomson[1] has presented a theory of light which makes it possible for the velocity to be a function of the velocity of the

[1] Philosophical Magazine, 19, p. 301, 1910.

source. This has been called by Kunz[1] the electromagnetic emission theory. The electrons in the light source, according to this theory have attached to them tubes of force. When an electron is accelerated a transverse vibration is propagated out each of its tubes with a velocity equal to c, the velocity of light. These transverse vibrations or waves are what we call light waves. The "medium" through which light is propagated is in this theory these Faraday-Thomson tubes. If the source has a motion of translation these tubes also have the same velocity, provided this velocity is uniform. The "medium" always has the same uniform velocity as the source. Hence if c is the velocity of light when the source is stationary, $c + v$ is the velocity when the source has a velocity whose component in the direction of the propagation of light is v. In other words, the velocity of light with *respect to the source* is always c.

Measurements of the Doppler Effect with a Grating.—It has been pointed out by both Tolman[2] and Kunz[3] that if the velocity of light changed in the manner just described, the length of a wave would not be changed by motion of the source. In the equation

$$v = n\lambda$$

both v and n are changed in such a way that λ is not changed. Moreover the same wave-length would be emitted in all directions by a moving source. This has led Tolman to object to this theory on the ground that gratings which always measure wavelengths would not detect the Doppler effect from stars or the limb of the sun as we know they do. However the theory of the grating assumes a *stationary* medium. For example, in the elementary theory of the transmission grating it is assumed that when a wavelet "spreads" out from each one of the slits the center of the disturbance in the medium remains in the slit. This is readily seen if one obtains the resultant wave-front by Huyghens's construction, that is, by means of wavelets emanating from the openings. Whether or not we could detect this Doppler effect

[1] Amer. Jour. Sci., 30, p. 313, 1910.
[2] PHYS. REV., 31, p. 26.
[3] *Loc. cit.*

with a grating would depend on the nature of the "medium" that transmits these waves.

If light waves are the vibrations of electric tubes moving with the source as supposed in Thomson's theory, this medium in the case of a moving star would be drifting through the grating with a velocity equal to the velocity of the star. In this event the grating would show the Doppler effect. Moreover, it may be shown that the observed effect would depend merely on the relative motion of the star and the observer, that is, it would be the same whether the observer is in motion or whether it is the star that is moving. In other words, this obeys the first postulate of relativity.[1] Thus if we change our conception of the medium as we must if we are to assume that the velocity of light depends on the velocity of the source, this difficulty disappears.

Tolman's Interference Experiment.—In the same paper Tolman describes an interesting experiment which leads him to raise another objection to the supposition that the velocity of light is changed by the motion of the source. The experiment consists in observing the interference fringes produced by Lloyd's mirror when first one limb of the sun and then the other is focused on the slit. Consider two beams of light both coming from the same limb of the sun one passing directly to the screen the other to the same point after being reflected at nearly grazing incidence. It is shown that if light came from the limb of the sun with a velocity $c \pm u$ where $\pm u$ is the velocity of the limb in the line of sight due to the rotation of the sun, and if light reflected by this mirror has its velocity changed to c, then the observed fringes should shift as one changed the source from one limb to the other. The negative result was regarded as proving that light from the limb of the sun did not have a velocity $c \pm u$. However, it was assumed that when light is reflected by a mirror at any angle the mirror becomes the source and the velocity of the reflected light depends only on the velocity of this mirror. The negative result may be regarded as merely

[1] The explanation of the Doppler effect in terms of the usually accepted theory does not agree with this postulate. If the source is moving and the observer fixed, the ether waves are said to be changed in length. On the other hand, if we consider the source as stationary and the observer moving there is no change in the wavelength.

proving that this assumption is in general not correct.   Much may be said in favor of the idea that when light is reflected, the mirror becomes a new source.   But in view of the fact that we must modify our concept of the ether and the electromagnetic theory, this is clearly an assumption.

Certain considerations, to be explained later, lead to the law that the velocity of light reflected by a mirror will depend on the velocity of the *image* of the source, that the velocity after reflection will be equal to $c + v$, where v is the component of the velocity of the image of the source in the direction of the reflected light.   Applying this to the experiment of Tolman, we find that the image of the moving source, when formed by reflection at nearly grazing incidence in the mirror, has the same velocity as the source.   Hence in this case the velocity of light would not be changed by the reflection and no shift of the fringes should be observed.

The Law of the Change of Velocity on Reflection.—This proposed reflection law is based on the electromagnetic emission theory of Thomson.   One of the essential features of this theory is the reality given to the electric tubes of force.   These are no longer to be geometrical fictions but to have a true objective existence.   Since light is a disturbance propagated along these tubes, we may regard reflection as produced by a bending or reflection of these tubes. If we now assume no longitudinal compression of the tubes on reflection, the energy per unit volume in these tubes will not be altered by reflection from a perfect mirror.[1]   Further, as there will be no change in the longitudinal dimensions of the tubes there will be no change in the length of a wave on reflection.   As a result of this assumption we have then two theorems: (*a*) No change in the energy density, (*b*) no change in the wave-length on reflection. Since this theory of Thomson shows that in the case of direct radiation, motion of the source produces no change in the energy density[2] or in the length of a wave these theorems are only parts of a

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[1] Absence of longitudinal compression would mean inability to transmit longitudinal waves.

[2] It is conceivable that there may in certain cases be changes in the distribution of the tubes of force on account of the motion of the source, for example when moving through a magnetic field.   In such a case there would be a change in the distribution of the intensity of radiation.   This effect is neglected above.

more general law which may be stated thus: Motion of a source, real or virtual, does not change either the energy density[1] or the wave-length.

This leads to a symmetrical condition which is shown by the consideration of a source A and a mirror B having relative motion. Whether we consider A at rest and B moving, or B at rest and A moving, we have identically the same condition, that is, the same energy in the reflected beam. The same symmetry is shown in regard to the length of waves. The wave-length is not changed by motion of the source nor by reflection. In this respect we have complete agreement with the first postulate of relativity. Hence the difficulty pointed out in the footnote on p. 421, which is found in the generally accepted theory, does not appear here.

We may by using either one of these theorems obtain the law of the velocity of reflected light. The constancy of the energy density will be used first. For brevity we shall take the case of normal incidence. Let the source have a velocity u, the mirror a velocity v, both velocities being in the same direction along the line joining the source and mirror. If the mirror were stationary it would receive energy at the rate of

$$eA(c + u)$$

per second, e being the energy per unit volume in the incident beam near the mirror, A the area of the beam, and $c + u$ the velocity of the incident light. But on account of the motion of the mirror not all of this energy is received, some being distributed over the space vacated by the mirror, the rate of loss being eAv. Hence the energy received per second is

$$eA(c + u - v).$$

As the energy density of the reflected beam is to remain constant, the light must be carried away from the mirror at a velocity $c+u-v$ measured with *respect to the mirror*. When referred to the system of coördinates from which both u and v are measured the velocity of the reflected beam is

$$c + u - 2v.$$

[1] This energy density is what is later called the *intrinsic* energy density.

It can readily be shown that the velocity of the image of the source seen in the mirror is

$$2v - u.$$

Hence the velocity of the reflected light is

$$c - (2v - u),$$

where $2v - u$ is the velocity of the image, in this case in a direction opposite to that of the propagation of the reflected light.[1]

This can be extended to the more general case where the light falls on the mirror at any angle of incidence. To do this it must be noted that the tangential component of the mirror velocity will have no effect and that only the component of the normal velocity in the direction of the incident or reflected light need be considered. A general rule follows: The velocity of light is always $c + v$ where v is the component in the direction of the propagation of light of the velocity of the source for direct light, of the image (virtual source) for reflected light.

The same result is obtained if we use the theorem that the wavelength is not changed by reflection. Since for any monochromatic beam of light λ is always a constant, we have

$$c'/n' = \lambda = \text{a constant,}$$

where c' and n' are respectively the velocity and frequency relative to any observer. When light is reflected from a moving mirror,

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[1] Some explanation of the use of the term energy density is necessary. As used above it may be defined as the energy absorbed from unit volume by a black body having the same velocity as the tubes of force, *i. e.*, the same velocity as the source, real or virtual as the case may be. We may call this the *intrinsic* energy density. In general, the energy density is relative, depending on the relative velocity of the source and the observer. In the case where the observer has not the same velocity as the source the energy density he measures may be said to be the intrinsic density plus or minus a term depending on the work done against radiation pressure. It is a matter of indifference whether we say this work is done by the moving tubes or whether we say the source is stationary and the work is done by the moving observer. This is very similar to problems that arise in mechanics where the energy a body has is relative, depending on the choice of a system of coördinates. In the formulas given above, the changes in the energy due to the work done on account of radiation pressure are omitted, so that we are always dealing with this intrinsic energy. It may be noted that whenever the velocity of light is changed, work must be done and the relative energy density changed.

both the velocity and the frequency may be changed and we will have

$$c/n = c'/n',$$

where c and n are the velocity and frequency before, and c' and n' the same quantities after reflection. Since n'/n may be observed, this may be written in the form

$$n'/n = c'/c.$$

If we take the case of a source stationary with respect to the observer and a mirror moving with a velocity v away from the source, a computation of the Doppler effect gives us

$$\frac{n'}{n} = \frac{c - 2v}{c}.$$

But this must equal the ratio c'/c.

Hence

$$c' = c - 2v.$$

Since the image has a velocity in this case equal to $-2v$ we find the same result as before.

The Doppler Effect.—As just shown the change in frequency due to motion either of the source or of the mirror is given by

$$n'/n = c'/c.$$

In applying this law n' may be regarded as the observed frequency and n the frequency when there is no relative motion with respect to the observer. The velocities c' and c must then be the velocities relative to this observer in the two cases. The law is independent of absolute motion. On the other hand in computing the change in frequency produced by the Doppler effect on the basis of a fixed medium, it is well known that the result is not independent of a choice of coördinates. For example, take the case of a source and an observer having relative motion. If we assume the source moving with a velocity v toward the stationary observer, the ratio of the frequencies is

$$\frac{n'}{n} = \frac{c}{c - v}.$$

While, if we assume that the observer is moving and the source fixed we have

$$\frac{n'}{n} = \frac{c+v}{c}.$$

That these two forms are different is in direct conflict with the first postulate of relativity.

In the electromagnetic emission theory, where motion produces no change in the wave-length, no such difference appears. No matter whether we regard the source moving or the observer we have for the case just cited

$$\frac{n'}{n} = \frac{c+v}{c}.$$

Kunz[1] has suggested that simultaneous measurements be made on the velocity of Kanal rays and the Doppler effect produced by their motion to distinguish between this last formula and the one predicted by the relativity principle which is still different from any of the above. This experiment could also be used to distinguish between the above conflicting laws, provided the spectroscopic work be carried to a very high degree of accuracy.

Two Applications of the Electromagnetic Emission Theory.—Two examples of the use of the electromagnetic emission theory will be given. The first cannot be explained by the generally accepted theory without bringing in some strange hypothesis. The second when interpreted by the same theory leads to a violation of the first postulate of relativity. Experiments show that the time required by light to go to a distant mirror and return is independent of any velocity given to the whole apparatus. Let A and B be the source and mirror respectively, and v their common velocity in the direction AB. According to this theory the light will leave A with a velocity $c + v$ and since the image of A in the mirror B has the same velocity as both A and B, the return velocity will be $c - v$. The time t_1 required for light to go from A to B is

$$t_1 = \frac{AB + vt_1}{c+v} = \frac{AB}{c}.$$

[1] Amer. Jour. Sci., 30, p. 319, 1910.

The time of the return will be

$$t_2 = \frac{AB - vt_2}{c - v} = \frac{AB}{c}.$$

Or the total time is $2AB/c$, a quantity independent of the vector v. The Michelson-Morley experiment is a very accurate method for comparing the time required to make the round trip when the line AB lies in the direction of the motion and when at right angles to it. The above expression is in complete agreement with their results in that it shows that the time is independent of any motion of AB.

The second case is that of two observers, who, while they have relative motion with respect to each other, exchange light signals. Let them agree to send at a predetermined instant a light flash to the other station, where it is to be reflected directly back. At first we may assume that A is at rest and that B has a velocity v directed away from A. The time required for light to go from A to B is

$$t_1 = \frac{AB + t_1v}{c} = \frac{AB}{c - v},$$

where AB is the distance between A and B at the instant the flash starts. The time of the return will be

$$t_2 = \frac{AB + t_1v}{c - 2v} = \frac{c}{(c - v)(c - 2v)} AB,$$

the velocity of the light reflected from B being $c - 2v$, as the image of A has a velocity $2v$. The time for the light flash starting at B to go to A will be

$$t_1 = \frac{AB}{c - v},$$

the velocity now being $c - v$. The return time from A to B will be

$$t_2 = \frac{AB + (t_1 + t_2)v}{c - v},$$

the image of B in A's mirror moving with a velocity $- v$. This time reduces to

$$t_2 = \frac{AB + t_1v}{c - 2v} = \frac{c}{(c - v)(c - 2v)} AB.$$

Hence both signals, starting at the same time, one from A, the other from a moving observer B, would reach the other station at the same instant and return at the same instant. There would be no discrepancy in their ideas about the simultaneity of two events and no trouble in their time standards such as arise when the details of this experiment are computed on the assumptions of the relativity principle. The same results are obtained whether we regard A as stationary and B moving, or A in motion and B stationary, that is, the result is independent of all conception of absolute motion. It is therefore in complete agreement with the *first* postulate of the relativity principle.

We thus find that if we chose a suitable "medium," one that will account for light having a velocity which is dependent on the velocity of the source, we can explain the objections raised by Tolman. The electromagnetic emission theory seems so far to agree with observation in those cases where the present theory fails. Further, it is in accord with the first postulate of relativity and leads if accepted to the overthrow of the second postulate.

University of Missouri,
     December, 1910.

TESTS ON CERTAIN ELECTRICAL INSULATORS
AT HIGH TEMPERATURES.

By W. W. Stifler.

A RECENT paper before the American Physical Society on "The Effect of Temperature on the Dielectric Strength of Porcelain Insulators"[1] has suggested that perhaps the results of some work recently done on two or three substances often used in the laboratory as insulators might be of interest. This work was incidental to another investigation and was undertaken in an attempt to find a substance which could be used to insulate a small coil at temperatures above 1000° C. This will account for the fact that the experiments were neither very extensive nor made with any great accuracy.

The insulators tested were mica, asbestos board, sodium silicate, a paste of magnesium oxide and alcohol mentioned by Terry in a recent article,[2] and a patent compound sold under the name of "cæmentium" or "liquid porcelain." The general method was as follows: The test sample was placed in a horizontal platinum resistance furnace and heated to 1100° C. At convenient temperature intervals its resistance was measured with a decade testing set. The temperatures were read by means of a platinum resistance thermometer and Whipple indicator.

The test samples were made as follows: Two nickel wires of sufficient length to reach to the center of the furnace were joined at one end by means of the insulator to be tested. In the case of cæmentium and sodium silicate, it was only necessary to clamp the two wires 2 or 3 mm. apart and apply the insulator for a distance of about 2 cm. After twenty-four hours the insulator had hardened, thus holding the wires in position. In the case of mica and asbestos, the ends of the wires were separated by a layer of the insulator

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[1] Weimer, Chicago, November 26, 1910.
[2] Phys. Rev., XXX., 2, p. 143, 1910.

from 2 to 4 mm. thick. A thicker layer was then wound around the junction and was held firmly in place by a nickel tie wire. In the case of magnesium oxide, a small quartz tube about 4 mm. in diameter and 5 cm. long was filled with the paste. A nickel wire was wedged into each end so that it was firmly imbedded in the paste.
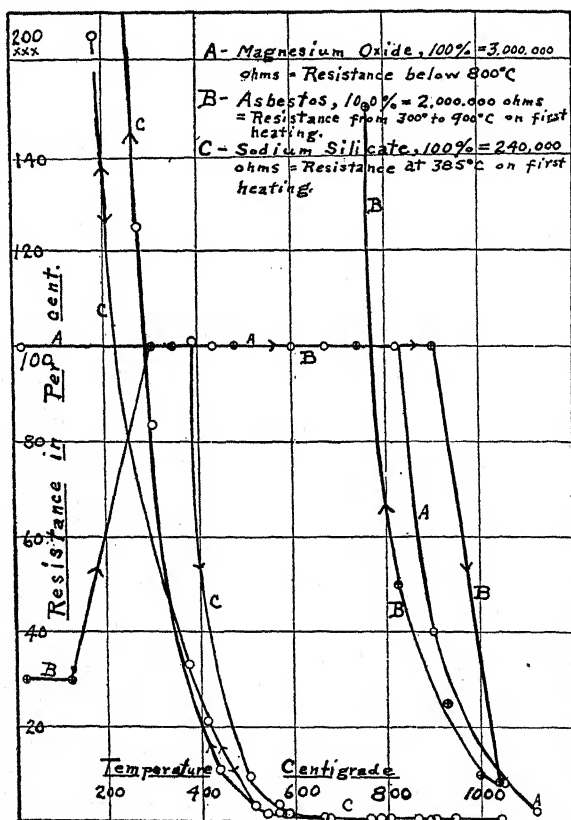
Fig. 1.

Owing to the use to which the insulator was to be put, the only object sought was to compare the resistance at high temperatures with that at room temperature. Hence no attempt was made to prepare samples of uniform size, and the numerical data from which the curves are plotted are meaningless except as showing the behavior of the particular sample from which it was obtained. Hence

all the results have been reduced to percents.   For the work in mind, the most convenient insulator would have been one which at the same time would serve as a cement to hold the coil in shape. Hence more attention was paid to cæmentium than to the other insulators.
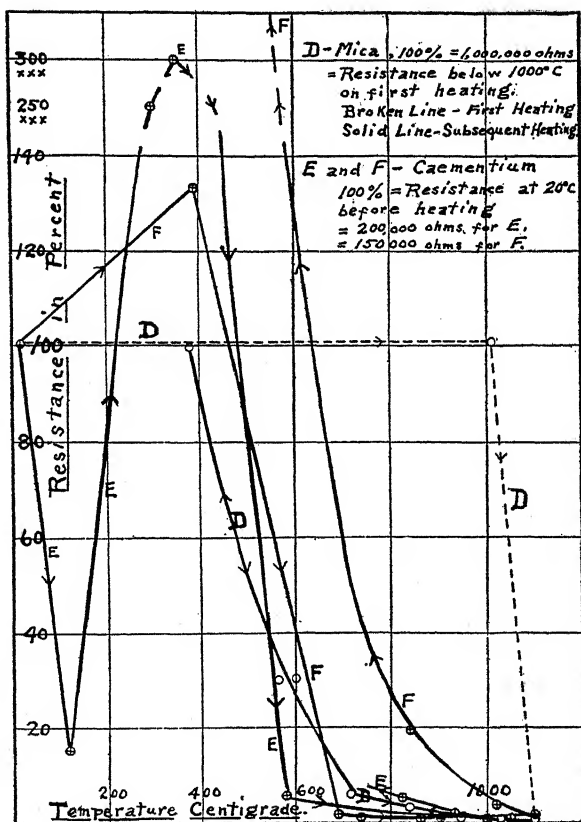
Fig. 2.

The condition of the samples after heating is worthy of notice. The asbestos had lost its fibrous structure to a great extent, and was quite crumbly.   The mica was somewhat deformed, had become opaque, and was quite brittle.   It could easily be crumbled between the fingers.   The cæmentium and sodium silicate were baked hard, much like porcelain.   The magnesium oxide seemed unaltered.

The results are shown graphically in the curves. While the data are too meager to permit of any very general conclusions, we may draw a few inferences.

1. The insulating power of asbestos seems to break down sharply at about 900° C. On cooling, it rapidly recovers its insulating properties.

2. Mica seems to remain an insulator up to about 1000° C. It then breaks down very rapidly with increase of temperature. On cooling, the insulating power is recovered quite slowly down to 700° C., but below this temperature it recovers rapidly. On heating again, it follows the curve of recovery.

3. Magnesium oxide is an excellent insulator up to 800° C. When heated above this temperature, it becomes quite conducting, but recovers its insulating power on cooling.

4. Sodium silicate breaks down very rapidly above 200° or 250° C.

5. The behavior of cæmentium is quite irregular. Apparently heating to 1000° C. improves its insulating properties to some extent, though it cannot be relied upon much above 500° C.

The results in the cases of sodium silicate and cæmentium were to be expected from the fact that glass and porcelain conduct electrolytically at high temperatures.

The author is indebted to Professor A. P. Carman for the facilities for this work.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
December 10, 1910.

THE FLOW OF A GAS IN A CAPILLARY TUBE WHEN BOYLE'S LAW IS NOT OBEYED.

By WILLARD J. FISHER.

THE formula of O. E. Meyer for determining the viscosity coefficients of gases and vapors ceases to be a good approximation when the temperature of the capillary is below or near the critical temperature of the substance, as it assumes the law of Boyle to be obeyed. The writer's attention was drawn to this as a result of observations made in the summer of 1910 on the viscosity of nitrous oxide. The apparatus used[1] was adapted to the measurement of the volume and the pressure at each end of the capillary, and it was found that there was a difference of some per cent. between the product pv for the inlet and the same product for the exit end of the tube, the latter being larger. This note treats the approximate theory of the gas flow when Boyle's law is not obeyed. For valuable suggestions regarding this solution the writer's thanks are due to Professor F. R. Sharpe, of the Mathematical Department of Cornell University.

Making the assumptions of O. E. Meyer regarding cylindrical flow, we arrive at the equation

$$u = \frac{1}{4\eta}(r^2 - a^2 - 2\zeta a)\frac{\partial p}{\partial x} \qquad (1)$$

for the velocity u parallel to the axis at a distance r from the axis. This is independent of the gas law assumed.

The results of Amagat and others show that the curvature of a (pv, p) isothermal for short distances is generally very small. For the moderate pressure differences involved in transpiration experiments we may assume that pv, or p/ρ, is linear in p, $i.\ e.$,

$$p/\rho = \alpha p + \beta. \qquad (2)$$

[1] PHYS. REV., 28, p. 73, 1909.

This amounts to using a tangent to the curve at the mean pressure of observation, instead of the curve itself; then α is the slope and β the y-intercept of the tangent. The use of the Van der Waals equation of the curve itself renders the necessary integrations very difficult.

Since the flow is cylindrical, ρu is independent of x. Multiplying the values of ρ and u together,

$$\rho u = \frac{r^2 - a^2 - 2\zeta a}{4\eta} \frac{p}{ap + \beta} \frac{\partial p}{\partial x}, \tag{3}$$

in which x is distance parallel to the axis. This may be directly integrated between the limits o and l for x, p_1 and p_2 for p, giving

$$\rho u = \frac{r^2 - a^2 - 2\zeta a}{4\eta \alpha^2 l} \left[\alpha(p_2 - p_1) - \beta \log \frac{\alpha p_2 + \beta}{\alpha p_1 + \beta} \right], \tag{4}$$

and this again integrated over a cross-section of the tube, giving the mass of gas passing per second, or

$$M = \frac{\pi a^4}{8\eta \alpha^2 l} \left(1 + 4 \frac{\zeta}{a} \right) \left[\alpha(p_1 - p_2) + \beta \log \frac{\alpha p_2 + \beta}{\alpha p_1 + \beta} \right]. \tag{5}$$

α and β are functions of the temperature of the capillary, and are to be determined experimentally. The slip-coefficient ζ is assumed constant; it undoubtedly varies more nearly inversely as the pressure, or $\zeta = k/p$; but this relation is hardly of importance, as errors in the value of ζ can hardly affect the result as much as other experimental errors. If ζ is computed for the mean of entrance and exit pressures the formula will be sufficiently accurate.

The method of calibrating a capillary tube for transpiration experiments, as described in a previous article, remains unchanged in principle, though some of the constants are altered. Equation (11),[1] becomes

$$\eta' = \frac{(v'' - v')^2 \left[\alpha(p_1 - p_2) + \beta \log \frac{\alpha p_2 + \beta}{\alpha p_1 + \beta} \right]}{8\pi^2 \alpha M \int_0^l \lambda^2 dl \left[\int_{v'}^{v'''} \frac{dl}{\lambda} \right]^2}, \tag{6}$$

in which pF is replaced by M, and the slip correction is neglected.

[1] Phys. Rev., 28, p. 91, 1909.

Equation (3) may also be integrated in another way, which has the advantage of showing the relation to the equation of O. E. Meyer. Solving (2) for ρ and expanding by the binomial formula, we obtain a series which is convergent for $|\alpha p/\beta| < 1$. Integrating term by term, we obtain (4) in the form

$$\rho u = \frac{1}{4\eta l\beta}(r^2 - a^2 - 2\zeta a) \times$$
$$\left[\frac{p_2^2 - p_1^2}{2} - \frac{\alpha}{\beta}\frac{p_2^3 - p_1^3}{3} + \frac{\alpha^2}{\beta^2}\frac{p_2^4 - p_1^4}{4} \cdots\right] \tag{7}$$

and (5) in the form

$$M = \frac{\pi a^4}{8\eta l\beta}\left(1 + \frac{4\zeta}{a}\right)\left[\frac{p_1^2 - p_2^2}{2} - \frac{\alpha}{\beta}\frac{p_1^3 - p_2^3}{3} + \frac{\alpha^2}{\beta^2}\frac{p_1^4 - p_2^4}{4} \cdots\right]. \tag{8}$$

Neglecting all but the first term in square brackets gives O. E Meyer's equation; the other terms may be regarded as corrections, depending upon the value of α/β. Taking only one of these correction terms, we obtain as an approximate formula

$$\eta = \frac{\pi a^4}{8Ml\beta}\left(1 + \frac{4\zeta}{a}\right)\left[\frac{p_1^2 - p_2^2}{2} - \frac{\alpha}{\beta}\frac{p_1^3 - p_2^3}{3}\right]. \tag{9}$$

Caution must be used in applying this formula to vapors in the neighborhood of liquefaction, as in reducing observations like those of P. Breitenbach on methyl chloride, or of A. Bestelmeyer on nitrogen at the temperature of liquid air. Equation (5) is more properly used under such circumstances, or more correction terms of (9). For high temperatures the importance of the correction terms diminishes, as the deviations from Boyle's law are then less. It is to be noted also that equation (8) gives a convergent result only if $|\alpha p_1/\beta| < 1$.

As the complexity of the molecule increases the deviations from Boyle's law become in general larger, and also the specific heat ratio C_p/C_v approaches 1. It is possible that the effect of neglecting the deviations from Boyle's law may be partly responsible for the curious apparent relation between viscosity-temperature coefficients and specific heat to which the writer has called attention.[1]

[1] PHYS. REV., 29, p. 147, 1909.

The writer regards it as not improbable that on the curve-sheets which he has plotted[1] with $\theta^{\frac{3}{2}}/\eta$ and θ as coördinates, for gases of high molecular weight and low specific heat ratio all the curves are thrown to the right from their true positions and given an increased slope, owing to neglect of the deviations from Boyle's law in computation; while for gases of low molecular weight and high specific heat ratio the curves are probably thrown to the left by non-isothermal expansion.[2] But neither of these conclusions can at present be regarded as final.

CORNELL UNIVERSITY PHYSICAL LABORATORY,
January, 1911.

[1] PHYS. REV., 24, pp. 395, 400, 1907; 28, p. 105, 1909.
[2] PHYS. REV., 30, p. 269, 1910.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE FIFTY-SIXTH MEETING.

A REGULAR meeting of the Physical Society was held in Fayerweather Hall, Columbia University, New York City, Saturday, March 4, 1911, President Magie occupying the chair.

The following papers were presented:

The Determination of Melting Points at High Temperatures.   C. W. KANOLT.

The Pressure Shift of the Arc and Spark Lines of Titanium.   HENRY G. GALE and WALTER S. ADAMS.

The Effect of Pressure upon Electric Furnace Spectra.   ARTHUR S. KING.

A New Spectrum of Hydrogen.   THEODORE LYMAN.

The Relation between the Fluorescence Spectrum and the Absorption Spectrum of Uranyl-Potassium Sulphate.   E. L. NICHOLS and ERNEST MERRITT.

The Selective Reflection of Phosphates.   GRACE LANGFORD.

Entladungsstrahlen in Different Gases and in Air at Low Pressures. ELIZABETH R. LAIRD

Bjerknes' Hydrodynamic Analogy to Fields of Force.   J. G. COFFIN.

The Selective Reflection of Salts of Chromium Acids.   HERBERT A. CLARK.

Kinetic Buoyancy in Sound Waves.   BERGEN DAVIS.

Artificial Rotatory Polarization.   (By title.)   A. W. EWELL.

Further Data on Water of Crystallization.   (By title.)   W. W. COBLENTZ.

ERNEST MERRITT,
Secretary.

THE PRESSURE SHIFT OF THE ARC AND SPARK LINES OF TITANIUM.[1]

BY HENRY G. GALE AND WALTER S. ADAMS.

IN examining the spectrum of the sun's limb Mr. Adams has found that the enchanced lines (lines stronger in the spark than in the arc) are shifted roward the red, presumably by pressure, markedly more than the arc lines. The investigation here described was undertaken in the Laboratory of the Mt. Wilson Solar Obervatory to see whether or not laboratory plates would show a greater pressure shift for the enhanced lines than for the arc lines.

A considerable number of photographs of the spectrum of a powerful spark under pressures varying up to 300 pounds were taken in the Pasadena Laboratory by us last spring. These were supplemented by a number of arc plates of titanium in the blue and violet, taken in September. The spectra were photographed in the second order of the 30-foot vertical Littrow spectrograph, provided with a 7-inch Michelson grating of excellent quality.

The measurements of the arc and spark plates of titanium in the blue and violet have yielded the following results.

At a pressure of 120 pounds the average displacement of 97 arc lines in the arc spectrum between λ 3904 and λ 4629 is a 0.024 Angström. The displacement of 20 enhanced lines on the same plates is 0.035 Angström. The ratio is $E_a/A_a = 1.46$. Between λ 4287 and λ 4629 (the region covered by the spark plates) 56 arc lines have a mean displacement of 0.027 and 12 enhanced lines a mean displacement of 0.036. Here the ratio is $E_a/A_a = 1.33$.

On the plates taken with the spark between λ 4290 and λ 4572 the mean displacement of 28 arc lines is .033 and the mean displacement of 11 enhanced lines on the same plates is .052. Here the ratio is $E_s/A_s = 1.58$.

The measurements, especially on the enhanced lines in the spark spectrum, are difficult on account of the character of the lines and different lines have widely different displacements, but the number of plates measured, and the number of lines included in our lists make us feel confident of the order of magnitude of the result. We believe that at moderate pressures and in titanium at least, the enhanced lines lines are shifted about 1.5 times as much as the arc lines.

The measurements also appear to show that in most cases the same line is shifted more in the spark than in the arc by the same pressure. The mean displacement of 28 arc lines in the arc is 0.027 and the mean dis-

placement of the same lines in the spark is 0.033. The ratio is $A_s/A_a =$ 1.22. The mean of the ratios taken line by line is 1.22 ± 0.04. The mean displacement of 11 enhanced lines in the arc is 0.036 and the mean displacement of the same lines in the spark is 0.052. The ratio is 1.44. The mean ratio taken line by line is 1.54 ± 0.06. The mean of the ratios undoubtedly is more significant than the ratio of the means where so few lines are used, and where the actual displacements are so different for different lines.

The temperature of the arc (8 to 10 amperes) was probably not far from 3800° A. In some unpublished work done by us on Mount Wilson several years ago it was found that the relative intensity of solar lines and spark lines agreed remarkably well for a number of elements including titanium. We cannot be greatly in error in assuming the temperature of the reversing layer in the sum to be about 5800° A. If we assume this to be the temperature of the powerful spark used there may perhaps be some significance in the fact that $5800/3800 = 1.53$ and $E_s/E_a = 1.54 \pm 0.06$. Moreover,

$$\sqrt{\frac{5800}{3800}} = 1.24 \quad \text{and} \quad \frac{A_s}{A_a} = 1.22 \pm 0.04.$$

If future work with the arc spark and electric furnace should verify this apparent relationship we should have the interesting result that the pressure shift of enhanced lines is proportional to the temperature, and the pressure shift of arc lines in proportional to the square root of the temperature of the source.

It is evident if this relationship holds that the ratio of the shift of the enhanced lines to the shift of the arc lines should be a function of the temperature, for if

$$\frac{E_s}{E_a} = \frac{T_s}{T_a}$$

and if

$$\frac{A_s}{A_a} = \sqrt{\frac{T_s}{T_a}}$$

it follows that

$$\frac{\dfrac{E_s}{E_a}}{\dfrac{A_s}{A_a}} = \frac{\dfrac{E_s}{A_s}}{\dfrac{E_a}{A_a}} = \sqrt{\frac{T_s}{T_a}}.$$

For the same region of wave lengths we had

$$\frac{\dfrac{E_s}{A_s}}{\dfrac{E_a}{A_a}} = \frac{1.58}{1.33} = 1.19.$$

The ratio is perhaps sufficiently near to $\sqrt{5800/3800} = 1.24$ to be significant, especially when we recall that a comparatively few enhanced lines are included.

Also $\sqrt{E_s/E_a}$ should equal A_s/A_a. The first ratio is equal to $\sqrt{1.54} = 124$ and the second is 1.22, a very satisfactory agreement. This comparison is of course independent of the values assumed for T_s and T_a.

This hypothesis of the variation of pressure displacements with temperature is suggested in a purely tentative way as a possible explanation of the differences in pressure displacement observed between lines in the arc spectrum and the same lines in the spark spectrum of titanium. The apparent differences of displacement of the arc lines in the two sources require strong confirmation. The fact, however, that our results for lines measured in the arc spectrum agree closely with those of Humphreys as well as the fact that some lines show the same displacement in arc and spark, seems to eliminate the probability of the occurrence of systematic errors in the results.

It is perhaps hardly probable that the relation between pressure shift and temperature is as simple as here suggested, but it does not seem improbable that the simple relation suggested above may represent the facts for the average of groups of lines to a first degree of approximation.

We are indebted to Miss Lasby and to Miss Burwell for measurements of many of the plates.

The Effect of Pressure upon Electric Furnace Spectra.[1]

By Arthur S. King.

THE electric resistance furnace in the Pasadena laboratory was used to observe the effect of pressure upon spectra. A graphite resistance tube of $\frac{1}{2}$ inch inside diameter and 12 inches long, 9 inches being heated, contained the metal to be vaporized. The compressed gas was carbon dioxide at pressures ranging from 6 to 15 atmospheres. Most of the observations thus far have been for a total pressure of 9 atmospheres, to obtain the general effects given by the furnace and to compare these with the results for the arc under pressure.

The furnace has proved very efficient in showing the shift of spectrum lines, which are of good quality for measurement. The shifts are toward the red and of very different magnitude for different lines. The relative displacement of lines in the iron spectrum follows very closely that observed for the arc under pressure. The magnitude of the displacement, however, is much larger for the furnace than for the arc under the same

pressure, the conditions of the furnace as normally used giving concordant values about 100 per cent. larger than those obtained by Duffield for the arc under about the same pressure, conclusive evidence of the reality of the effect being given by the fact that groups of iron lines are displaced in the ratio of 1:2:4 which was found for the same lines in the arc.

It was found possible to obtain different shifts of furnace lines for the same pressure by varying the conditions of the luminous vapor. These changes, as well as the general difference between the furnace and the arc have not as yet been traced either to difference of temperature or of vapor density; although an increase of temperature was found to bring the shifts nearer those of the arc. The luminous vapor in the furnace appears to offer less resistance to the displacing influence of pressure than is offered by the vapor in the arc. A fundamental difference in the two sources is found in the fact that in the furnace the compressed gas is heated to the same temperature as the particles of radiating metallic vapor, which cannot be true to the same degree for the discharge conditions of the arc. It is hoped that further investigations will explain the differences observed. These seem to preclude basing any direct comparison of the pressure in different light sources on the shift of spectrum lines, especially for low pressures. Measurements and detailed discussion of the effects will be published in an early number of the *Astrophysical Journal.*

MOUNT WILSON SOLAR OBSERVATORY,
    February 27, 1911.

SPECTRAL LUMINOSITY CURVES OBTAINED BY THE EQUALITY OF BRIGHTNESS AND FLICKER PHOTOMETERS.[1]

BY HERBERT E. IVES.

THE photometry of lights of different colors is complicated by physiological and psychological factors. These enter to different degrees in the various methods of photometry which have been employed for such comparisons. The most accurate methods available are those of equality of brightness and flicker photometry. In the latter the psychological element of estimation is eliminated, but ignorance of the behavior of the flicker photometer under various conditions has delayed its general use.

The present investigation consists of a comparison of the two methods of photometry under a wide range of conditions. Spectral luminosity curves have been obtained by comparing the brightness of the prism face of a spectometer with the illumination of a white sector illuminated

[1] Abstract of a paper presented at the New York meeting of the Physical Society, October 14, 1910,

by a standard incandescent lamp.   The sector in rotation forms a flicker photometer.   Curves have been obtained for a large range of illuminations and three sizes of photometric field.   It has been found that:

1. The sensibility and reproducibility of the flicker method are much greater than the method by equality of brightness.

2. With decrease of illumination the maximum of luminosity shifts toward the blue with the method of equality of brightness (Purkinje effect); toward the red with the flicker method.

3. With decreased size of field the maximum of luminosity shifts toward the red for the equality of brightness method (yellow spot effect); toward the blue for the flicker method.

4. At low illuminations the two methods give results considerably different, but at high illuminations the spectral luminosity curves closely approach each other.

5. Measurements made at two illuminations (high and low) by five different observers of normal color vision show marked differences in the position of both kinds of curves; all show the opposite shift of luminosity maximum by the two methods with change of illumination; different relative positions of the two curves at high illuminations are found with each observer but the mean high illumination curves by each method closely agree, supporting the conclusion that the methods approach equivalence at high illuminations for an average eye.

A mean luminosity curve is obtained and reduced to a normal spectrum. Its maximum lies at $.545\,\mu$.

Conclusions are drawn that the standard condition for heterochromatic photometry should be a high illumination permitting the use of the more accurate flicker photometer.   Further investigations on problems arising in this connection are mentioned.

The Selective Reflection of Salts of Chromium Acids.[1]

By Herbert A. Clark.

THE approximate linear relation, in the case of certain inorganic salts, between the atomic weight of the element in the acid radical combined with O_3, and the wave-length of the reflection maximum in the infrared spectrum, as pointed out by Morse,[2] seemed worth testing in the case of other salts.   In the present investigation, reflection curves for wave-lengths from 1.5μ to 15.0 or 16.0μ, were determined for all the available salts of the chromium acids.   Since many of these salts occur

[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 4, 1911.

[2] Astrophys. Jour., Vol. 26, p. 242 (1907).

as very fine crystals or as powders, each surface was made by grinding the salt, then compressing it into a cake under a pressure of about 40 tons to the square inch, and polishing one face.

The available chromites and chromates show single complex reflection maxima in the neighborhood of 15.0μ and 11.5μ, respectively. The dichromates show two complex reflection maxima, near 11.0μ and 13.5μ, respectively. The approximate relation of Morse holds in most cases for the salts of any one acid. Of these salts, one (potassium chromate) has a maximum so sharp and intense that it might be used as a source of monochromatic energy.

The Determination of Melting Points at High Temperatures.[1]

By C. W. Kanolt.

THE melting points of a number of very refractory substances, including magnesia and alumina, have been determined by means of heating curves or cooling curves obtained with a Holborn-Kurlbaum optical pyrometer. The substances were heated in an Arsem graphite resistance furnace, usually in a vacuum.

The pyrometer was sighted through a glass window in the top of the furnace. A correction was applied for the reflection and absorption of the glass. This correction was small and could be determined accurately.

The substance to be melted was placed in a crucible of graphite, tungsten, platinum, magnesia or kaolin, with a cover provided with a hole about three millimeters in diameter. The pyrometer was sighted into the inside of the crucible through this hole, thus obtaining black body conditions. In case the substance was likely to be contaminated by the carbon or other substances in the furnace, it was placed within a tube of magnesia or kaolin, which was connected by a nearly air-tight joint to the top of the furnace in such a way that the pyrometer could be sighted down the tube. Sometimes two concentric tubes were used. In case it was necessary to avoid any trace of the carbon monoxide produced from the graphite heater, a feeble current of air was admitted to the magnesia or kaolin tube, and allowed to pass out through the porous walls of the tube into the furnace, the pressure in the furnace being kept down to two or three millimeters of mercury by running the vacuum pump continually.

Usually about five grams of material was employed, although good results have been obtained with one half gram. Successive determinations upon the same substance usually agreed within five degrees. As a check, the method has been applied to a number of those substances whose melting

points are most accurately known, and the results obtained are in close agreement with the accepted values.

It is proposed to apply the method to the determination of the melting points of a number of refractory oxides and other substances in the pure condition.

Bureau of Standards.

Further Data on Water of Crystallization.[1]

By W. W. Coblentz.

THE present investigation was undertaken to bridge over the gap between 7 per cent. and 2 per cent. of water, in minerals, to determine whether there is a sudden change in the absorption spectrum with decrease in the water content. Opal was examined.

On dehydrating, through successive stages, from 3.6 per cent. to less than 1 per cent. of water, it was found that the absorption bands of water gradually decrease in intensity.

Since the tremolite, previously examined, contained 2 per cent. of water but showed no absorption bands, it is to be inferred that the water is present as "water of constitution."

[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 4, 1911.

THE

PHYSICAL REVIEW.

ON THE FREE VIBRATIONS OF A LECHER SYSTEM, USING A BLONDLOT OSCILLATOR.[1]

BY F. C. BLAKE AND EMMA A. RUPPERSBERG.

INTRODUCTION.

THE distribution of energy along Lecher wires in air was first quantitatively studied by Rubens.[1a] By means of the bolometer he showed that the wave system set up in two parallel wires under certain conditions consists of the free vibrations of those wires. Later Apt[2] studied the influence of the exciter upon the form and intensity of the electrical vibrations of the Lecher system. Among other things he showed that the lead wires from the induction coil to the oscillator played an important part. He obtained the strongest effects when complete resonance existed among the four circuits of the system concerned: viz., secondary binding posts of induction coil to oscillator, oscillator to primary Lecher plates, secondary Lecher plates to bridge of Lecher system, bridge to end Leyden jars. Apt states, moreover, that the intensity of the waves of a given length set up in the parallel wires depends upon the form of the oscillator and the conditions controlling the production of the primary spark. He discusses the relative merits of the two forms of oscillators, the Lecher and Blondlot types, and comes to certain conclusions to which Drude[3] has taken exception. In stating his objections Drude discusses the mode of action of the Lecher

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[1] Read at the Chicago meeting of the American Physical Society, Nov. 26, 1910.
[1a] Rubens, Wied. Ann., 42, p. 154, 1891.
[2] Apt, Wied. Ann., 61, p. 293, 1897.
[3] Drude, Wied. Ann., 61, p. 631, 1897.

system and shows that the character of the wave system is far more important than the production of a large potential difference in the first loop behind (*i. e.*, toward the bolometer end of the Lecher system) the fixed bridge. In other words, he says that system of waves is best and most characteristic which makes the ratio of intensities for a loop and for a node the greatest possible. Drude was, of course, aware that not all induction coils work equally well in the production of wire-waves, a thing many experimenters have found. He showed, however, that if one connects the wires which lead from the induction coil to the vibrator as close as possible to the spark-gap he then becomes in some measure free from the effects of the induction coil upon the wave-system in the parallel wires.

Coolidge[1] modified the original Blondlot oscillator by inserting a sheet of mica between the primary and secondary turns, thereby enabling him to obtain very much stronger oscillations by reason of the greater induction. He was disturbed, however, he says, by the presence of strong mechanical vibrations in the secondary if an influence machine was used as a source of power. He tried to get rid of this disturbance by introducing a second spark-gap in oil, but did not succeed.

The experiments described in this paper were undertaken in an endeavor to study the wave system in the Lecher wires using a Blondlot oscillator as exciter and a Rubens bolometer as detector. To be as free as possible from the influence of the induction coil upon the oscillator it was thought wise to introduce two side sparks in air after the manner of the Righi vibrator. This has of course the advantage of symmetry over the Drude-Coolidge single side-spark in oil.

One cannot study the literature on this subject of wire waves without wondering whether the different experimenters[2] were always sure in their own minds of the character of the oscillations with which they were working. We find it difficult, for instance, to tell in certain cases from the published accounts whether the oscillator was forcing its own system of vibrations upon the parallel wires or whether they were swinging freely. It seems best to defer

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[1] Coolidge, Wied. Ann., 67, p. 578, 1899; 69, p. 125, 1899. See also Hormell, Phil. Mag., 3, p. 52, 1902, or Am. Journ. Sci., 12, p. 433, 1901.

[2] Coolidge, *l. c.*

a critical discussion of this and other points however until after our own experimental results are presented. As the experimental development may be of interest the results are presented in the order in which they were obtained.

DESCRIPTION OF APPARATUS.

The Blondlot oscillator (Fig. 1) was a modification of the form used by Coolidge and by Hormell.[1] The primary consisted of two semi-circular brass wires PP' (diameter 2.08 mm.) enclosing a circle of 4.8 cm. diameter, with an opening of 2 mm. at k and a spark-gap at a. The balls bb' were of brass of 6.36 mm. diameter with a
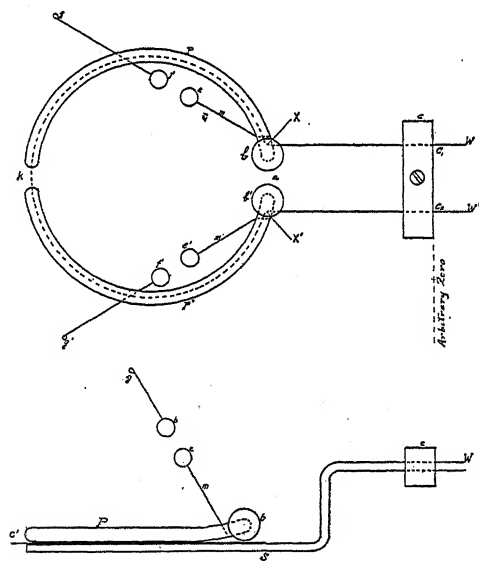
Fig. 1.

hole bored in each radially and made to fit the ground ends of the primary wires PP' very tightly. In this way each ball could be turned on its diameter as an axis, thus leaving a new surface to be exposed whenever necessary and yet remaining immovable when the spark was passing, as well as retaining a constant position relative to the secondary circuit after being turned. Before any one curve was taken a new portion of the balls was turned toward

[1] Hormell, *l. c.*

the spark-gap. For such a spherical ball spark-gap the length of the gap rapidly increases at first as the discharge passes and tears off the brass and hence at first the oscillator deterioration is more rapid than after the oscillator has been in use a short while. Accordingly, to avoid this rapid initial deterioration when a new portion of the brass was exposed the circuit was always closed at least twenty times before readings were commenced. After that the oscillator deterioration was quite slow and uniform and could readily be taken into account. This was done for all the curves except where specified by choosing some position on the Lecher wires as a check point at which every sixth reading was taken.

The air-gap balls ee' and ff' were smaller in size, being 4.2 mm. in diameter, ee' being attached to brass wires mm' 14 mm. long and 1.3 mm. diameter. Following Drude's suggestion of partly freeing the vibrator from the influence of the induction coil, the wires mm' were riveted into PP' just as near the balls bb' as possible. The balls ff' were soldered to wires similar to mm' whose other ends gg' were looped for attaching the copper lead wires from the secondary of the induction coil.

The secondary circuit SS' of the oscillator consisted of brass wire of 1.4 mm. diameter and total length c_1kc_2 of 29 cm. up to the point where the meter stick readings began, that is, the point where the Lecher wires proper WW' were soldered, the whole being supported at the junction by a clamp of soft wood. This secondary circuit had the same diameter as the primary, forming a closed circle with the exception of 1.5 cm. just below the spark-gap of the primary. It was firmly cemented to the bottom of a shallow glass vessel by means of water-glass and was separated from the primary circuit by a sheet of mica 0.016 mm. thick. By means of a wooden support pressure could be applied from above and thus hold the two circuits rigidly touching the mica, the wires PP' being bent slightly upward near the balls to permit this. A small support equal in thickness to the secondary wire of the oscillator had to be provided for the mica sheet at the center to prevent its bending under the applied pressure. The glass vessel was supplied with a running stream of kerosene oil. To prevent oil splashing up on the air-gaps when the discharge passed they were protected by glass tubes.

The Lecher system consisted of copper wires 0.67 mm. in diameter which were kept scrupulously bright and polished and at a fixed distance of 2 cm. apart.   Their free ends were passed through a support of soft wood 2 cm. thick against which the small Leyden jars pressed.   These jars were made of glass tubes 2.9 cm. long of inner bore slightly greater than the diameter of the Lecher wires; their outer diameter was 7 mm.   The wires leading from the jars to the bolometer were wrapped as a single turn around the jars at their center.   The parallel wires were kept under a constant but unknown tension during the early part of the work.   Later a known tension of 2,500 grams was used.   A preliminary experiment showed that this tension could not be reduced much below 2 kilograms and have the readings constant for a fixed position of the bridge.   Of course, in applying it all metal had to be avoided in the neighborhood of the Lecher wires, so it was done by means of a strong fishline working over pulleys.   The bridges were all of the same diameter as the Lecher wires.   They were 3 cm. long and bent downward at the ends at right angles to their length.   To insure good contact they were weighted at the middle by a cord and plumb-bob.

As a source of power a 110-volt, 60-cycle alternating lighting circuit was used supplying by means of a control rheostat about 8 amperes to the primary of the 8-inch induction coil used.   Except when expressly stated otherwise the coil was run directly on this circuit without condenser or interrupter.   Two coils of about equal size (4-inch) were available and when for any reason one did not suffice the two were used.   In this paper the two together will often be spoken of as a single coil except where possible confusion might result.

The bolometer used was very similar to the Paalzow-Rubens'[1] instrument, the quadrilaterals being constructed of iron wire of diameter 0.0208 mm.   Following Rubens and Ritter[2] its sensitiveness was measured as 0.0001934° C.   The usual precautions were taken against temperature variation by enclosing it entirely in a soft wooden box filled with cotton packing.

[1] Wied. Ann., 37, p. 529, 1890.
[2] Wied. Ann., 40, p. 55, 1890.

The galvanometer was a Leeds and Northrup four-coil Thomson instrument with a resistance of 7.78 ohms in parallel, and a sensitiveness of 1.55×10^{-8} amperes for a complete swing of 4.4 seconds.

It was found by trial that when the oscillator was set into action and the galvanometer was not shunted a rather large deflection (in some cases as high as 50 scale divisions) could be obtained when the wires leading to the bolometer from the Lecher system were removed. Further testing showed that the effect was not due to any direct action of the induction coil on the galvanometer but instead to diffuse radiation influencing the bolometer. As the walls of the room in which the work was carried out were constructed of metal lathing unusual care had to be exercised to be free from extraneous disturbances. By enclosing the bolometer with its accessories in a metal box the effect was considerably diminished. There was still some effect present when the bolometer was fully enclosed and it was located finally as due to stray radiation running back along the wires between bolometer and galvanometer, thus upsetting the bolometer balance and producing a throw. This was got rid of by wrapping these wires with tinfoil and earthing it together with the galvanometer case. Of course, for actual work there had to be an opening in the bolometer box for the wires from the Leyden jars of the Lecher system. As soon as these wires projected even a few inches above the box a large galvanometer throw was obtained when the oscillator was running. It was at once evident that the wires leading from the Lecher system to the bolometer had to be protected from the stray radiation. Trial showed that 10-inch furnace pipe allowed about 95 per cent. of the energy to travel along the wires if they were held in place along the center of the pipe by wooden supports, so this was done. By means of elbow joints in the pipe all the wire except about a foot immediately above the Leyden jars was thus protected. Not to be disturbed by capacity effects we deemed it unwise to run the protection piping any closer. We express it as our belief that a part of the deflection shown at the minima of the best curves shown below is due to stray radiation. We are not able to say, however, how much of the deflection at such minima is legitimate, that is, due to the energy following the Lecher system on to the bolometer wires, and how

much is extraneous.   Accordingly we were unable wholly to correct for this error though we feel sure it was in all cases well within 5 mm.

<div align="center">RESULTS.</div>

In one of the early experiments using an oscillator somewhat larger than that described above and employing a Wehnelt interrupter attached to a single induction coil an attempt was made to obtain a system of nodes and loops on the Lecher wires in the case where the wires leading from the coil were metallically connected to the oscillator close to the spark-gap.   How successful the attempt was is shown in curve *I*, Fig. 2.   Here the Lecher wires were about 175 cm. long measured from the arbitrary point near the oscillator.   The wire system was searched by means of a single bridge with the detecting Leyden jars at the ends.   Plainly there is present nothing marked, although the galvanometer deflection without a bridge on was 200 divisions.   Measurements were not continued beyond 104 cm. for it was evident that nothing characteristic would develop.   Then a side-spark in air was introduced on either side of the oscillator as described above, and, still using the Wehnelt interrupter, Curve *II* of the same figure was obtained.   Each point shown is the mean of from two to three readings that agreed fairly well (*e. g.*, 176, 166, 170) with each other.
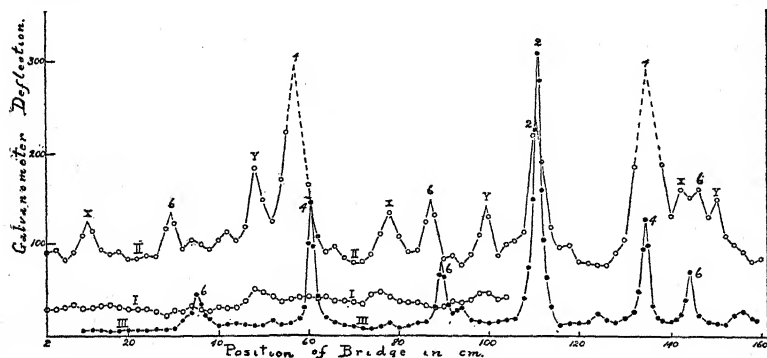
Fig. 2.

The two highest peaks are drawn partly dotted as the actual readings were off the scale.   There can be no question as to their location, however, for enough of the slope for each was obtained to make their position approximatey correct.

Now it is well known in such work as this that the employment of a second and third bridge will enable one to determine what nodes respond to each other, and so other bridges were tried. This process revealed response between those nodes marked with the same number or letter. It was tried in the following way. The first maximum of a given type nearest the oscillator was carefully redetermined using smaller steps than 2 cm. and taking more readings for a given position; the true location of the maximum was thereby determined. Then a second bridge was similarly used to explore the wires further along and its correct position having been found a third bridge was used. Without exception in every case in which this was done it was noted that by employing a second bridge the second maximum of a given type was displaced slightly toward the oscillator compared with its position when a single bridge was used. This was true also of the third maximum. In other words, the internodal space is slightly shortened by the use of a second bridge. Unfortunately the maxima *Y* of Curve *II* were not tried for responses but there can be little doubt, we think, that they would respond to one another and so we have marked them as belonging to the same system. Further discussion of Curve *II* is temporarily reserved.

As the Wehnelt interrupter requires considerable attention from time to time it was thought worth while, following Blake and Fountain,[1] to use the alternating current direct upon the induction coil, without condenser and interrupter. When this was done, however, instead of giving a good system of nodes and loops such as Curve *II* shows, the nodes practically disappeared and we had Curve *I* over again.[2] Before resorting again to the Wehnelt interrupter, however, it seemed worth while to try a few variations.

Accordingly, following Berg,[3] three changes were made, the second of which proved of far greater importance than either of the other two, though each seemed helpful. First, a condenser was used between the induction coil and the exciter as shown in

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[1] Blake and Fountain, PHYS. REV., XXIII., p. 257, 1906.

[2] The air-gaps were kept the same as for Curve *II*, 5 mm. say. At this stage the importance of knowing them accurately was not appreciated. They were probably between 4 and 6 mm. in length.

[3] Berg, Ann. d. Phys., 15, pp. 317, 318, 1904.

Fig. 3. Only a single condenser was used as shown, whereas Berg used two, one in each lead wire of the secondary coil. The condenser consisted of an ordinary piece of sheet glass with an area
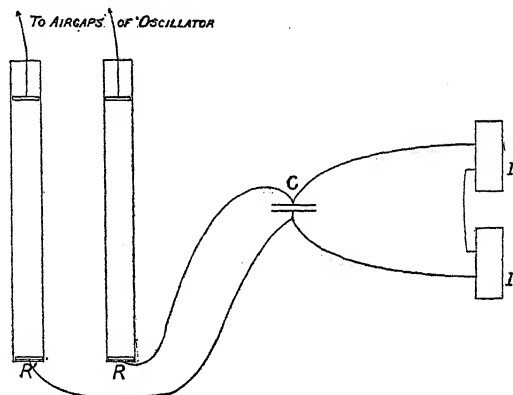
Fig. 3.

of lead foil 28 by 8 cm. in size on each side. Secondly, a variable water resistance was introduced between the condenser and the oscillator as shown in the same figure. In the third place, a metal screen with a circular aperture 18 cm. in diameter was introduced around the Lecher wire system but near the oscillator. The plane of the screen was at right angles to the parallel wires and symmetrically placed with respect to them. Although put in quite a different place from the two screens used by Berg it served the same purpose as his, viz., to lessen the direct action of the oscillator upon the detecting system.

With these changes made the node system was sought for on the parallel wires of length 154 cm. Good nodes were located at 31.2 and 83.5 cm.; the latter by use of a second bridge giving apparently 52.3 for the internodal space. Thinking these nodes[1] due to the oscillator and trying to allow for the end capacity due to the Leyden jars it was figured that the parallel wires should have a length of 160.4 cm. to respond best to the oscillator. Accordingly they were lengthened to 165 cm. since if too long they could be readily shortened. Further experimentation showed at

[1] The evidence was not yet at hand for knowing the relationship between the various "systems of nodes" of Curve *II*, Fig. 2.

once that the nodes shifted with every change in the length of the parallel wires. This meant of course that some of the nodes at least were those due to the free vibrations of the Lecher system and so a thorough search was begun.

Leaving the system 165 cm. long a series of curves was taken whose object was to find the influence of the air-gap length upon the Lecher vibrations. For measuring the gap-lengths a Brown and Sharpe thickness gauge was used. The method was of course not very correct but for the air-gaps at least it answered well enough as small changes in their length apparently did not appreciably affect the vibration system.

A typical curve obtained in this way is shown in Curve *III*, Fig. 2. Here the air-gaps were each 1.4 mm. long, while the oil-gap was 0.1 mm. long. The water resistance was each a column of rain water 30 cm. long and 2.5 cm. in diameter.

Remembering that a Lecher system using a Blondlot oscillator is closed at the vibrator end, we should expect[1] it to vibrate freely as shown in Fig. 4. Interpreted in the light of our detecting system Fig. 4 becomes Fig. 4a.[2] Plainly Curve *III*, Fig. 2, is an
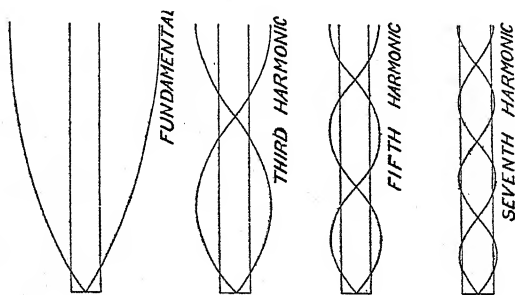
Fig. 4.

approximation to this ideal case. The fundamental tone is, of course, that represented by the galvanometer deflection when no

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[1] See M. Abraham, Theorie der Elektrizitat, Vol. 1, p. 352, second edition.

[2] In Fig. 4, following the current practice in alternating current work the harmonics are denoted as *odd* harmonics. In Figs. 2 and 4a these same harmonics are marked as *even* harmonics. For instance the seventh harmonic of Fig. 4 is marked 6, 6, 6 in Fig. 4a. Unquestionably the latter notation is the only logical one, but for the sake of certain readers it seemed advisable to use here the double notation. Personally we prefer to have the word "harmonic" *exclude* the fundamental tone.

bridge is used on the Lecher system. Using a bridge the highest maximum at 111.3 was 40 per cent. of the reading for no bridge. Further theoretical discussion is reserved till later. The principal maxima of Curve *III* correspond then to the free vibrations of the Lecher system; the first overtone at 111.3, the second overtones at 60.7 and 134.7, the third overtones at 35.3, 89.7 and 144.3.
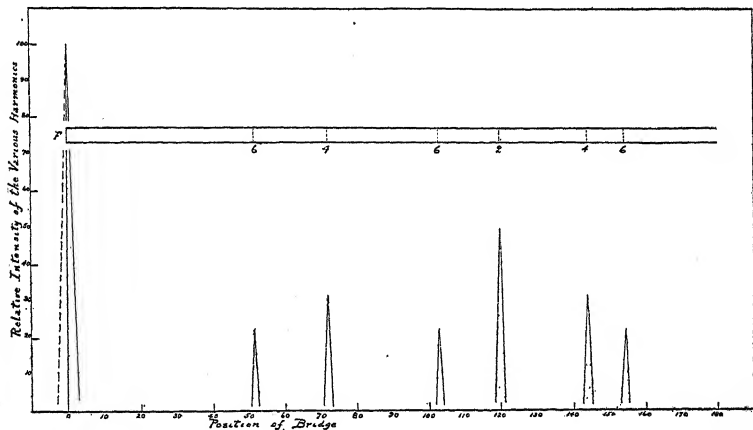
Fig. 4*a*.

Ratio of frequencies 1 : 3 : 5 : 7.
Ratio of intensities 1 : 0.50 : 0.31 : 0.22.

This curve gives us the clue to the explanation of Curve *II*. The numbered maxima in the two curves correspond. In making a comparison it should be borne in mind that the total wire system for Curve *II* was several centimeters longer than for Curve *III*. In *II* undoubtedly the maxima marked XXX and YYY are due to the oscillator. In figuring the internodal space for the X and Y maxima, the work of Blake and Sheard[1] will show that the X and Y maxima at 142 and 150 respectively are influenced by the free overtone marked 6 at 146 and vice versa, and so only the other maxima are used. This gives the half wave length for the X oscillation as 67 cm. and for the Y oscillation 51 cm. An inspection of the figure will show that the Y maximum is some greater in intensity than the X maximum, though not much greater (10 per cent.). It is to be remarked, however, that in general for all the

[1] See note, page 474.

curves shown in this paper there seems to be a general tendency for the maxima quite near the oscillator end to be smaller than the others due to some action not yet clear, and hence since the X maximum at 11 is so much closer to the oscillator than the Y maximum at 48 one can safely say that the X and Y maxima are equal in intensity. These maxima will be spoken of again.

Having ascertained under what conditions practically only the free vibrations of the Lecher system were present it became of interest, as already stated, to ascertain how the wave system was affected by a change in the length of the air-gap. Accordingly, setting the oil-gap length at a certain constant value before each curve was taken and starting with each column of water resistance 30 cm. long, a series of curves was taken with different lengths of air-gap. Seven such curves were taken but to avoid confusion in the figure only four are shown in Fig. 5. It is seen at once that there is not even a slight displacement of any of the maxima as the air-gap is lengthened or shortened. The curves of this figure are recalculated to open circuit on the galvanometer variable shunt. Specifications for the different curves of this paper are collected

TABLE I.

Specification.		Length of Air-gap in mm.	Length of Oil-gap in mm.	Length of Water-resistance in cm.	Condenser (C, Fig. 3).
No. of Fig.	No. of Curve.				
2	I	0	?		
2	II	2 about	?		
2	III	1.53	0.10	30	Inserted.
5	I	1.30	0.10	30	Inserted.
5	II	0.64	0.10	30	Inserted.
5	III	0.31	0.10	30	Inserted.
5	IV	0.13	0.10	30	Inserted.
5	V	0	0.10	30	Inserted.
5	VI	0	0.10	30	Inserted.
8		1.22	0.10	30	Inserted.
11	I	1.22	0.10	0	Removed.
11	II	1.22	0.10	0	Inserted.
11	III	1.22	0.10	30	Removed.

together into Table I. The three curves taken but not shown lie in between the others. Plainly as the air-gap is lessened the size

of a reading for a given bridge position increases up to a certain point, but the ratios between maxima and minima decrease to a marked extent. This means, we think, that more and more energy overflows from the induction coil past the oscillator on to the Lecher wires, thus always giving a fairly large potential difference at the bolometer end of the wires no matter what the bridge position. An explanation of such action is perhaps afforded by thinking of the whole system from the secondary terminals of the induction coil clear through the oscillator to the free end of the Lecher wires as a single swinging system with inductance and capacity. The action with the Blondlot oscillator with primary and secondary very close together and with no air-gaps present can be thought of as similar to that with the Lecher oscillator with the primary and secondary plates very close, when instead of two separately swinging systems there is but one.[1] With but a single system one cannot expect the potential difference at the end capacity to change much as a bridge is moved along over a part of that system. If we have air-gaps present, however, then the short oscillator system may be said to set the longer Lecher system into vibration and as the air-gap is increased in length the more independent does the Lecher system become both of the "slopping-over" of energy from the induction coil and of the forced vibrations of the oscillator.

A study of the curves of Fig. 5 together with Curve *III* of Fig. 2 and two or three others already spoken of but not shown shows the more important maxima to be located at the positions given below. A second bridge showed what maxima responded and although it was found that the use of a second bridge invariably displaced the maximum toward the oscillator end thus shortening the internodal space, still the displacement was always small and hence there can be no question as to what maxima belong together. The strongest maximum occurred at 111.3 cm., the next two at 134.7 and 60.7, the next three at 35.2, 89.7 and 144.3 cm. These figures are accurate to within 2 mm., obtained as the mean of all the curves on the assumption of no displacement due to shortening the air-gaps. It was found experimentally that the deflection for no bridge compared to that for a single bridge at the 111.3 cm.

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[1] See Drude, Wied. Ann., p. 632, 1897.

maximum was 2.5[1] times as great and it became at once apparent that we have here a first approximation to the system shown in Fig. 4*a*. It was evident, however, that it was only an approximation as the following calculation shows. The end of the Lecher wires (flush with the outer end of the small Leyden jars) was 165.0 cm.

Fig. 5.

The internodal space for the maxima at 134.7 and 60.7 is 74.0. Multiplying this by 10 gives for the fundamental wave-length 740 cm. Also, the three other maxima above give 764 cm. for the fundamental wave-length. Furthermore, using these two overtones for calculating the end of the Lecher wire including the end-capacity we obtain 171.6 cm. (*e. g.*, 134.7 + ½ of 74 = 171.7). Subtracting 111.3 from 171.6 gives 60.3. Multiplying by 12 gives 723.6 as the

[1] For all the curves this ratio never differed more than 4 per cent. from 2.5. The mean of all was just 2.50. However when the air-gaps were very small this ratio decreased until for zero air-gap it was only 1.3.

fundamental wave-length, λ_0.   If the ideal case pictured in Figs.
4 and 4a were experimentally obtained these numbers for λ_0 should
be identical.   Plotting these numbers against the corresponding
frequencies gives the curve shown as I, Fig. 6.   Extrapolating the
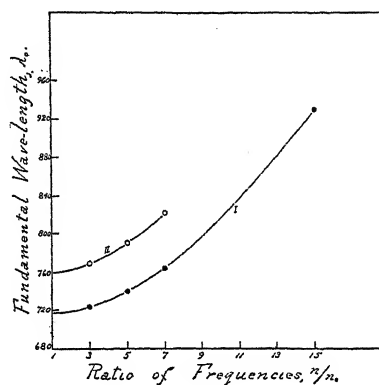
Fig. 6.

curve we obtain for the true fundamental wave-length 713 cm.
Similarly from Curve II of Fig. 2 we obtain Curve II of Fig. 6,
giving 760 as the extrapolated fundamental wave-length.   Assum-
ing the internodal spaces all equal for any given tone of the curves
of Fig. 5 we obtain for the location of the node at the oscillator end
these figures for the sixth, fourth, second harmonics and funda-
mental respectively, $-$ 19.5, $-$ 13.3, $-$ 9.3, $-$ 6.6 cm.   Now the
points XX' of the secondary (Fig. 1) were located at $-$ 6.7 cm.
while the point k (measured along the circumference) had the
position of $-$ 14.5 cm.   This argues strongly that the electrical
vibrations for the fundamental tone at least does not follow along
the circumference of the secondary but jumps across the neck of
the secondary at XX' (Fig. 1).[1]   For this reason it seems as if it
would be better to have the spark-gap of the oscillator at k rather
than at a, although Drude[2] objects to this arrangement.

[1] In another paper by Blake and Sheard it will be shown that the fact that λ_0 ob-
tained from the overtones increases with the frequency is due to the reaction of the
oscillator upon the Lecher system on account of the closeness of the coupling.   When
the coupling is very loose it is found that the overtones are strictly harmonic.   See
note at bottom of page 474.

[2] Drude, *loc. cit.*

An attempt was made to classify the less intense overtones of the curves *I–IV* of Fig. 5 and Curve *III* of Fig. 2, though not with much success perhaps. Secondary maxima common to all these curves occur at 32, 39, 52, 63, 78, 94, 104, 125 and 156 cm. in round numbers. It would seem that the maxima at 32, 63, 94, 125, 156 belong together, for the common internodal space is 31 cm. (= 2 (171.6 − 156)). They probably belong to the seventh overtone (*i. e.*, the fourteenth harmonic modified by the close coupling). On this basis the seventh overtone is to be seen in Curve *I* of Fig. 6. A discussion of the remaining secondary maxima at 39, 52, 78 and 104 cm. cannot be given till later. Curves *V* and *VI* were taken with zero[1] air-gap length and should duplicate each other, a thing they do as to *type* certainly. For all the curves of Fig. 5 the actual observations are given uncorrected for oscillator deterioration and since for zero air-gap the spark was more disruptive than usual the duplication is sufficiently satisfactory, especially when the crudeness of the method for measuring the oil-gap and air-gaps is remembered.

An appreciation of the effect of the air-gap length upon the vibrating system in the Lecher wires can be gained from Fig. 7, obtained indirectly from Fig. 5 by getting the ratio of the first overtone maximum at 111.3 cm. to the mean minimum at 18 to 26 cm. inclusive. We say "indirectly," for in Fig. 7 the oscillator
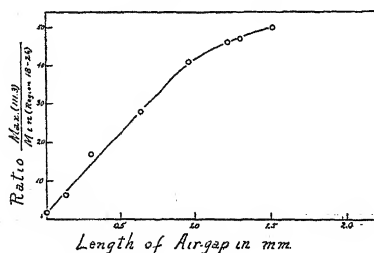
Fig. 7.

deterioration has been corrected for as far as possible. Moreover, the results of curves not shown are included in Fig. 7. For the largest air-gaps obtainable with the induction coils[2] at our disposal

[1] After closing the gaps by pushing the balls *ff'* against *ee'* fine copper wire was soldered across each gap back of the balls.

[2] It should be borne in mind that they were operated without condenser or interrupter.

a ratio of 50 to 1 was obtained. Still larger air-gaps would have given a somewhat larger ratio. The relation between this ratio and the length of air-gap is seen to be a linear one up to 1 mm. length. Beyond that an increase of air-gap length has less effect. By a rough extrapolation one could say that a ratio of 60 to 1 could be obtained for a 3-mm. air-gap.

Since it has been proven that the oscillator was able at times to force some of its own vibrations upon the Lecher system it seemed worth while to try to see if some of the secondary maxima of curves *I–IV* of Fig. 5 might not be attributed to the oscillator. They were too feeble, we thought, to make the method of responses by employing additional bridges seem satisfactory when we knew that each additional bridge displaced its maximum somewhat, the effect being cumulative. It seemed wiser to search for resonance between the oscillator and Lecher system by employing no bridge and shortening the Lecher system. Starting with the length 165 we cut off 2 cm. at a time down to 45 cm., taking three readings of the galvanometer after each cut. The results are shown as circles (O) in Fig. 8. Continuing this work we thought we were bothered with difficulties due to the tension[1] of the Lecher wires changing. Accordingly, in order to apply a large known tension a loop of wire was soldered to each Lecher wire back of the Leyden jar and a cord ran from this loop over a pulley to a scale pan carrying 2.5 kilos. This variation rendered the work of shortening the Lecher system much more tedious and proved unwise in the end. For by reason of the increase in the time necessary to run a curve it probably introduced more error than was eliminated by the assurance of a constant tension. Nevertheless it was persisted in and many curves and parts of curves were taken for longer and shorter Lecher systems. The observations are all plotted in Fig. 8, each new symbol displayed in the figure showing a new wire system that was cut. The curves indicated by the various symbols ought to be compared as to type more particularly, for the actual value of the ordinate for any given abscissa is dependent upon several factors, more particularly oscillator deterioration, over which we had no control; and hence we could not make the curves shown by the

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[1] Up to this time the wires were held taut by wooden plugs driven into a wooden support.

various symbols directly comparable except in a qualitative way. However, each set of symbols, in so far as it goes, shows various maxima and minima, so that taking all the symbols together and averaging, a curve can be drawn about as shown in the figure,
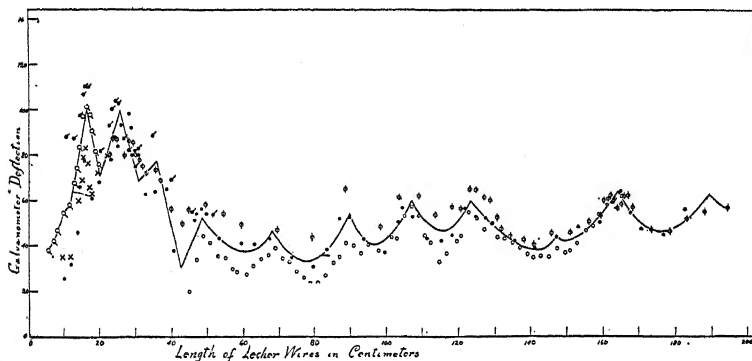
Fig. 8.

each maximum being accurate within 2 cm. at most.   If more faith be put in the circles (O) as having been obtained under the best conditions, one could say that the maxima and minima are about equally sharp, the curve having the maxima purposely somewhat too sharp.   Two strong maxima of equal intensity are seen at 16 and 26, close to the oscillator, with other maxima, but less strong at 49, 68, 90, 107, 123, 164, 189, enough points not having been taken definitely to locate the last one.   Secondary maxima are seen at 10, 36 and 147.

We believe we correctly interpret Fig. 8 as follows: The two maxima at 16 and 26 represent the *two* lengths of the Lecher system that respond to the vibrator because of the closeness of the coupling.[1]   Choosing the maxima 26, 68, 107, 147, and 189 one may call the common interval 41 cm., giving as the wave-length 82 cm.   With the other maxima 16, 49, 90, 123, 164 it is seen that the interval is either 33 or 41.   These figures of 66 and 82 cm. as the two fundamental oscillator wave-lengths are confirmed in two ways.   Their mean, 74.2 cm., calculated[2] from $T_1^2 = T^2 + \theta^2$ and

[1] See A. Oberbeck, Ann. d. Phys., Vol. 55, p. 624, 1895, or Fleming, Principles of Wireless Telegraphy, p. 209, first edition.
[2] See Fleming, *loc. cit.*

$T_2{}^2 = T^2 - \theta^2$ agrees with Coolidge's value of 74 cm. for a Blondlot oscillator of 5 cm. diameter.   Moreover, if one figures back to the point k of the oscillator he obtains the same values for the half wavelengths thus: the wires leading from the Lecher Leyden jars to the bolometer were 1 cm. from the outer end of the jars, an amount to be subtracted from the maximum position.   The distance from the arbitrary zero to the petroleum free surface was 6 cm., while the length of the secondary under the oil up to k (Fig. 1) was 8 cm. We have then, $15 + 6 + 8 \times 1.43$ (index of refraction of petroleum) $= 32.5$.   Similarly, $25 + 6 + 8 \times 1.43 = 42.5$ cm., thus giving numbers in substantial agreement with those obtained above.   In the same way we get 26.5 cm. for the secondary maximum at 10 and 36.5, though beyond that on the Lecher system this submultiple of the oscillator seems too weak to detect.   Since $82/53 = 3/2$ it would seem that this weak oscillation is the upper fifth of the longer of the two oscillator fundamentals.   It would appear now that the unexplained secondary maxima of Fig. 5 at 52, 78 and 104 cm. are due to this vibration of the oscillator.

Now Curve *II* of Fig. 2 was taken with an oscillator[1] other than the one used for most of the curves of this paper.   In the light of the presence of two fundamental vibrations of equal intensity, one higher in frequency, the other practically the same amount lower than the true fundamental tone of the oscillator, it seems likely that the maxima X and Y of Curve *II*, Fig. 2 represent these two fundamental tones.   If it be objected that the one circuit was much longer than the other and hence that the conditions were not right for producing two fundamental tones of different period, let the reader remember that we are here dealing with *forced* oscillations.

The maximum marked Z at 66 cm. in Curves V and VI of Fig. 5 we are unable to interpret.   It has begun to show itself in Curve *IV* of the same figure.   We think it has a counterpart or corresponding maximum Z' concealed somewhere between 140 and 150, for Curve V shows the maximum at 144 to be too broad at the base to contain only the maximum due to the third free overtone at 144.3.   Moreover, the other points for this third overtone are seen to be prac-

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[1] Its true dimensions are not now available.

tically absent at 35 and 90 in Curves *V* and *VI*, which would strengthen this idea. Calling $Z' = 142$ we thus get the internode 76 cm., or the wave-length 152 cm., a little more than twice the true fundamental of the oscillator. On this view it would seem that for no air-gaps in the vibrator circuit a portion of the lead wires form a part of the oscillating circuit, whereas when suitable air-gaps are used not even the wires mm' (Fig. 1) take part in the oscillation.

Keeping the air-gaps constant at 1 mm. length (accurately 0.94 mm.) we tried the effect of changing the length of the oil-gap. The Lecher circuit was somewhat longer than 165 cm., for the first overtone maximum was at 113 instead of 111.3 cm. The results obtained are plotted in Fig. 9. A glance at Curve *III*, Fig.
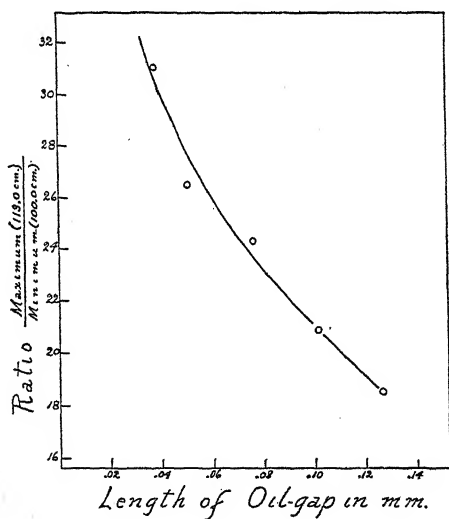
Fig. 9.

11, shows that a minimum occurs at 100, and so, instead of taking full curves, alternate readings were taken at 113 and 100, usually seven in number. Necessarily the true location of the curve of Fig. 9 was by far the most difficult of all the curves to obtain. The thickness gauge was used to measure the length of oil-gap and at best the method is exceedingly inaccurate. Fig. 9 is the mean result of four trials; in three of these the oil-gap was first increased

and then decreased by steps.   There can be no doubt of the general nature of the curve.   The minimum limit of the gauge was 0.0015 inch, so still smaller lengths than shown could not be investigated. While undoubtedly the smaller oil-gaps gave better ratio values, nevertheless for them the oscillator was much less steady in its action, so that in general we preferred to work with a gap 0.1 mm. long.

In the same way we tried the effect of changing the length of the water resistance back of the oscillator and the results are shown in Fig. 10.   Here again the curve is the mean of several trials,
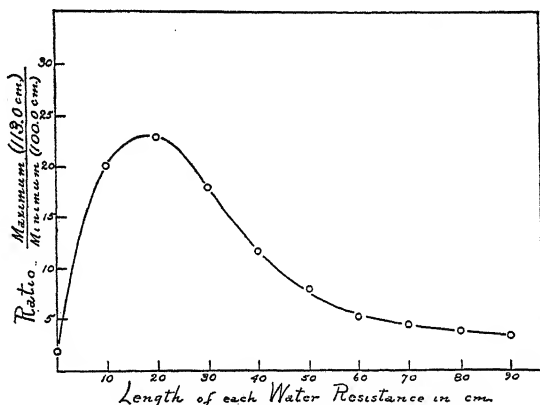
Fig. 10.

first increasing and then decreasing the length.   For a length below 20 cm. for each water column the spark became so snappy and disruptive as to render the oscillator action unsteady.   Accordingly we cannot say that the maximum at 20 cm. is correct to within 5 cm. either way.   For the sake of steadiness of action 30 cm. was chosen in most of the work of this paper.

Then we tried the effect of making the water resistance zero. But the spark gap at once became too disruptive to work with so we removed the condenser also (C, Fig. 3).   As a result we obtained Curve I, Fig. 11.   It was at once evident that the free vibrations of the Lecher system were the only vibrations present and they were *exceedingly feeble*.   Then to see whether this great change in the character of the wave system was due to the condenser or to

the absence of water resistance we again inserted the condenser. But we had proceeded with the curve (*II*, Fig. 11) only 50 cm. along the wires when the mica insulation between the primary and secondary of the oscillator was punctured by the spark and hence
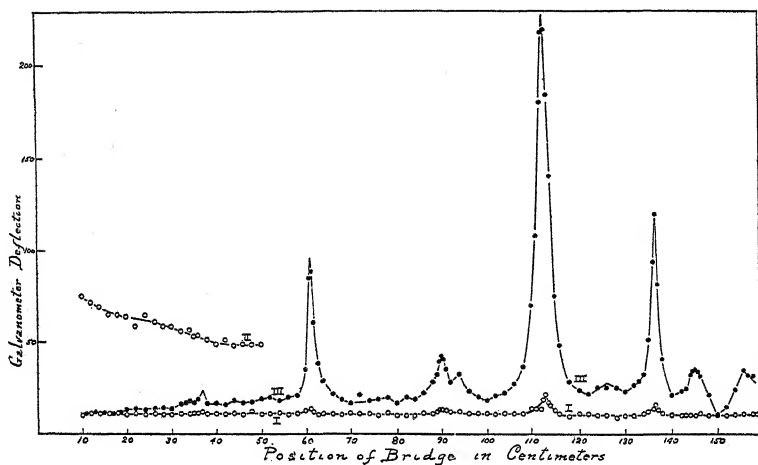
Fig. 11.

we didn't try to proceed. However, there is no evidence of the maximum at 37 and so we concluded that the introduction of the free vibrations to any marked extent must have been due to the presence of the water resistance. We then proceeded to confirm this view by running a curve with a 30-cm. water resistance but without the condenser. Thus we obtained Curve *III*, Fig. 11. Its similarity to many of the other curves shown is at once apparent.

CRITICAL DISCUSSION.

We have shown by the results here presented that there are two very necessary conditions that have to be fulfilled before the free vibrations of the Lecher system are able to establish themselves in all their simplicity (except as they are influenced by the closeness of the coupling of the oscillator system). These two conditions are the introduction of water resistance back of the oscillator and in the same place but nearer the oscillator two symmetrical air-gaps. We have tried in the curves of Fig. 5 to show the functioning of the air-gaps and in those of Figs. 10 and 11 that of the water

resistance.   The point on the Y axis of Fig. 10 is obtained of course from Fig. 11.   Unquestionably, the general nature of the curve of Fig. 10 is right, but the difficulties under which it was obtained were great and the number of points in the region of short lengths were too few to locate it accurately.   It is hoped in a later report to be able to discuss more intelligently than is now possible the exact functioning of the water resistance in such work.   The importance of the fulfilling of the above conditions will, we believe, be at once recognized.

We surmise it must have long been a source of wonder to many an experimenter why the curves of Rubens,[1] which undoubtedly represent the free vibrations of the Lecher system, had relative intensities for the different tones so at variance with what one might expect from a knowledge of acoustical vibrations.   We hope this paper may help to clear up the matter.   It would appear that it is necessary to prevent the free vibrations from being swamped by those of the induction coil and oscillator lead wires, and that the introduction of water resistance and air-gap resistance affords the necessary prevention.

It becomes interesting then in the light of such investigations to see in how far the above results lend themselves to known theories, particularly to that developed in essence by Kirchhoff[2] and in full detail by Abraham.[3]   Employing the latter's notation we have for the potential difference $\Phi_1 - \Phi_2$ between the wires

$$\Phi_1 - \Phi_2 = \frac{2|A|}{K} \cos \left(\frac{\nu x}{w} - \gamma \right) \cos \nu t,$$

and for the current

$$J_1 - J_2 = w \cdot 2|A| \sin \left(\frac{\nu x}{w} - \gamma \right) \sin \nu t,$$

where x is the distance measured along the wires, K the capacity of the wires per unit length and γ the half phase change upon reflection from the free end of the wires due to end-capacity (the detecting Leyden jars).   If a bridge is employed, there is of course

[1] Rubens, *loc. cit.*

[2] Kirchhoff, Pogg. Ann., 121, p. 551, 1864.

[3] Abraham, *loc. cit.*

a node of potential difference at the bridge. The above equations express the possible free vibrations of the Leyden jars when connected by the parallel wires and the bridge. Take the end capacity at $x = 0$ and the bridge position at $x = -l$, then for a given frequency ν, the values of l where potential nodes should be found are determined from the equation

$$-\frac{2\pi l}{\lambda} - \gamma = -\frac{\pi}{2} + m\pi,$$

where $\tan \gamma = 2\pi K_0/\lambda K$, w being the velocity of free radiation and m a whole number. On the other hand if the length of the wires is given reckoned from the bridge position to the end-capacity, the frequency of the possible free vibrations is determined from the transcendental equation $\xi \tan \xi = \alpha$ where $\xi = 2\pi l/\lambda$ and $\alpha = Kl/K_0$. Now the end of the wires was at 165 cm. and the wire end including end-capacity was shown above to be 171.6, giving $K_0 = 6.6K$. Extrapolating the fundamental wave-length from Fig. 6 to be 712.8 and using this value throughout we get the results indicated in Table II. with the percentage error shown in column 8, assuming

<div align="center">TABLE II.</div>

No. of Overtone.	Observed x.	λ	$\xi \tan \xi$	λ' from Fig. 6.	$\xi' \tan \xi'$	α	Per Cent. Error $\dfrac{\xi \tan \xi - \alpha}{\alpha}$.	Per Cent. Error $\dfrac{\xi' \tan \xi' - \alpha}{\alpha}$.
0 Fund.	171.6	712.8	26.00	712.8	26.00	26.00	1	
1	53.7	237.6	9.34	241.2	8.05	8.14	+ 14.8	− 1.1
2	30.3	142.5	5.55	148.0	4.38	4.59	+ 20.4	− 4.6
3	20.7	101.8	4.25	109.1	3.01	3.14	+ 35.4	− 4.1
7	9.0	47.5	2.98	62.0	1.19	1.36	+119.0	−12.5

α correct. It is evident at once from the table that the theory is far from being satisfied. If on the other hand one uses instead of λ in the formula a value λ' obtained from Fig. 6, the agreement may be said to be quite satisfactory. It is so good in fact that one is safe in saying that to a first approximation the effect of the close coupling is one of increasing the wave-length of the overtones and the higher the tone the greater the effect. This confirms the viewpoint gained in drawing the curve of Fig. 6.

The result of a comparison of theory with experiment as to the

[1] $\xi \tan \xi$ was put equal to α here in order to extrapolate the value of λ_0.

relative intensities of the various maxima is summed up in Table
III.   The relative theoretical value of any maximum is equal to

TABLE III.

No. of Overtone.	$\sin\left(\frac{\pi}{2}\cdot\frac{1}{2n+1}-2\gamma\right).$	Relative Intensity (Fund. = 100 Per Cent.)	$\sin\left(\frac{\pi}{2}\cdot\frac{1}{2n+1}-2\gamma'\right).$	Relative Intensity.	Observed Intensity.
0 Fund.	0.994	100.0	0.994	100.0	100
1	0.397	39.8	0.399	40.0	40
2	0.197	19.8	0.193	19.4	20
3	0.108	10.9	0.100	10.0	9
7	−0.012	−1.2	0.016	1.6	2

$$\sin\left(\frac{\pi}{2}\cdot\frac{1}{2n+1}-2\gamma\right)$$

where n is the number of the overtone and 2γ is the phase change
on reflection due to the end-capacity.   Now

$$\gamma = \tan^{-1}\frac{2\pi}{\lambda_0}\frac{K_0}{K}$$

and hence is 3° 19′ 50″.   From Table III. it is seen that the in-
tensity of the seventh overtone has a negative sign which shows that
the close coupling of oscillator and Lecher system plays another
part besides that of changing the wave-length already mentioned.
If one introduces γ' for γ where

$$\gamma' = \gamma\frac{\lambda_0}{\lambda'},$$

the seventh overtone comes out positive.   Now it is evident for
weaker overtones that the average minimum of the curve should be
subtracted from the average observed deflection for a given over-
tone before a fair comparison can be made.   It will be observed
from the curves that there is a general tendency for the minima
to increase in intensity as also the maxima as their distance from
the oscillator increases.   It is clear, moreover, that when a maxi-
mum belonging to a given overtone occurs on the slope of a maxi-
mum of another and stronger overtone, this fact must be taken into
account in estimating the mean value of the maxima for that
overtone.   Attempting such corrections for all the curves taken
which have proper water resistance and air-gap we obtain the
figures shown in the sixth column of Table III.   A glance at any

of the curves will convince one of the substantial correctness of the figures there shown. From Tables II. and III. the agreement between the theory, modified because of close coupling, and experiment may be said to be quite satisfactory.

After the presentation of these results it would seem wise, perhaps, to enter into a critical discussion of the results of other experimenters. To avoid repetition it seems best, however, to defer such discussion until after the results of Blake and Sheard[1] are presented and these will follow in due course.

In closing this paper only one other thing need be pointed out. The ever-present question of the proper relation between wire-length and wave-length is still unsettled.[2] These experiments throw some light on the question we think. We confess to some surprise that in comparing theory (modified) and experiment we found it necessary to use the outer end (away from the oscillator) of the Leyden jars rather than their inner end in calculating the phase change. Rather, perhaps, can we more safely say that the free end of the wire had to be used. This gives us $K_0 = 6.6K$. Now the actual wire-length of the Lecher system was 2 (165.0 + 6.6) up to the neck XX' of the secondary giving 343.2 cm., neglecting the distance XX' across the neck.[3] Dividing this into 712.8, the extrapolated value for λ_0, we obtain 2.08. If we include the neck this ratio becomes 2.07. We interpret the fact that in estimating γ, the free end of the wire has to be used to mean that the theoretical value of 2.00 obtained by Poincaré, Abraham and others can never be obtained in practice. Just as in acoustics there is always an "end correction" for resounding air-columns so in electricity there is a corresponding end correction which is *always* present. One might say that the very momentum of the electric wave carries it *beyond* the end of the wire before reflection occurs. It is seen, then, that the value of the end correction is say at most 4 per cent. If

[1] The experiments described in this paper were performed in 1907–8, but for various reasons their publication has been deferred. In the meantime the results obtained by Blake and Sheard have become known to the writers. Their results have largely influenced the point of view of this paper, especially in regard to the influence of the oscillator coupling upon the results enabling us to point out the way in which Abraham's theory of free oscillations must be modified to fit the facts for a closely coupled circuit. It is with pleasure that we acknowledge all credit due them in this and other regards.

[2] See for instance an article by Ives in PHYS. REV., September, 1910.

[3] As a matter of fact XX' was somewhat less than 2 cm., say 1.5 cm.

the *entire* length of the secondary were added to the Lecher system the total metallic circuit would be 359 and if the value of λ_0 were extrapolated as high as 718 this would give exactly 2 for the above ratio. This cannot possibly be right, however, as the work of Blake and Sheard will later show. A value as high as 718 is impossible from Fig. 6, moreover.

The theoretical value of 2.00 makes no allowance for the end correction. Assuming for the moment its entire absence we may say that the Poincaré-Abraham value of 2.00 holds, *but for a non-radiant system.* In this connection it is interesting to note that the recent values obtained by Ives vary from 2.03 to 2.08, for a system also *non-radiant*, though for an entirely different reason.

SUMMARY.

It has been shown in this paper how it is possible to obtain on a set of parallel Lecher wires a system of free vibrations uncontaminated by oscillator influence except that of coupling. It was found that the unmodified theory fitted even the case of close coupling to a first approximation. The manner in which the theory has to be modified for a second approximation was pointed out. The conditions for freedom from contamination were found to be two, the introduction of suitable air-gaps into the oscillator circuit and suitable water-resistance between the induction coil and oscillator. The results of varying these conditions were studied. A ratio between fundamental maximum and minimum equal to 125 to 1 was experimentally obtained and it was thought that this ratio could be materially increased by observing the best conditions for controlling the various factors that enter into the problem. The ratio between Lecher wire-length and wave-length was found to be 2.07 and the difference between this value and the theoretical value 2.00 was shown to be due to an "end-correction" which is necessarily always present.

In the earlier part of the paper where the Wehnelt interrupter was employed the work was done by one [1] of us under the supervision of Professor A. D. Cole and we take pleasure in expressing our best thanks to him for his suggestions and advice.

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[1] Miss Ruppersberg.

ON THE RELATION BETWEEN THE DENSITY AND CONCENTRATION OF AQUEOUS SOLUTIONS.

By Paul Frederick Gaehr.

IN the winter of 1907–8, in connection with an experiment to determine the curve between the density and concentration of a solution of common salt, a very simple equation was found. I then set out to see if this equation is also good for the solutions of other salts, and of acids.

The proposed equation is

$$\log \delta = mc,$$

wherein δ = density and c = concentration. This simple equation was first found for solutions of common salt. I had attempted to derive an equation based on the following assumption. Let V be the specific volume of a solution of concentration c per cent. Then in 1 gram of the solution are contained $c/100$ grams of the salt, and $(100 - c)/100$ grams of water. If on mixing these, the sum of their individual volumes is equal to the specific volume, we have a very simple case. Unfortunately, the facts are otherwise. If we represent the density of the salt by s, and the density of water by w, and the sum of the individual volumes of salt and water v, then we can write

$$v = \frac{c}{100s} + \frac{100 - c}{100w}.$$

Now if there were a simple relation between v and V like $V = mv + n$, wherein m and n are constants, we would be led to a linear relation between V and c. But such was not the case with NaCl.

Now in the case of no "shrinkage," *i. e.*, where $V = v$, we should have

$$V = \left(\frac{1}{s} - \frac{1}{w}\right)\frac{c}{100} + \frac{1}{w}.$$

So I plotted this line. The difference between corresponding

ordinates of this and the preceding curve represent the "shrinkage" and I had hoped that perhaps this could be more easily handled. It then occurred to me that the curve between V and c very much
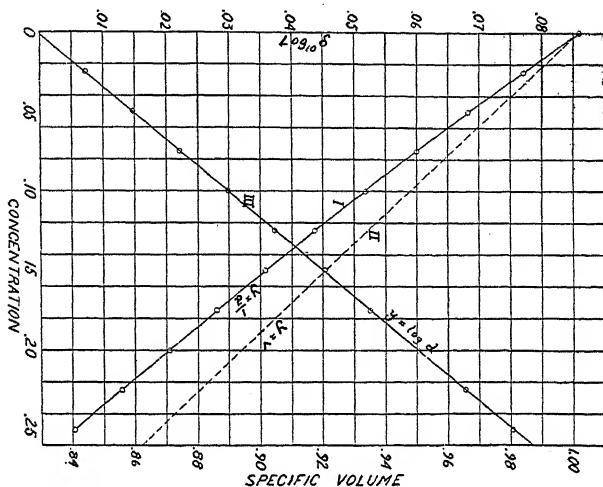
Fig. 1.

resembled a "decay-curve." I found that equally spaced ordinates formed a geometric progression, my conjecture thus proving true. Therefore I knew that the equation must be $V = 10^{-mc}$ or $1/V = 10^{mc}$.
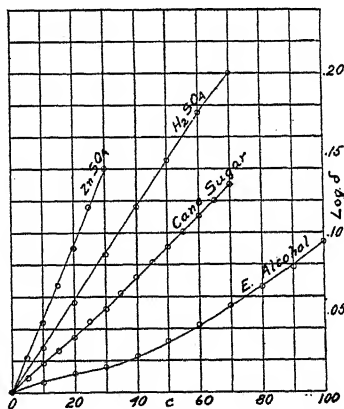
Fig. 2.

Hence $\log \delta = mc$. The reason for choosing 10 as the base of the exponential is thus apparent. The reason for not multiplying the

exponential by a constant is that $\delta = 1$ when $c = 0$.   The justification
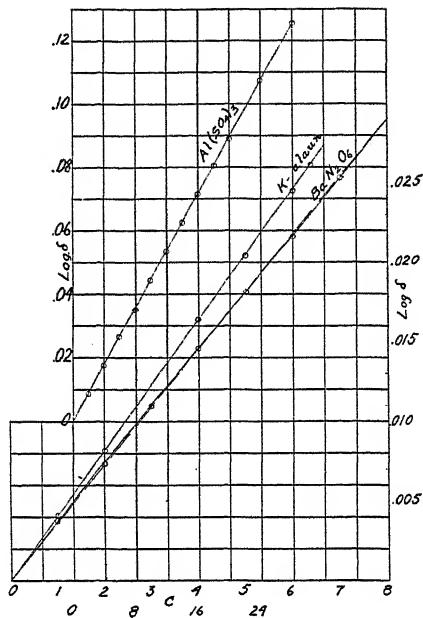
Fig. 3.

for this equation as applied to solutions of NaCl will be found in Fig. 1.
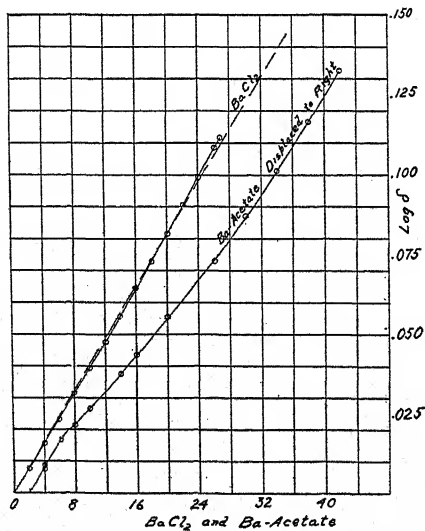
Fig. 4.

This equation was tried for a number of other salts, and also for some acids (see Fig. 2). To some it applied, to others not. It seemed worth while however to make an inventory of the inorganic
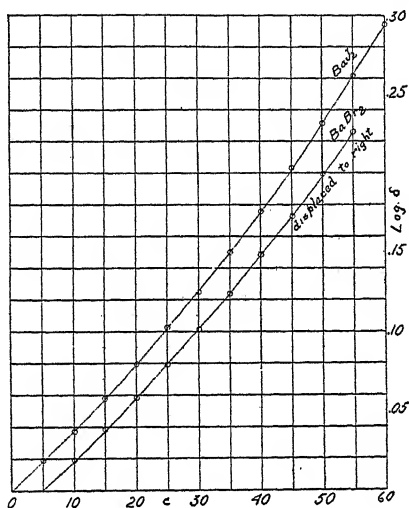
Fig. 5.

salts to begin with, to see if it might be possible to settle any one of these points: (1) Do the salts that obey, obey also some other common law, or have they some common properties? (2) Are
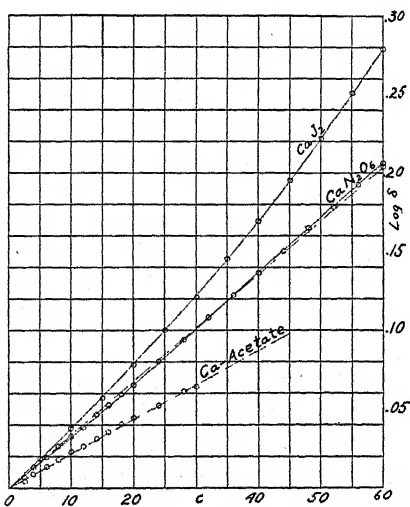
Fig. 6.

the slopes related to the atomic weight so as to be easily computed from them? (3) Can the deviations from the law proposed above be explained on any ground? (4) Does the law throw any light on the molecular behavior of those salts that obey it? I have attempted no answers to any of these questions, but have confined myself to a statement of facts, largely because I was unable to finish the list of salts. The data for the curves were taken partly from Landolt and Boernstein, partly from Kohlrausch's Handbuch der Prak- tischen Physik, and partly from the Chemiker Kalender for 1908.
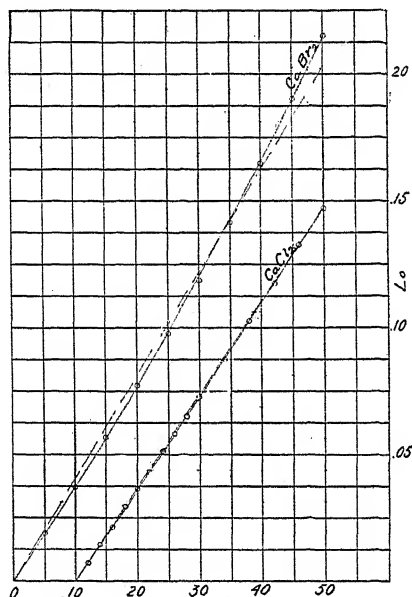
Fig. 7.

In the last named book I found a few typographical errors, which led me to suspect, when a bad irregularity occurred, that I had an error before me. So a few salts were dropped for that reason. Also, I soon discovered that when a curve is short, it is straight; so salts whose saturation came at concentrations less than 20 per cent. were dropped.

There are three general classes of curves to be found:

(*a*) Straight lines; this includes those curves which differ but little from a straight line. The criterion for this was that the

density as taken from the curve should at no point differ from the density as taken from the average straight line by more than 1.2 part in two hundred, *i. e.*, by not more than six tenths per cent.
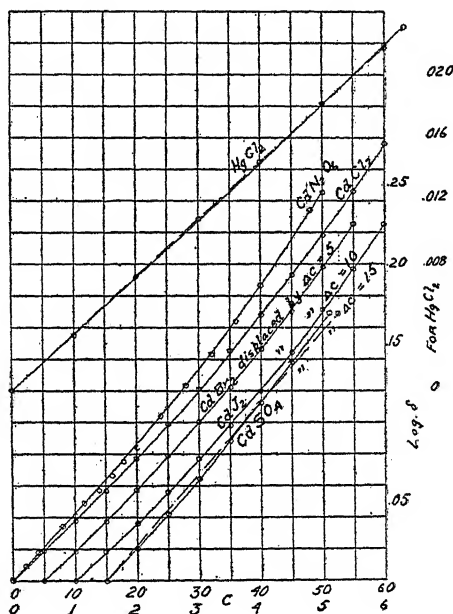
Fig. 8.

(*b*) Curves sloping uniformly upward or uniformly downward.

(*c*) Curves with double curvature.

The easiest way to test the applicability of the law was to plot values of *c* in per cent. as abscissæ, and corresponding logarithms of the density as ordinates. The average slope was computed from the data, according to the formula: slope = (sum of ordinates) divided by (sum of abscissæ). The least square formula for the case that the *y*-intercept is equal to zero reduces to this. It were better to say that the curves of class (*b*) were concave or convex toward the *x*-axis. A few among class (*b*), and more among the third class, exhibited the peculiar phenomenon of being perfectly straight over a considerable length.

In view of this fact, especially, and also of the double curvatures, it must be evident that the fitting of a single equation to all salts

is not an easy matter. We might have tried to fit a parabola of the nth order to the curves, and have said that in the case of certain salts the coefficients of the x's are such as to make it the series for 10^{mc}. However the computation of the coefficients is apt to be a very long task, and in the cases of some salts, whose densities are given only to the third decimal place, next to impossible. Besides this, there is a very good reason for sticking to the logarithmic equation, for differentiation gives

$$\frac{d\delta}{\delta} = \frac{m \cdot dc}{\log_{10} \epsilon},$$

an equation whose significance will be pointed out later.

At this time it may be useful to recall how the tables of densities are made up. As a rule, the observer did not have precisely 5,
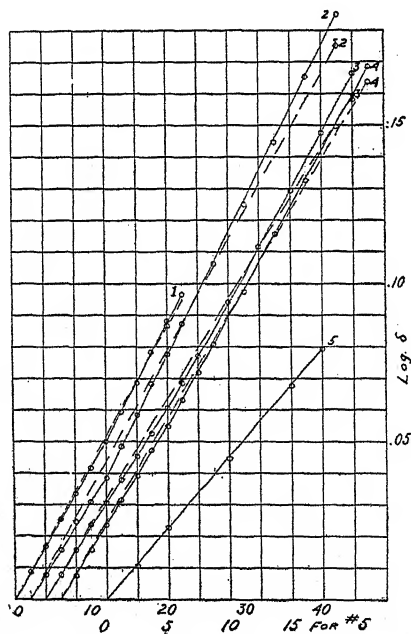
Fig. 9.

10, 15, etc., per cent. He made up his solutions as near these values as possible, plotted a curve, and then by interpolation got the values for the concentrations given in the table. Or else, the observer

might find the densities corresponding to certain degrees on, say, the Beaumé hydrometer, and the compiler of tables then does the interpolating. That this may lead to errors of greater or less magnitude must be clear.

It is difficult to believe that men whose data are quoted in such works of compilation as Landolt and Boernstein's, should have omitted any precautions in their measurements. Yet the writer
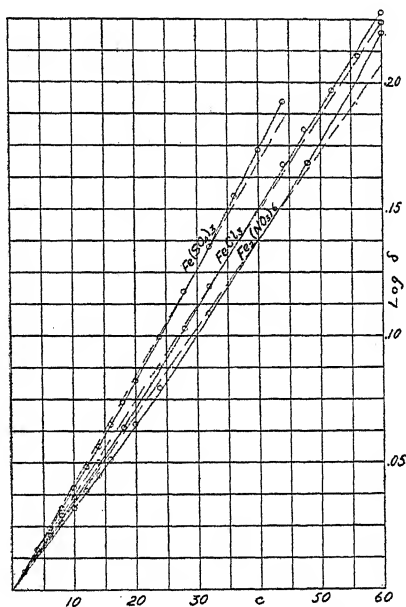
Fig. 10.

ventures to suggest that perhaps some unknown error was committed which in the case of certain salts varies with the concentration.

In the diagrams presented in this paper, I have in some cases, to prevent the overlapping, displaced curves to the right or upward. Remembering that the log of the density is zero for zero concentration, one easily finds the origin for each curve; the coördinates of any point on the curve must then be corrected for the displacement. The straight lines drawn with the curves represent the line of average slope, the computation of the slope being performed as explained on a preceding page.

In the tables, the salts are arranged in accordance with the curva-

ture as indicated on a previous page. For the salts of class *A*, I have also tabulated the molecular weights, the slopes, and the maximum relative error (in per cents) made made by taking the calculated density instead of that found in the tables. This per cent. error was not calculated for all the salts; where a curve deviated considerably from a straight line, or else not noticeably, the error was not calculated. There is given for all the solutions the maximum density and the corresponding concentration.

It was stated above that several curves contained straight segments. Such are marked by an *s* in the last column. Those curves whose error is so small as to admit them into class *A*, but which nevertheless show a clear deviation from the straight line, are marked by a *b* or *c* in the fifth column. It is remarkable how cadmium
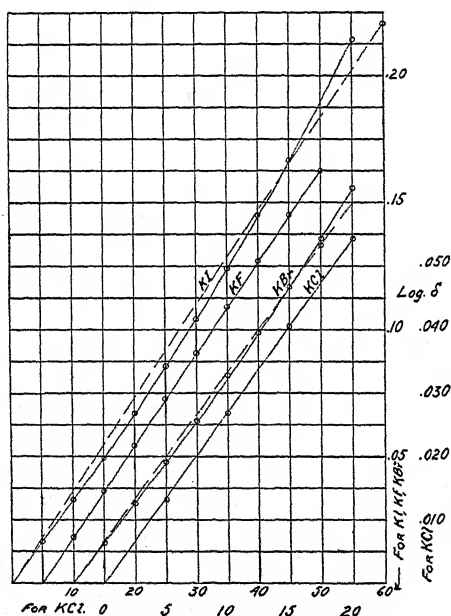
Fig. 11.

nitrate and barium acetate show an irregularity near the origin. Alcohol (ethyl) shows this same tendency. I have wondered whether some of this may not be due to the fact that the tables as used by me were made up from data taken from two different observers. I found two tables for aluminium chloride, both for the

same temperature, but the densities differ from each other by per cents varying up to 2 per cent.

It is impossible in this paper to include the curves for anywhere near all the solutions I have undertaken to study. But I may say I found the curves for sugar, sulphuric acid and zinc sulphate to be perfectly straight, and those for NaOH and nitric acid concave towards the x-axis, thus differing from all the B curves in this report. It is thus clear that we do not as yet possess sufficient data to draw any very general conclusions.

<div align="center">

TABLE I.

Class A (Curves Straight or Nearly Straight).

</div>

	I.	II.	III.	IV.	V.	VI.
AgNO$_3$	169.94	$^1.33721 \times 10^{-2}$	20	1.15	c	.2
Al$_2$Cl$_6$	196	.31316	40	1.34	b	.5
Al$_2$(SO$_4$)$_3$	142.38	.44873	28	1.34	b	
KAl(SO$_4$)$_2$	258.37	.40946	6	1.06		
BaN$_2$O$_6$	261.42	.3627	8	1.07	c	.03
BaCl$_2$	208.30	.40656	27	1.29	b	.6
CaN$_2$O$_6$	164.12	.33861	60	1.61	c	.5
CaCl$_2$	111.0	.36591	40	1.40	b	.2
Ca(C$_2$H$_3$O$_2$)$_2$	158.15	.21807	30	1.59	c	.3
CdSO$_4$	208.46	.44936	36	1.47	b	.6
FeCl$_3$	162.25	.37271	60	1.67	c	.5
HgCl$_2$	270.9	.36208	6	1.05	b	.04
CoCl$_2$	129.90	.4287	20	1.23	s	.5
CuSO$_4$	159.66	.44767	17	1.2		.17
KBr	119.11	.36241	45	1.46	b	.6
KF	58.15	.33216	45	1.43		.04
KCl	74.60	.26996	20	1.34	b	.1
KNO$_3$	101.16	.27688	20	1.14		
K$_2$CrO$_4$	194.40	.35784	40	1.40	b	.6
LiCl	42.48	.24408	60	1.78	b	.3

The column I. gives the molecular weights, II. gives the slope of the line log $\delta = mc$, III. and IV. give respectively the concentration and density for the highest point (approximately). The last column gives the relative error (maximum) made by taking the density from the logarithmic law rather than from the tables. In the following tables, II. and III., column II. gives the slope of the average straight line. Column VI. is omitted; the per cent. errors range from .8 to about 5.

TABLE II.

Class B (Sloping Uniformly Upward or Downward).

	I.	II.	III.	IV.	V.
$Ba(C_2H_3O_2)_2$	255.45		40	1.36	irreg.
$BaBr_2$	297.32		50	1.69	
BaI_2	391.34		60	1.97	
$CaBr_2$	200.02	$.4062 \times 10^{-2}$	50	1.64	
CaI_2	294.04		50	1.91	
$CdCl_2$	183.30		60	1.89	
$CdBr_2$	272.32		50	1.68	
CdI_2	366.34		50	1.68	
$CuCl_2$	134.60	$.43767 \times 10^{-2}$	40	1.53	
CuN_2O_6	187.62	.40938	44	1.54	2 st. lines.
$FeSO_4$	151.96	.41947	44	1.56	
$Fe_2(NO_3)_6$	483.86	.34666	60	1.66	
$Fe_2(SO_4)_3$	400.	.42159	44	1.56	max. error 1.5.
KI	166.12	.36735	60	1.73	
LiI	134.04	.3848	55	1.58	
$LiBr$	86.99	.33575	40	1.26	

TABLE III.

Class C (with Double Curvature).

	I.	II.	III.	IV.	V.
$CO(N_2O_6)$	56.16	$.39603 \times 10^{-2}$	40	1.47	*s*
KOH	183.02	.3673	54	1.60	
CdN_2O_6	236.42		50	1.76	irreg.

A survey of the tables shows (1); that the majority of the class *A* (which we may call linear salts) are electrolytes. Also a majority of the non-linear salts are non-electrolytes. From the salts which do not run to concentrations above 20 per cent. we can draw no conclusions, as they are practically linear.

2. If the salts of class *A* are plotted, with molecular weights as abscissæ and slopes of the straight lines as ordinates, we obtain a "target diagram" from which we can draw no conclusion.

3. Among the non-linear salts we find that most of them run up to rather higher concentrations and densities than most of the linear salts. However we should not forget that sugar and sulphuric acid, both of them linear, run up to concentrations of 65 and 100 per cent. respectively.

4. It does not appear that any element is confined to one class or the other; nor does it appear that an acid radical appears in only one class.

5. We may suggest also that organic salts will be found among the linear class, judging from the fact that sugar is linear.

It is evident that the present question is closely connected with that of the freedom of motion of molecules. In gas theory this is the starting point for many a theoretical discussion. But then gas molecules move about with more freedom than do molecules in a
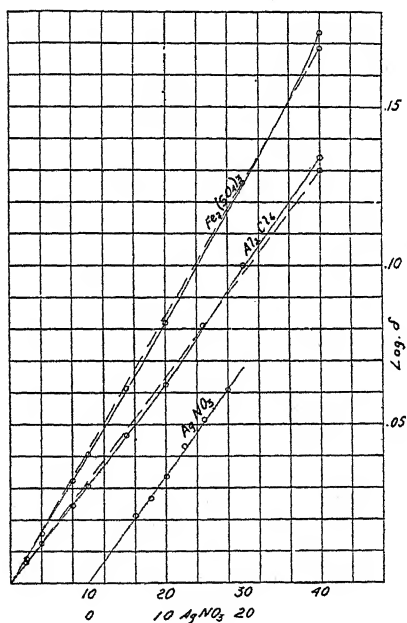
Fig. 12.

solution. However we can imagine that at first molecules enter with ease. But after a while this is not so much the case. Still we cannot make any suppositions as to how the freedom of these molecules depends on the number already present but must derive any equations of that sort from known equations. We can certainly see that as a chemical whose density is greater than that of the solvent is introduced, the density of the solution must be between that of the chemical and that of the solvent, and that as more and

more of the chemical is introduced, the density increases faster. And anything that differs from this points to something strange.

It seems reasonable that the relative increase in the density, produced by the addition of a chemical, should be simply related to the increase in the concentration. Our proposed equation, when differentiated, gives

$$\frac{d\delta}{\delta} = \log_e \mathrm{10} \cdot m \cdot dc.$$

Is it possible that the question is related to that of dissociation? Perhaps the density of a solution in which all molecules are dissociated is different from that in which none are dissociated. And
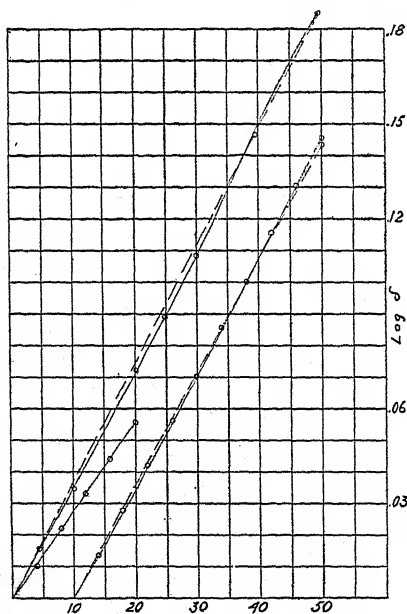
Fig. 13.

if the dissociation depends on the number of molecules already present (*i. e.*, on the concentration) then we can see why the shrinkage is not proportional to either density or concentration. Perhaps it will be possible from the correct equation for any one salt to arrive at an equation which shall say how dissociation depends on the concentration.

One of the things we should be able to calculate from our equation is the density of the salt itself.   The law may hold for the solution up to saturation, but if it holds up to a concentration of 100 per cent. it is more serviceable.   Now in the case of common salt, the
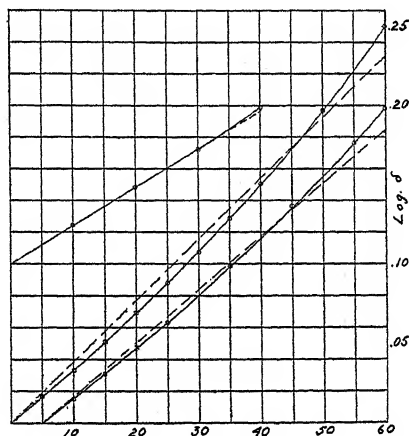
Fig. 14.

density for a concentration of 25 per cent. is 1.1897, and the logarithm of the density is .075438.   The logarithm for a concentration of 100 per cent. (=4×25) would be four times as great, hence .301752, whose antilogarithm is a trifle over 2; the density of common salt is about 2.15, not so very different when one considers that the computation was made by an extrapolation for a point whose distance from the last point of the curve is three times the length of the curve.

Another property which our equation should possess is that from it we should be able to determine at what density or concentration the solution is saturated.

The law, as now proposed, seems to need modifications of some sort.   Perhaps in its completed form it will be to solutions what Dalton's law is for the physical mixture of gases.   There are many things to be said in favor of the present equation, however.   Like Boyle's law, it answers only approximately to the actual facts, but it was discovered, like Boyle's law, by the study of a case where it applied certainly to a degree of accuracy as great as the measurements from which it was derived.   Its great advantage is simplicity,

and from it we can derive many important facts. Its deviation from the true facts, as in the case of Boyle's law, is not the same for every substance nor for every temperature. On the other hand, the errors of Boyle's law have led to the discovery of the very things that made it possible to improve on it, as well as the discovery of other important things.

Then there are two ways in which a simple equation may serve: If it be true for an ideal case, it will serve as the basis of many a theoretical discussion leading to more or less correct results. If it be not true, it may serve as the starting point for finding a better

Fig. 15.

equation. It is not claimed that my equation is any better than Boyle's law. It is merely presented as a first step. Then if it is shown to be true, it may perhaps lead to the discovery of phenomena at present unknown.

The curves in Figs. 3–14 are those catalogued in Tables I., II. and III. of this paper. Fig. 15 is the target diagram already referred to.

RÉSUMÉ.

The densities and concentrations of solutions of common salt in water are connected very accurately by the very simple equation $\log = mc$, in which m is a constant. This gave rise to the question whether perhaps this law also applies to the solution of other chemicals. It is with this question that this paper is chiefly concerned.

Of the solutions studied so far, it was found that the solutions whose maximum concentration is less than 15 per cent. obey the law very well; of the others, about one half obey the law within .6 per cent.   Of those that are listed in this paper, about half are electrolytes, and the other half are not; and the electrolytes seem to be mostly confined to the linear solutions.   The number of salts studied is not great enough to permit any very definite and general conclusions to be drawn.

It is not known now whether the law proposed is only an approximate law, or whether it is a law applying to an ideal condition. In either case it may lead to the discovery of important facts.

It is furthermore suggested that the law may throw light on various other questions: (1) On the problem of the molecular behavior of chemicals, (2) on the problem of osmosis, (3) on the laws governing the mixture of more than two substances.

The law when properly modified may be used to calculate: (1) the density of a solution for any concentration; (2) hence also the density for a concentration of 100 per cent., *i. e.*, the density of the pure chemicals, (3) the concentration of the saturated solution.

That the present law, though as yet imperfect, is rich in possibilities, is at once evident.   It is hoped that a further contribution on the subject may be made soon.

DISCHARGE FROM HOT CAO.

By C. D. Child.

IT was suggested a few years ago by Sir J. J. Thomson[1] that certain phenomena of electrical discharge could be explained by what we may call ionization by *repeated* impact. He had found that luminous discharge could be produced from a cathode of hot CaO with potential differences much smaller than those which are needed when a cold cathode is used, and that at a certain critical point it needed but a very small increase in the voltage to produce a large increase in the current and to change from the non-luminous to a luminous form of discharge. He believed that the ionization in the luminous form of discharge was not due to the breaking up of an atom by a single impact with an electron, but rather to an explosion of the atom because of its having absorbed so much internal energy from repeated impacts with electrons that its equilibrium had become unstable.

If this idea is correct, it is important not only because it explains this particular phenomenon, but also because it offers an explanation of the ionization of the arc and gives us some information concerning the character of the atom. I have, therefore, given some study to the subject, but find that there are several reasons for rejecting this explanation.

Improbability of Repeated Impacts.—The first of these objections is that there are far too few electrons present in the tube at any time before the discharge becomes luminous to occasion repeated impacts. For example, in one experiment performed by myself the largest current that could be passed through the tube without having a luminous discharge was 5×10^{-9} ampere, which is 150 electrostatic units. The charge carried by each ion[2] is 3.4×10^{-10}. The cross section of the tube used was approximately 5 sq. cm. So that the number of electrons passing through each square

[1] Nature, 73, 496, 1906.
[2] Thomson's Conduction of Electricity through Gases, 2d ed., p. 158.

centimeter per second was 8.85×10^{10}. The pressure of the gas was .014 mm. and the mean free path of the electron at this pressure is approximately 3.76 cm.[1] So that in each cubic centimeter only 2.34×10^{10} electrons would hit molecules per sec.

According to Myers[2] there will be 11.2×10^{14} molecules per c.c. under these condition of pressure and temperature. From this we find that each molecule will on the average be hit once in 48,000 sec., or once in thirteen hours, and it only requires a fraction of a second to produce the luminous discharge when the conditions are right for it.

This is, of course, not an argument against ionization by repeated impact after the luminous discharge has commenced, for then the number of electrons will be very much greater, but the idea of such impact was used by Thomson as an explanation of the beginning of the luminous discharge, and it is an argument against such an explanation. There will, however, be other data given later which indicate that not at any time is there ionization by repeated impact.

Critical Condition Does not Depend on the Amount of Current, but on the Condition of the Cathode.—In the second place experiments indicate that the voltage required to change from the non-luminous to the luminous discharge depends but slightly, if at all, on the amount of current flowing, but does depend very greatly on the kind and condition of the cathode. That is, the number of electrons passing through the tube and hitting upon the molecules does not determine the point where the change occurs, while other conditions do.

The form of apparatus used for showing this is given in Fig. 1. T is a tube 2.8 cm. in diameter. P is the connection to the vacuum pump, McLeod gauge, and drying tube. C is the cathode consisting of platinum foil, approximately 2 mm. in width, 12 mm. long, and .02 mm. thick, the bottom being covered with CaO. This was welded to aluminum wires which led out of the tube and was heated by an alternating current connected at c and c'.

A is an iron anode, fastened to an iron wire which is brought down through the tube a into a mercury cup, so that the anode

[1] Idem, p. 476.
[2] Myers' Kinetic Theory of Gases, p. 333.

could be raised or lowered. A shunt of 50 ohms resistance was placed across cc'. The middle of this, lettered b, had the same potential as the middle of the foil C. A potential difference was maintained between b and A by connecting them to d and d', two points on a variable resistance through which a current was passed from a dynamo. By this means any potential difference from 0 to 130 could be used. V is a Weston voltmeter measuring this potential difference. G is a galvanometer measuring the current through the tube.
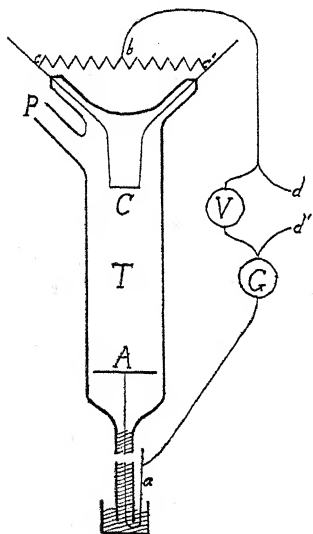
Fig. 1.

The first experiment was a comparison of the discharge from clean hot platinum with that from hot platinum covered with CaO. With clean platinum it is possible to get a large current without changing the character of the discharge, while with CaO it requires but a very small current to produce this change, the potential difference between the electrodes being the same in the two cases.

In order to get as large a current from the platinum as possible it is desirable to perform the experiment before the occluded gas has been entirely driven off. If this is done, the current is continually changing, so that no two sets of readings are the same, but this does not interfere with our present purpose.

The readings as taken are given in Table I., the first column giving the potential difference between the electrodes and the second the current. The platinum foil was a little more than red hot and the pressure of the gas was approximately .04 mm.

TABLE I.

Potential Difference in Volts.	Current in Amperes Times 10^{-7}.
71	.1
80	.5
84	2.1
90	6.5
125	48.2

There was here no abrupt change in the amount of current and the discharge was not at any time luminous. There is in some cases a slight luminosity with the discharge from clean platinum, but even then there appears to be no sudden increase in current nor sudden change from non-luminous to luminous discharge.

On the other hand with hot CaO the discharge changes to the luminous form before anything like this amount of current passes through the tube. This is especially true when the cathode is quite hot and has been used until the occluded gases have been driven off. In one case it was impossible to pass more than 5×10^{-9} ampere with a potential difference of 95 volts without the discharge becoming luminous. When the voltage was raised above this, the current became as large as an ampere or more, if the resistance in series with the tube was small.

Thus we see that the discharge remained non-luminous quite irrespective of the amount of current passing, provided the cathode was clean platinum and that it very quickly changed to the luminous form, if the cathode was very hot CaO.

Thomson states that he found the voltage at which the change took place to depend on the current, but in his experiments the current was changed by raising the tempera-ture of the cathode. When that is done, it is not possible to decide whether the critical point depends on the current or on the tem-perature of the CaO. In the experiments described above it is made clear that it is the condition and temperature of the cathode that determines this point and not the amount of current.

The Potential Difference between the Begin-ning and End of a Striation.—The potential difference between the beginning and end of a striation was examined, hoping that it would throw some light on this question.

Fig. 2.

The tube was exchanged to the form shown in Fig. 2. Both A and C are here fixed; e and e' are two movable exploring electrodes which are connected to an electrometer. The
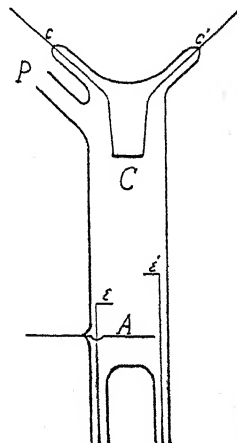
vertical parts of these were covered with small glass tubes for a few centimeters near the top, so that only the horizontal ends would receive charges. These ends were platinum wires .5 mm. in diameter. They were placed as near as possible to the sides of the tube and did not appear to distort the striations. The main tube was 4.2 cm. in diameter. The distance between A and C was 8 cm. The length of a striation varied as the pressure of the gas was varied, but was in general in the neighborhood of 2.5 cm.

It was first of all observed that the potential difference which was being studied varied as the pressure of the gas was changed. This is shown in Table II. The first column gives the pressure of the air left in the tube, and the second the potential difference between the beginning of one striation and that of the next. The current was .02 ampere.

TABLE II.

Pressure of Gas in Millimeters.	Potential Difference in Volts.
.06	11
.08	12.9
.164	16
.38	23.5

It was also found that the striation potential difference depended on the amount of current flowing through the tube. This is shown in Table III., where the first column gives the current and the second the potential difference. The pressure in the gas was .06 mm. of mercury.

TABLE III.

Current in Amperes.	Potential Difference in Volts.
.00032	14
.00064	13.2
.01	12
.2	10

Since these experiments were performed an article has been published by Wehner,[1] giving the results of a full investigation of the striation potential difference when there was discharge from a cold cathode. His experiments on the change produced in this quantity when the pressure of the gas was changed were performed in hydrogen and the potential differences which he gives are lower than the ones given above, but they show the same kind of a change.

[1] Ann. d. Phys., 32, 49, 1910.

In general he also found the same kind of a change when the current was changed. The changes, however, were through a much smaller range than that given here and were accordingly less noticeable. The lowest value which he found for the potential difference was 8.56 volts.

At first sight Table III. appears to give some reason for believing that ionization may take place more easily where there is a chance for repeated impact. Certainly it is true that where there is the greatest number of electrons passing through a gas, there is the smallest striation potential difference, and it is probable that this quantity is closely related to the potential difference needed to produce ionization by impact, as is suggested by Wehner.

But a second thought will hardly encourage such a view. The current was six hundred times greater in the last case given in the table than in the first. Each molecule would be hit six hundred times in one case where it would be hit but once in the other, and yet the potential difference decreased but a few per cent. It is certainly highly improbable that such a small change would occur, if repeated impact causes the molecules to be more easily ionized.

Certainly it is possible to suggest some more probable explanation. For example, when the larger current is flowing, the temperature of the gas must be much above that of the room. A small percentage of the electrical energy dissipated in the tube would be sufficient to raise the temperature of the gas several hundred degrees in a few seconds and it is altogether probable that at high temperatures the gas is more easily ionized than at low ones.

However, we will not be ready to give any explanation of these facts until we know more about what the facts are. For example, the lowest voltage here recorded was 10 volts, but Thomson states[1] that he found the potential difference between one striation and the next to be as low as 2.7 volts under certain conditions. He does not state what these conditions are, and I have not been able to get such a value. But until the phenomena have been more thoroughly examined, it is not possible to state what the cause of the relation between the voltage and the current is, and certainly we cannot say that these phenomena uphold in any way the idea that ionization occurs more easily when there is repeated impact.

[1] Phil. Mag., 6, 18, 449, 1909.

Increase in Current not due Alone to Ionization in the Gas.—We have given reasons for believing that the change from the non-luminous to the luminous form of discharge is not due to ionization by repeated impact. We may go a step further and say that the change does not appear to be due alone to ionization of any kind in the gas. There is no question but that ionization by impact occurs, but there are reasons for believing that it is not the only cause producing the change.

There are two ways in which ionization in the gas can increase the current. It can increase the number of positive ions moving toward the cathode, or it can change the field near the cathode, so that all the electrons shall be drawn from it, instead of being driven back by the electrostatic repulsion of those which had previously been emitted.

That the great increase in the current cannot be accounted for by a movement of positive ions toward the cathode is shown by two lines of reasoning. The first is that the electrostatic effect on the positive ions would check any large increase in the current due to them alone.

The Electrostatic Effect Produced by Ions.—An electrostatic effect occurs whenever there are more ions of one kind than of the other in a given volume. This effect may become so large as to reverse the previous direction of the field and to limit very greatly the amount of current flowing. This is especially apt to occur when there are positive ions present, for their mass is much greater than that of the negative ions and their motion correspondingly slower.

The ratio of the mass of positive ions to their charge is approximately 20,000 times as great as the corresponding ratio for the negative ions.[1] As a result one positive ion going through a given space per second would neutralize the effect of 20,000 electrons. If there were more positive ions than this they would raise the potential of the region and tend to check their own movement.

It is not possible to compute just how many positive ions it would take to raise the potential so as to check the current when the cathode is an irregular piece of foil, but we may get some idea of what is to be expected by considering the current density between

[1] Thomson's Conduction of Electricity through Gases, 2d ed., p. 149.

parallel plates of infinite extent, when there are only positive ions present. The less the ions are stopped by collisions with molecules, the greater will be their velocity and the less the electrostatic effect produced by the movement through the field of a given number. In order to find the maximum current, we may assume that there are no collisions. I have not been able to find any computation applying to this case, and hence have given it in the following paragraphs.

Let us assume that the plates A and C are of infinite extent, separated by a distance x_1. Let the potential of C be zero and that of A be V_1, and assume that there are at A an indefinitely large number of positive ions. The electric force will cause these to move toward C, producing a current.

Fig. 3.

Let I = the current flowing through a unit cross section,

n = the number of ions in a unit volume,

ϵ = the charge in electrostatic units carried by each ion,

m = the mass of each ion,

V = the potential in electrostatic units at any point at a distance x from C,

v = the velocity of the ions at a distance x from C,

ρ = the density of the electricity at a distance x from C.

$I = n\epsilon v$ = a constant, and the acceleration of the ions equals the charge times the electric force divided by the mass, or

$$\frac{dv}{dt} = -\frac{\epsilon}{m}\frac{dV}{dx}.$$

If we multiply this equation by the equation, $vdt = dx$, and integrate, remembering that the velocity is zero when V is V_1, we have

$$\frac{1}{2}v^2 = \frac{\epsilon}{m}(V_1 - V) \qquad \text{or} \qquad v = \left\{\frac{2\epsilon}{m}(V_1 - V)\right\}^{\frac{1}{2}}.$$

But

$$\rho = n\epsilon = \frac{I}{v} = \frac{I}{\left\{\dfrac{2\epsilon}{m}(V_1 - V)\right\}^{\frac{1}{2}}}$$

and if we assume that there is no variation in potential in directions
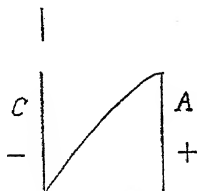
parallel to the plate we have

$$\frac{d^2V}{dx^2} = -4\pi\rho = -\frac{4\pi I}{\left\{\frac{2\epsilon}{m}(V_1 - V)\right\}^{\frac{1}{2}}} = -\frac{k}{(V_1 - V)^{\frac{1}{2}}}$$

where k is a constant equal to $4\pi I/\sqrt{2\epsilon/m}$.

Multiplying this by the equation

$$\frac{dV}{dx}dx = dV$$

and integrating, we have

$$\frac{1}{2}\left(\frac{dV}{dx}\right)^2 = 2k(V_1 - V)^{\frac{1}{2}} + C.$$

The constant, C, equals the value of $\frac{1}{2}(dV/dx)^2$ at A. As the space between A and C becomes more and more filled with positive ions, the value of dV/dx approaches zero, and for the largest current which it is possible to have it would equal zero. For such a current we have

$$\frac{dV}{dx} = 2\sqrt{\bar{k}}(V_1 - V)^{\frac{1}{4}}.$$

Further integration gives us

$$V = V_1 - (V_1^{\frac{3}{4}} - \tfrac{3}{2}\sqrt{\bar{k}}\,x)^{\frac{4}{3}},$$

since $V = 0$ when $x = 0$. The curve in Fig. 3 between A and C represents such a distribution of potential as this.

Since V_1 is the value of V when $x = x_1$, we have

$$\tfrac{3}{2}\sqrt{\bar{k}}\,x_1 = V_1^{\frac{3}{4}}$$

or

$$I = \frac{1}{9\pi}\sqrt{\frac{2\epsilon}{m}}\frac{V_1^{\frac{3}{2}}}{x_1^2}.$$

In other words, this is the largest current which it is possible to have carried by positive ions with the given distance and the given potential difference between the plates.

The value of ϵ/m for positive ions in electrostatic units is approximately 12×10^{12}.[1] If for example we take one third of an electro-

[1] Thomson's Conduction of Electricity through Gases, p. 149.

static unit (100 volts) for the potential difference between A and C and the distance 4 cm., the current would be 2.08×10^3 electro-static units, or 7×10^{-7} ampere.   This is the amount of current which could be carried if there were only positive ions, or the amount carried by the excess of positive ions.   A certain amount might be carried by negative ions, a second small amount by the positive ions which would be needed to neutralize the electrostatic effect of these negative ions and the amount given above would be the greatest possible additional amount which the excess of positive ions could carry.

The potential difference and distance assumed are similar to those existing in the experiments which have been described.   The shapes of the electrodes in the experiment were indeed quite dif-ferent from those considered in the mathematical treatment.   But when we remember that it was possible to have an increase of nearly an ampere per sq.cm. a few centimeters away from the cathode and of many times this density of current near the cathode, we can see how improbable it is that any such increase in the current was carried by positive ions coming from the gas.

But an even more convincing reason for believing that the increase in current is not produced by a movement of the positive ions toward the cathode is given by the appearance of the luminous discharge.   With non-luminous discharge there were, of course, no streams of cathode rays which could be detected.   With the lumi-nous discharge there were very noticeable cathode rays, appearing as brilliant streamers extending from the cathode to the sides of the tube or down into the gas.   That these were cathode rays was shown by the effect which a magnet had upon them and by the phosphorescence which they produced.   One could hardly see these streamers without realizing that something had happened to in-crease enormously the number of electrons leaving the cathode.

Increase in Current not due to Change in Field.—While the increase in current cannot be due to the movement of positive ions, it is conceivable that it is due to a change in field produced by them. When there is no ionization of the gas, the electrons coming from the cathode may be so numerous as to reverse the direction of the field in part of the space and hold back other electrons which may

be leaving the CaO. That this actually could occur is shown by the data given in Table IV. When there are also positive ions in the field, they may so neutralize the effect of the electrons, that all the electrons which escape from within the CaO will pass to the anode. In order to consider this possibility the potential between the electrodes was examined both with non-luminous and with luminous discharge.

Potential between the Electrodes.—The apparatus shown in Fig. 2 was used to measure the potential between the electrodes, and since only one exploring wire was needed, e' was removed.

There are given in Table IV. a series of readings of the potential between A and C taken when the cathode was a dull red and a non-luminous current of 1×10^{-8} ampere was passing through the tube. In Table V. there is given a similar set taken with a luminous current of .03 ampere. In both cases the pressure of the gas was .02 mm. This was so low that there were no striations nor anode glow between the electrodes. The potential difference between A and C was 90 volts, and the distance 8 cm. Column one gives the distance from C and column two the potential difference between C and the exploring electrode as measured by an electrometer.

TABLE IV.

Distance from Cathode in cm.	Pd between C and e in Volts.
.3	−1.
2.	0.
4.	3.2
6.	7.7
7.	14.3

TABLE V.

Distance from Cathode in cm.	Pd between C and e in Volts.
.3	71
2.	71
4.	69
6.	69
7.	66

The first value in Table IV. indicates that the potential of the gas near the cathode was slightly lower than that of the cathode itself. This is due to the tendency of the CaO to give out electrons faster than the field draws them away, in fact to emit them against a small electric field, as has been pointed out by Richardson.[1]

[1] Phil. Mag., 6, 16, 354, 1908.

In the second case the same kind of an effect occurs except that here the electric force is reversed not near the cathode but a few centimeters from it. This fact has been observed by Thomson.[1] The momentum of the ions is such that it carries them for some distance against the opposing field.

The potentials here given are much lower than those given by Westphal.[2] He was undoubtedly working with much higher temperatures than those existing in these experiments.

There results are plotted in Fig. 4, the first curve showing the
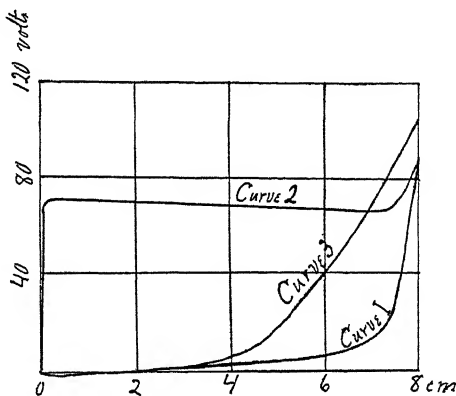
Fig. 4.

potential when the discharge was non-luminous and the second when it was luminous. With the first one there is an excess of negative ions between the electrodes, with the second an excess of positive. In the first case there is little or no ionization in the gas, while in the second there is a large amount. This might at first lead one to think that the difference between these two forms of discharge was entirely due to the presence of the positive ions near the cathode and the resulting drop in potential at that point. For this would cause all of the electrons which might escape from within the CaO to pass through the tube, but further light is thrown on the matter by examining the potential between the electrodes when discharge is passing from clean platinum.

For this purpose clean platinum was substituted for that covered

[1] Phil. Mag., 6, 18, 442, 1909.

[2] Deutsch. Phys. Gesell. Vehr., 10, 11, 401, 1908, and Science Abs., 11, p. 519.

with CaO and observations were taken with the platinum heated to a bright red. The potential difference between the electrodes was 90 volts and the current through the tube was 5×10^{-7} ampere. The potential at different points was found to be approximately the same as those given in Table IV., when there was non-luminous discharge from CaO.

TABLE VI.

Distance from Cathode in cm.	Pd between C and e in Volts.
.3	$-1.$
2	$-.4$
4	4.5
6	42.
7	70.

These data are plotted in curve three, Fig. 4. There is here little change between this curve and that found with small currents as far as the space near the cathode is concerned, but a large change near the anode. There is apparently ionization of the gas near the anode, but not enough positive ions are produced to change the potential of all the space.

By raising the temperature of the platinum still higher a current of 4.8×10^{-5} ampere was passed through the tube. The potential in the neighborhood of the cathode was unchanged, while that in the neighborhood of the anode was still further raised. For example, the potential 6 cm. from the cathode became 68 instead of 42 volts.

It is apparent that this form of discharge is not the same as either the non-luminous or the luminous form of discharge from CaO, since we have some ionization of the gas and yet there is no great change near the cathode. Apparently something besides ionization in the gas is needed in order to produce this change. *Some Ionization of the Gas Necessary.*—While something besides ionization of the gas appears to be necessary in order to change from the non-luminous to the luminous discharge, it is also necessary that there should be this ionization, as is shown by the following experiment.

Fig. 5.

Two pieces of platinum wire, C and C' in Fig. 5, are surrounded by brass cylinders of different diameters and placed in a vacuum

tube. Both wires were .5 mm. in diameter, had the same length and were heated by the same current. They were coated with CaO and were made the cathode with the cylinders as anodes. The smaller cylinder was 4.85 mm. and the larger 23 mm. in diameter. Both were 4.5 cm. in length. The potential difference between the wires and the cylinders was in all cases 90 volts. There are given in the following table the currents to the cylinders with two different pressures of the gas.

TABLE VII.

Pressure of Gas in mm.	Current to Small Cylinder in Amperes.	Current to Small Cylinder in Amperes.
.35	.01	8×10^{-3}
.014	5.8×10^{-6}	1.6×10^{-3}

This shows that the current to the small cylinder decreased very much when the pressure of the gas was diminished below a certain amount, and that at the same time the current to the large cylinder increased. The number of electrons coming from within the CaO was no doubt the same for both wires, and the electric force was, of course, in both cases larger for the small cylinder. In the first case the discharge to the small cylinder was the larger, as one would expect, since the electric force was larger. Apparently the small discharge to the small cylinder in the second case is due to the fact that there were then so few molecules within the cylinder that there was less opportunity for impact and consequently there was little ionization. It would, therefore, appear that while ionization of the gas is not the only requirement, it is at least a necessary requirement.

The Correct Explanation.—It has already been stated that much smaller potential differences are adequate to produce luminous discharge from hot CaO than from any cold cathode, but this is practically the only difference between the two cases. In both the luminous discharge commences with great suddenness and in both there are streams of electrons shooting out from the cathode. It would, therefore, seem as if some modification of the explanation which Thomson has given for the ordinary discharge in a vacuum would come nearer the truth than that which was suggested.

His explanation for such discharge is briefly as follows: The current through the tube is carried by ions. The production of these is a two-fold action. Negative ions which have been driven off from the cathode hit the molecules of the gas and ionize them by their impact. The positive ions thus formed are drawn up to the cathode and ionize the molecules at the surface of the cathode by their impact on them. The negative ions thus formed repeat the process by ionizing more molecules of the gas. The electric force in the tube must be sufficient to produce ionization at both of these places, but since it requires a much higher potential difference to ionize by the impact of the positive ions, it comes about that the critical potential difference is reached when such ionization begins.

It is altogether probable that the same thing occurs in the case of discharge from hot CaO, the only difference being that the molecules on the surface of the hot CaO are more easily ionized than those on the cold cathode, and there are reasons for believing that this is true, as will be given shortly.

If we make this assumption the phenomena may be explained as follows: As soon as the electric force is great enough to ionize the gas, we have a slight increase in current, such as is shown with the discharge from clean platinum. This increase does not become large, unless the positive ions thus formed are able in turn to produce ions by impact on the surface of the CaO. When this occurs, the greater the number of positive ions formed in the gas, the greater the number bombarding the cathode with the corresponding further increase in the number of electrons sent off. There is thus produced the sudden change in current which occurs at the critical point. This change is limited by the rise in potential between the electrodes which is caused by the presence of the positive ions. When this increase becomes so large that the ions are not carried to the electrode as fast as formed, the current remains stationary.

This sudden increase is probably helped by the fact that the rise in potential in the tube causes the principal drop in potential to be near the cathode, instead of near the anode. The electrons coming from the cathode have a certain velocity when first emitted. A drop in potential in its immediate neighborhood increases this and produces a greater final velocity than if applied to electrons which might be starting from rest at some point in the gas.

There are the following reasons for believing that the molecules on the surface of the cathode are more easily ionized by impact of positive ions when the cathode is hot than when cold. Due to their high temperature they are already in unstable equilibrium, as is shown by their sending out electrons even when not being bombarded. One would in fact be surprised, if it did not require a smaller momentum in this case to produce ionization than when the molecules are at a low temperature.

Secondly it has been shown by Hittorf[1] and Cunningham[2] that heating the cathode causes the cathode drop to become smaller.

Furthermore I have found that the higher the temperature of the CaO the less the cathode drop in its neighborhood. The drop given in Table V. was 71 volts, but when the temperature of the CaO was raised, this decreased rapidly. When heated as much as practical without melting the platinum foil, the cathode drop was only 8 volts.

Increasing the current through the tube also decreased the cathode drop but only to a slight extent. Thus in one case the current was varied from .02 to .4 ampere by changing the voltage at the terminals of the tube, the heating current through the foil remaining constant, and the cathode drop varied from 30 to 21 volts. The same change could be made by a very slight increase in the current heating the foil, and it is altogether probable that this change was due to the increase in the temperature at the surface of the cathode caused by the increased bombardment of the cathode and not by repeated impact on the molecules of the CaO.

Production of Electrons by Bombardment of CaO with "Canal-strahlen."—In addition to this it was shown that electrons may very easily be produced when CaO is bombarded with positive ions in the form of "canal-strahlen." It has already been shown by Austin[3] that when such rays strike a metal plate electrons are produced, but it seemed well to investigate the phenomena which exist when they strike hot CaO under conditions somewhat similar to those which held in the preceding experiments.

[1] Wied. Ann., 21, 133, 1884.
[2] Phil. Mag., 6, 4, 684, 1902.
[3] Phys. Rev., 22, 312, 1906.

For this purpose the tube given in Fig. 6 was used. An opening was made in the side of the tube shown in Fig. 1, and a second tube, T', was sealed into this. This had a cathode, C', and an anode, A', the distance between them being 9 cm. The anode, A', was a metal disk filling the inside of the small tube and having an opening of 2 mm. in diameter in the center. A discharge could be sent through this tube by means of a Wimhurst machine or an induction coil at I and with proper pressure of the gas "canal- strahlen" would hit on the cathode, C. The anode A' was kept at the same potential as C by connecting it to the point b shown in Fig. 1.

Fig. 6.

With this arrangement C was heated by passing a current through it as in the preceding experiments and a potential difference was established between A and C somewhat below that needed to cause the luminous discharge. If then a discharge was caused to pass from A' to C' so that the "canal-strahlen" hit upon C, luminous discharge would start up in T. With potential differences of about 70 volts this discharge would pass as long as C was being hit by these rays and would stop as soon as the rays would stop.
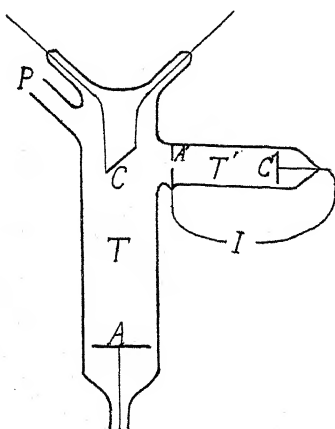
With larger voltages a continuous arc was often formed. With lower voltages the discharge between A and C was non-luminous but still considerably larger than what it was when there were no "canalstrahlen."

It was found that it was not even necessary to heat C. When C was at the room temperature, and the voltage was approximately 100 volts, the positive rays striking on C caused a luminous discharge between A and C to flash out for an instant. This luminous discharge showed striations the same as those usually produced by hot lime, and if the resistance in series with A was sufficiently small, the discharge became an arc.

Relation between this Work and the Theory of the Electric Arc.—

The work which has been described has an important bearing on the theory of the electric arc. There is no question but that the ionization at the cathode is the essential phenomenon of the arc, but there are two views concerning the cause of this ionization. According to Stark[1] and Thomson[2] the electrons are driven out of the cathode because of its high temperature in the same way as discharge is produced from hot platinum wire. The bombardment of the cathode by the positive ions heats it, but does not directly cause the ionization.

According to the other view the ionization at the cathode is produced directly by the bombardment of the positive ions on the molecules on the surface of the cathode, the same as in the Geissler tube discharge, the cathode drop in the arc being smaller than elsewhere, because a smaller potential difference is needed to produce ionization when the molecules of cathode are hot.

There are some rather serious objections to the first view. For example, mercury can be the cathode of an arc in a vacuum, although it cannot be raised to anything like a high temperature in a vacuum without changing it to a vapor. On the other hand, iron can easily be heated to a point where it will give off electrons, and yet it cannot be made the cathode of an arc in a vacuum.[3]

In addition to these arguments the work which is here described furnishes a further argument in favor of the second view. The arc is apparently the same form of discharge as that which we have been studying, for it has been shown[4] that it is possible to pass by gradual changes from the luminous discharge produced by hot CaO to that of the electric arc. The only essential difference between the two is that in the arc the current through the gas heats the cathode sufficiently to maintain the temperature of the cathode, while in the other form the temperature must be maintained by some outside source, and this difference has nothing to do with the manner in which the ions are produced.

We are, therefore, safe in saying that the weight of evidence is in favor of the view that the ionization at the cathode of the arc

[1] Ann. d. Phys., 12, 673, 1903.

[2] Conduction of Electricity through Gases, 2d ed., p. 612.

[3] Phys. Rev., 20, 369, 1905.

[4] Phys. Rev., 29, 361, 1909.

is not caused by the high temperature of the cathode, but by the impact of positive ions on the hot surface.

Summary.—When the potential difference between the electrodes is increased the discharge in a vacuum from hot CaO passes through a critical condition, the current being very much greater after the critical point is passed and the discharge becoming luminous. It has been suggested by Sir. J. J. Thomson that this sudden change is caused by the molecules having been hit so often by the electrons that many of them are in a state of unstable equilibrium, and that when in this state but a small increase in the electric force is needed to cause ionization.

There are the following reasons for thinking that this is not the correct explanation. First, the scarcity of electrons present in the tube render such repeated collisions very improbable. Secondly, the potential difference necessary to cause this sudden change does not depend on the number of electrons passing through the tube but does depend on the condition and temperature of the cathode. Third, an examination of the potential gradient through the tube leads us to believe that the sudden increase in current is caused by something other than an increase in the ionization in the gas. Lastly the appearance of the discharge indicates very plainly that the number of electrons streaming from the cathode increases enormously when the discharge becomes luminous, becoming in fact very noticeable streams of cathode rays. On the other hand, some ionization of the gas appears to be necessary as is shown by the fact that, when the pressure of the gas is very low, the discharge from a wire coated with CaO to a small surrounding cylinder is much smaller than that to a large cylinder.

The phenomena may be explained by asssuming that ionizaton is produced at the cathode by the bombardment of its surface by positive ions and that such ionization occurs very much more easily with a very hot cathode than with one which is cold.

The potential difference between the beginning and end of a striation was found to increase as the pressure of the gas increased and to decrease slightly when the current was increased. It was in certain cases as low as 10 volts.

Because of the similarity between this form of discharge and

the electric arc, it is reasonable to assume that the electrons at the cathode of the arc are also produced by bombardment of the cathode by the positive ions which come from the gas, and that they are not to any great extent emitted from within the cathode because of its high temperature. Such an explanation accounts for certain difficulties which may be raised against any other explanation.

COLGATE UNIVERSITY,
 January, 1911.

ON POSITIVE ATOMIC CHARGES.

BY FERNANDO SANFORD.

SINCE Faraday's discovery that the same definite quantity of electricity was apparently set free by the electrolytic separation of the combining weight of any univalent element from any compound, it has been believed that univalent ions all carry the same definite charge of negative or positive electricity, and that ions of higher valency carry some simple multiple of this unit charge. Since the discovery and isolation of the unit negative charge great efforts have been made to isolate the corresponding positive charge, but so far without results. This is not surprising, since our present understanding of the phenomena of electrolysis makes the assumption of the unit positive charge unnecessary, as we know of no case where an ion gives off a positive charge to or receives a positive charge from either electrode.

So far as our knowledge extends at the present time, an atom consists of a positive part (subatom) combined with some unknown number of electrons. The positive subatom is generally regarded as the strictly material part of the atom, since the atomic mass seems to be associated with it. A gas molecule, when not monatomic, apparently consists of two or more positive subatoms held together by their mutual attraction for the same electron or group of electrons. These positive subatoms accordingly seem to have permanent electrical charges, and the difference in the force with which they cling to electrons indicates that these charges are different for different atoms.

Many attempts have been made to measure the specific charges of the positive subatoms, but apparently the assumption has always been made that the characteristic charges were simple multiples of charges equal and opposite to the electronic charge, and the results reached have accordingly been difficult of interpretation. The determination of the charge of the alpha particle by Rutherford

and Geiger and by Regener have strengthened the notion of the unit positive charge, since the alpha particles were found to be electrically equal and opposite to two electrons.   Accordingly, when the alpha particle combines with two electrons and becomes a helium atom it gives an electrically neutral atom, one which has virtually no affinity or cohesion for other atoms.   This is not true to the same extent of any other known atom, and it would accordingly seem that no other atom is so nearly electrically neutral as the helium atom.

There are several methods of separating an atom or molecule into its ions and of estimating the relative charges of these ions. Probably the best known of these methods has been by dissociation in water.   In the case of the positive ions in electrolysis we know only the number of electrons which they may take up from the cathode and their average velocity of drift through the liquid under the impulse of a known electric field.   We do not know the resistance with which they meet in their movement through the solution, nor through what distances they travel between successive dissociations and recombinations.   We know only that if the ions of a given group all have equal positive charges they are acted upon by the same force.   If the speeds measured are assumed to be the maximum speeds which the ions can reach in the solution, then the slower ions are slower because they are retarded more than the faster ones, and it is natural to expect the smaller ions to have the higher speeds.   The attempts which have been made to apply Stokes's equation to their movement would make their velocities vary inversely as their diameters while, on the contrary, the larger ions of each group have the higher speeds.   If we assume, on the other hand, that they move only very small distances as free ions, they will move with accelerated velocities, and if their charges are equal the lighter ions will have the higher speeds.   This is again contrary to experience, since the heaviest ions of each group have the highest speeds.

On the other hand, if it be admitted that different subatoms have different electrical charges it is very easy to account for their differences in ionic velocity.   While we cannot with certainty calculate the magnitude of these hypothetical charges without knowing more

about the phenomena of electrolysis, we may be justified in making
the attempt to see where it will lead.

If the ions in the solution move only from molecule to molecule
as free ions their motion will be accelerated according to the equa-
tion $F = ma$. Since the force acting on an ion is proportional to its
charge, if the ion met with no frictional retardation its charge would
be proportional to the product of its mass into its ionic acceleration,
and the charges of different ions would be relatively proportional
to the products of their ionic masses into their ionic speeds. The
following table[1] gives for three well-known groups of elements the
ionic charges calculated in this way, the ionic speeds being taken
from Lehfeldt's tables. In the last column are given the ratios of
the hypothetical charges of corresponding elements in different
groups. It will be seen that for each pair of groups these ratios
are constant within the limits of our knowledge of ionic velocities,
and that within each group the elements are arranged according
to their order in the voltaic series.

Element.	Atomic Weight.	Ionic Velocity.	Ionic Charge.	Ratio of Charges.
Cs	133	78.8	10,480	Cs/I = 1.07
Rb	85.5	78.6	6,720	Rb/Br = 1.07
K	39	75.5	2,945	K/Cl = 1.09
Na	23	52.6	1,210	Na/F = 1.17
Li	7	42.6	298	
H	1	365	365	
I	127	77	9,779	I/Ba = 1.04
Br	80	78.1	6,248	Br/Sr = 1.06
Cl	35.5	75.1	2,666	Cl/Ca = 1.07
F	19	54.4	1,034	F/Mg = 1.02
Ba	137.4	68	9,343	Cs/Ba = 1.12
Sr	87.6	67	5,869	Rb/Sr = 1.14
Ca	40.1	66	2,646	K/Ca = 1.11
Mg	24.3	46	1,108	Na/Mg = 1.18

At the time when this table was prepared it was regarded as
having significance only in that it served to strengthen a belief

[1] This table was prepared for a paper read before the San Francisco meeting of the
American Chemical Society, July 16, 1910, and was published in an abstract in Science
of October 7, 1910.

already formed on other grounds that the electrical charge of an atom is as characteristic a physical constant as its atomic weight. Later developments have served to give the figures presented in the table a much higher significance. In a paper by Richardson and Hulbirt in the Phil. Mag., XX., 545, October, 1910, is given a list of values of e/m for a number of positive ions given off by hot metals. The authors express surprise that the values should vary so little for the different metals and attempt to account for this fact by assuming that the ions are not the true subatoms of the metal under investigation, but are due to alkaline impurities in the metals, apparently overlooking the fact that in a previous experiment they found that the ions from the alkali metals were given off with much greater speeds. If the ions are the subatoms of the metals which were heated, the data given by Richardson and Hulbirt enable us to calculate their characteristic charges.

Ten metals were investigated, but of these four gave unsatisfactory results while another, gold, gave two values of e/m. Thus iron gave two values for different samples, and the individual values for the same sample differed by more than 100 per cent. Only one measurement was made on osmium, the single values for tantalum ranged from 115 to 253 and for tungsten from 47 to 541. It seems extremely probable that these metals contained traces of some metals whose ions were more easily given off.

The other six metals belong in two well-marked groups. Their values of e/m and values of e obtained from them by multiplying by the atomic weight are given below. The values of e/m used are those which were determined before they were multiplied by a constant to correct for the error made in determining the value of e/m for an electron by the same method, but as only relative values are considered here this correction is unnecessary.

Element.	e/m	e	Element.	e/m	e
Pt	243	474	Au	139	376
Pd	212	225		280	552
Ni	239	130	Ag	215	232
			Cu	230	146

It will be seen at once that there is a striking similarity between

the relations of the atomic charges calculated in this way and the relation shown in the table calculated from the ionic velocities in electrolysis.   It will also be seen that by multiplying the electrolytic charges given by three they become strictly comparable, group by group, with the charges calculated from Richardson and Hulbirt's data.   Further, it was seen that in each group the atomic charge varies approximately as the square root of the atomic weight.

In the table below the elements whose charges have been calculated by both methods are given in groups according to their arrangement in the periodic tables.   Below each element are given the square root of its atomic weight and its relative atomic charge as calculated above.

H	Li			
1.	2.5			
11.	9.			
F	Na	Mg		
4.4	4.8	4.9		
31.	36.	30.		
Cl	K	Ca		
6.	6.2	6.3		
80.	88.	79.	Ni	Cu
			7.6	8.—
Br	Rb	Sr	130.	146.
9.	9.25	9.4		
188.	202.	176.	Pd	Ag
			10.3	10.4
I	Cs	Ba	225.	232.
11.3	11.5	11.7		
293.	314.	280.	?	?
			Pt	Au
			14.—	14.+
			474.	546. and 273.

The relation is still better shown in the accompanying curve where the atomic charges are used as abscissas and the square roots of the atomic weights as ordinates.   It will be seen that with the exceptions of lithium and hydrogen the charges are very approximately proportional to the square roots of the atomic weights, and that the charges calculated from the two entirely different

sets of data fit the curve equally well. Strictly speaking, the elements of each group seem to lie on a curve of their own, but these curves are parallel and very close together.
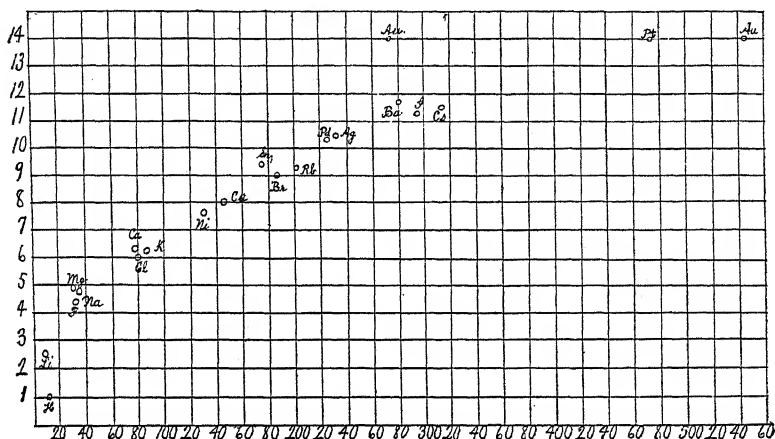
Fig. 1.

Other arguments which seem to the writer to strengthen the hypothesis that the positive subatoms of the elements have characteristic charges which vary as the square roots of their atomic weights may be found in the deflection of the positive ions of a Bunsen flame by an electric field, in the conductivity of flames charged with various salts and especially in the phenomena of chemical valence. The subject is discussed at greater length in a paper soon to appear in the Leland Stanford Jr. University publications.

STANFORD UNIVERSITY,
    December 3, 1910.

ATOMIC CHARGES AND COHESION.

By Fernando Sanford.

IN a paper on positive atomic charges which has been accepted but has not been published by the Physical Review it is shown that the positive subatoms apparently carry characteristic charges which are proportional to the square roots of their atomic weight, or more precisely, which may be calculated for atoms having an atomic weight greater than 13 from the equation $e = (\sqrt{W} - a)k$, where w is the atomic weight, a is a constant having a value of approximately 3.6 and k is a constant depending upon the system of units used.

Since writing the above paper I have become acquainted with the results of two other investigations which have an important bearing upon the theory of characteristic atomic charges. The first is a paper by A. von Antropoff in Roy. Soc. Proc., A, 83, 474, April 14, 1910, on the solubility in water of the gases of the Argon group. v. Antropoff's numbers for the solubility of these gases at 20 degrees are as follows: He, .0138; Ne, .0147; Ar, .0379; Kr, .0729; Xe, .0119.

Since cohesion is undoubtedly an electric attraction, and since we may expect the solubility of gases in water at a given temperature to be proportional to cohesion, these numbers suggest a method of calculating the relative magnitude of the charges carried by these monatomic molecules. If these numbers are proportional to the atomic charges, we should be able to calculate them from the formula $S = (\sqrt{w} - 3.6)k$, where S is the solubility factor and w the atomic weight of the element. The following table contains v. Antropoff's observed values of S and the values calculated from the above formula, letting $k = .014$.

It seems from the above table that with the exception of helium, whose charge cannot be calculated from our formula, but which is shown in the preceding paper to be much larger than is indicated

by theory, the solubility can be calculated from the above formula as closely as it can be measured.

Element.	w	S Obs.	S Calc.	Per Cent. Error.
He	4	.0138		
Ne	20	.0147	.0122	17
Ar	40	.0379	.0378	—
Kr	82	.0729	.0763	4.6
Xe	128	.1109	.1078	2.8

Average error = 6.1 per cent.

Another investigation which is related to the subject under consideration is one on the viscosity of the gases of the argon group, by Rankine, in Phys. Zeitsch., 11, 746, September, 1910. The bearing of Rankine's work on the subject under consideration rests on the fact that Sutherland, in a paper on "The Viscosity of Gases and Molecular Force," Phil. Mag., 36, 507, 1893, gives a formula for calculating a cohesion factor from the viscosity of gases. Rankine calculated Sutherland's cohesion factor, C, from his data and found the following values: He, 70; Ne, 56; Ar, 142; Kr, 188; Xe, 252. He calls attention to the evident misfit of helium in the series, and shows that for the other gases Sutherland's factor, $C = T_c/1.12$, where T_c is the critical temperature (absolute) of the gas. Calculated in this way, C should be very small for helium.

Since the cohesion between gas molecules must be due to the attraction of the positive subatoms of one molecule for the electrons of another molecule, these attractions would seem to be approximately proportional to the atomic charges, though here, since the electric fields of the electrons in the molecules of different gases cannot be assumed to be equal, as they can in the case of molecules of a single kind, we cannot look for the close agreement with the law which we find in the case of cohesion between gas molecules and water molecules. That is, in the case of solubility, each molecule is attracted by a group of water molecules which may be assumed to be equally charged in all cases, while here the attraction is between the electrical fields of similar molecules which are probably different for each gas.

The observed and calculated values of C are given below where, as before, $a = 3.6$ and where $k = 37$.

Element.	C Obs.	C Calc.	Per Cent. Error.
He	70		
Ne	56	32	43
Ar	142	100	30
Kr	188	202	7
Xe	252	285	13

Average error = 23 per cent.

The above values can be shown to vary much closer with the square root of the atomic weight than is indicated by the above table. Thus if $a = 2$ and $k = 27$, the following values are calculated:

Element.	C Obs.	C Calc.	Per Cent. Error.
He	70	0	
Ne	56	66	20
Ar	142	113	20
Kr	188	190	2
Xe	252	251	0

Average error = 5 per cent.

The above data seem to indicate that both the viscosity of gases and their solubility in water depend upon the characteristic charges carried by their molecules. Since most gaseous molecules are compound, consisting of two or more positive subatoms held together by their mutual attractions for one or more electrons, their charges are the resultant of the positive and negative charges of which they are composed and consequently cannot be calculated by the formula given above; but here the relation between their resultant charges and their critical temperature and solubility would probably hold. Hence, so long as the gas molecules or the water molecules are not dissociated by their mutual cohesion we would expect the solubility of gases in water to be related to their critical temperatures.[1]

[1] At the time when this paper was written I had not read a paper by R. D. Kleeman, entitled On the Nature of the Forces of Attraction between Atoms and Molecules which was published in the Phil. Mag. for May, 1910. In this paper Kleeman has calculated for a number of atoms a constant C_a, which determines the surface tension of the molecules in a liquid form to which the atoms belong, and has found this constant to vary as the square root of the atomic weight. Also, since this paper was in the hands of the Editors of the Physical Review Rankine has,

himself, published an article in the Phil. Mag. of Jan., 1911, in which he relates the viscosity of the inert gases to their atomic weight and gives the equation $N^2/A = 3.93 \times 10^{-10}$ in which N represents the viscosity factor and A the atomic weight. In this paper Rankine also gives the critical temperature of these gases as far as known and calculates it for neon and emanation. The values as given make the critical temperature almost exactly proportional to the square root of the atomic weights.

Since the melting points of the elements, as well as their critical temperatures depend upon cohesion, the melting points of the elements of a given group should vary as the square root of their atomic weights. Thus the melting points of the alkali metals can be calculated to within less than one per cent. from the equation

$$T = \frac{531.5}{\sqrt{w}} + 252.5$$ in which T is the melting point (absolute) and w the atomic weight.

The compressibilities of several groups of elements can be calculated from similar formulæ as closely as they can be measured. The atomic refraction constants for a number of the elements, both as calculated by Edwards in Vols. 16 and 17 of the American Chemical Journal and by Eisenlohr in Zeit. phys. Chem., LXXV., 585, varies as the square root of the atomic weights. There can be no longer any doubt that this is the prominent factor in determining cohesion, nor that cohesion is an electrical force, hence it seems that the electrical charges of the atoms should also vary in some way with the square root of their atomic weights.

March 27, 1911.

STANFORD UNIVERSITY,
    December 31, 1910.

A FORM OF NEUTRAL TINT ABSORBING SCREEN FOR PHOTOMETRIC USE.

By Herbert E. Ives and M. Luckiesh.

PERFECTLY neutral tint absorbing screens, suitable for reducing intensities with the photometer, or spectrophotometer, are not easily obtained. Certain neutral tint glasses on the market appear on measurement to be a mixture of "smoke" and cobalt blue. The "smoke" alone produces a yellow tinge, addition of cobalt blue produces a neutral tint, which is neutral only when the total transmission is viewed by the eye. It is a subjective gray, which on measurement reveals the characteristic absorption bands of cobalt blue, and such glasses are, as a consequence, useless for the spectrophotometer. Photographic screens have been spoken of as entirely non-selective. Those that we have prepared did not prove so on critical measurement, their defect being a progressive deficiency of transmission toward the shorter wave-lengths. Furthermore the large amount of scattering of light with them made their effective transmission vary with position to an unsatisfactory degree. Wherever possible there is one simple escape from these difficulties and that is the use of a rotating sectored disc. At other times the distance of the light source from the photometric screen may be changed. Occasions sometimes arise when neither the disc nor varying distance can be used, and in such cases the screens described below may be found useful.

In an investigation on the flicker photometer with spectral colors need was found for some means of reducing intensity in an entirely non-selective way, by known amounts. The dimensions of the source of light, and the distance at which it is used prohibit the use of the law of inverse squares. The sectored disc is barred because of the stroboscopic effects caused by it in conjunction with the periodicity of the flickering device of the photometer. Resort was had to a special form of neutral tint screen, namely an opaque-line grating on glass, of the type used by photo-engravers.

These screens are made by ruling fine lines through a wax layer, etching the glass and finally filling the grooves with opaque material, such as plumbago paste. They may be obtained as fine as 1,200 lines and as coarse as 40 or less lines to the inch. The relative sizes of the opaque and clear spaces may be varied in the ruling from equality to one and four (approximately) either way, or, if the ruling is coarse, the ratio may be larger. The transmission of such screens may therefore be anything from about 80 down to 20 per cent., and by using two together crossed, as low as three or four per cent. Examination with the spectrophotometer of screens of dimensions properly chosen, as described below, fails to detect any selective absorption.

The question of calibrating these gratings as absorbing screens brings up some points not to be overlooked in their use. The most important point is that the transmission depends not only upon the relative size of opaque and transparent spaces but upon the position, fineness, and dimensions of the grating. These variations in the transmission only become of considerable amount under certain conditions which may be termed extreme. It is shown below that proper choice of spacing may make the consequent errors negligibly small in most practical cases.

The light received upon the photometric screen consists of the central image, and a certain number of lateral spectra. The state of affairs is easily seen by holding the grating between the eye and a light source. The number of spectra visible depends upon the size of the grating and its position with respect to the eye and the light, and of course, on the fineness of the spacing. The amount of light falling at any point on the photometer screen is obtained by a knowledge of the number of spectra received from the grating and the quantity of light in each.

If θ is the angle of incidence, and ϕ the angle of emergence, the number of the order is given by

$$m = \frac{d}{\lambda}(\sin\theta + \sin\phi),$$

where d = grating spacing, λ = wave-length used.

Let D = distance between light source and photometer screen.

αD = distance between grating and screen.

βD = half width of grating.

Then

$$m = \beta \frac{d}{\lambda} \left(\frac{1}{\sqrt{\beta^2 + \alpha^2}} + \frac{1}{\sqrt{1 + \alpha^2 + \beta^2 - 2\alpha}} \right).$$

In Fig. 1 are plotted values of $m\lambda/d$ for various values of α and
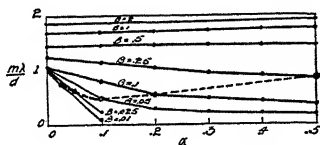
Fig. 1.

β.   In Table I. are given the values of d/λ for various values of d at different wave-lengths.   The product $m\lambda/d \cdot d/\lambda$ is at once the order, m, for the particular d studied.   It is seen that when the grating is small with respect to the distance between source and screen (β small), the number of orders illuminating the latter varies markedly with the distance.

To determine what differences in illumination these differences in the number of orders cause, a formula given by Rayleigh[1] for the light in each order has been used.   Let a and d be the widths of the transparent and opaque spaces respectively, m the order, B_0 the brightness of the central image, B_m the brightness of the mth spectrum, then

$$B_m : B_0 = \left(\frac{a + d}{am\pi} \right)^2 \sin^2 \frac{am\pi}{a + d},$$

also, if B is the brightness of the central image when the whole of the space occupied by the grating is transparent,

$$B_0 : B = a^2 : (a + d)^2.$$

These equations properly apply only to plane waves, *i. e.*, to the Fraunhofer class, while in using the gratings in the manner here considered we are dealing with the Fresnel class of diffraction phenomena.   The errors will however be small.   The light received on any linear element of a surface behind the grating, from a

[1] Lord Rayleigh, Scientific Papers, Vol. I., p. 213.

narrow line of light in front will be

$$I = B_0 \left[1 + \sum_1^m \left(\frac{a+d}{am\pi} \right)^2 \sin^2 \frac{am\pi}{a+d} \right],$$

where the summation includes the orders on each side. The total light transmitted for $m = \infty$ will be $\dfrac{a}{a+d} B$, a quantity which the above expression approaches as a limit; the light transmitted up to any order m may then be expressed in percentage of $\dfrac{a}{a+d} B$. In Fig. 2 are plotted as curves the number of orders included,
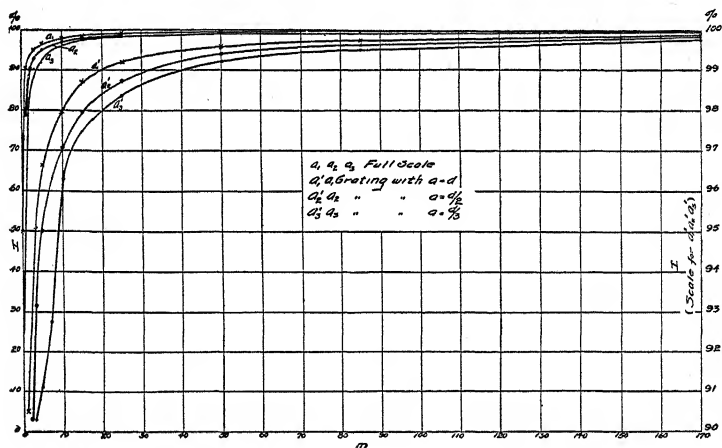
Fig. 2.

against the quantity I in per cent. of $\dfrac{a}{a+d} B$, for gratings in which $a = d$, $a = d/2$, $a = d/3$. The curves in Fig. 2 are plotted to two scales; a_1', a_2', a_3' on the larger scale are respectively a_1, a_2, a_3 magnified ten times.

For the present purpose it is not so important to study the absolute value of the total transmission,—this must be determined by experimental calibration. It is more important to study how the transmission varies with the variations in size and position which might occur in the practical use of the grating. Inspection of the curves shows that the transmission becomes constant to well within one per cent. after the fiftieth order, for the ratios of a

to d considered. It is only necessary that the extremes of the number of orders acting should both be high to secure any degree of constancy of transmission desired. By combining these data with those of Fig. 1 and Table I. we have everything necessary to solve any practicable case. Several such cases are next considered.

TABLE I.

Lines per Inch.	Lines per mm.	d (mm.)	d/λ ($\lambda=.45\mu$)	d/λ ($\lambda=.55\mu$)	d/λ ($\lambda=.65\mu$)	d/λ ($\lambda=.58\mu$)
50	1.97	.507	1126	921	780	874
100	3.94	.254	565	462	391	438
150	5.91	.169	376	307	260	292
200	7.88	.127	282	231	196	219
240	9.45	.1075	239	195	166	185
250	9.85	.1015	226	184	156	175
500	19.7	.0507	113	92.1	78	87.4

A grating of 7 cm. diameter, 240 lines to the inch, $a = d/2$ was available for experiment, and the following test was made to check the agreement between the calculated and experimental values of the transmission. The grating was placed between an incandescent lamp and the screen of a Lummer-Brodhun contrast photometer. The illumination on the screen was then measured for various
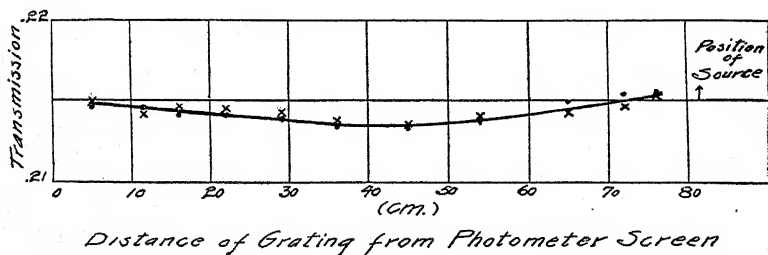
Fig. 3.

positions of the grating along the photometer track. A measurement of the illumination without the grating made possible a determination of the absolute values of the transmission. These are given in Fig. 3 and show a variation from 21.35 per cent. to 21.55 per cent., or a total variation of about 0.7 per cent. between the central position and the points 5 cm. from the lamp and screen.

The minimum at the center is in agreement with the curves of Fig. 1.

In order to find this variation by calculation we have $\beta = 3.5/81.5 = .043$. α varies from .5 at the center to $5/81.5 = .061$, at the points nearest to lamp and screen. Fig. 1 gives for the orders acting, $.17\ d/\lambda$ for $\alpha = .5$, and $.57\ d/\lambda$ for $\alpha = .061$. For λ we may take $.58\mu$, which closely represents the wave-length of maximum brightness in the present case; the orders are then (240 lines to inch) 31.5 and 105. from Table I. From Fig. 2 we obtain for $a = d/2$ values of I of 99.0 and 99.7. The difference between which agrees with the experimentally determined value.

This test indicates the reliability of the method of calculation, and so justifies applying it to the case of coarser or larger gratings, *i. e.*, to ones in which the variation in transmission is less than the one above measured. The case just considered is an extreme one, because in the first place the grating spacing is at least 5 times finer than is necessary for complete absence of line shadows on the photometer screen; in the second place, because one would not place a grating of this size so far from either light or screen under any practical conditions, as is the central position. Use of a grating of 50 lines to the inch would divide the error found by more than 5, *i. e.*, make it as small as .1 per cent. As the errors of the most careful photometric measurement is of the order of .5 per cent., an error of this size is small enough to be disregarded.

A case of some practical interest may be treated before drawing conclusions. Suppose a grating were used attached to a Lummer-Brodhun photometer head, that is, at a distance of about 5 cm. from the screen. Suppose it measured for absorption with the lamp at 100 cm.; what error would be introduced by using this value with the lamp at other distances? In this case α and β both vary. We must therefore take our values of $m\lambda/d$ from successive curves in Fig. 1. Thus, if the grating is 5 cm. wide we have at $D = 100$, $\alpha = .05$, $\beta = .025$; at 10 cm., $\alpha = .5$, $\beta = .25$, etc. These points are shown by the dotted line in the figure. These show a maximum value at $\alpha = .5$ (50 cm.) of .9, a minimum at $\alpha = .1$ (10 cm.) of .5. Applying these figures to the grating of 240 lines to the inch we have:

$$a = d/2, \quad \lambda = .58\mu,$$

$$m_{max.} = .9 \times 185 = 166.5, \quad m_{min.} = .5 \times 185 = 92.5.$$

$$I_{max.} = 99.8 \text{ per cent.}, \quad I_{min.} = 99.65 \text{ per cent.},$$

$$\text{difference} = .15 \text{ per cent.}$$

To within this degree of accuracy, therefore, the position of the light source is unimportant. A calibration made for one position holds for any position. With a coarser grating the calculated error is less, as discussed above.

This calculation includes two of the most usual cases of the use of absorbing screens; first, use on a photometer bar with the grating near the photometer screen and the light source at some distance; and second, the condition obtaining often with the spectrophotometer, namely, the light source quite near the diffusing surface. It appears then that a neutral tint screen of this character may be chosen which can be calibrated on an ordinary photometer bar, using sector discs or the law of inverse squares, and afterward used with negligible error with a spectrophotometer.

In the case of use with spectrophotometer it becomes of interest to investigate the possible selective transmission due to the different number of orders of different wave-lengths. If we take the minimum m from the last calculation, namely, $.5d/\lambda$ for the extreme wave-lengths $.65\mu$ and $.45\mu$ we obtain 83 and 119. From Fig. 2 we obtain for I_{83} and I_{119} respectively 99.6 per cent. and 99.7 per cent. or a selectiveness of 0.1 per cent. Here again the perfectly practicable coarser grating of say 100 lines to the inch would make this error still more negligible.

In the case of light sources and photometer screens of finite size the grating dimensions as calculated here need to be increased to secure very perfect uniformity of illumination. These calculations are however perfectly straightforward with the data given. Usually calculations as made above, on the basis of point source and point photometer screen, will be sufficient, especially if one allows a considerable "factor of safety" as has been done above by calculating the errors for a 240-line grating where a 100-line grating is quite fine enough.

Obviously gratings can be used for neutral tint screens only where the light is received on a diffusing reflecting surface, or diffusing glass. Where a bright filament is viewed or projected, gratings would be useless. However, a large part of practical photometric work conforms to the first condition. For certain special cases, where other means are not applicable these gratings should prove of use. Should the flicker photometer come into wide use, black line gratings would fulfil with it some of the functions of the sector disc with other forms of photometers.

A MICROSCOPE PLATE MICROMETER.

BY JOHN ZELENY AND L. W. McKEEHAN.

IN the plate micrometer due to J. H. Poynting, for use with micro-
scopes or telescopes, a plane parallel glass plate is tilted be-
tween the object and the eye-piece, shifting the image on the eye-
piece cross-hairs in a direction perpendicular to the axis of rotation
of the plate.   By a suitable calibration, the angle of tilt necessary
to place the images of two points in succession on a cross-hair
parallel to this axis gives the distance between the points in the
object.   Poynting's adaptation of this micrometer to the microscope
is described by A. H. R. Buller.[1]   A plane parallel plate was in-
serted through a hole cut in the microscope tube.   A rigid pointer
gave the angular position of the plate on an attached scale.

The modified form of instrument to be described possesses certain
advantages, and is especially adapted to the measurement of small
objects or distances when a large number of such measurements is
required.

The plate A of plane parallel glass is set just above the objective
B, in the short tube C.   To the ring D in which the plate A fits
are attached the trunnions E, E, turning easily in bearings F, F,
which are screwed into the sides of the tube C until they just touch
D.   At one end of the rotating system is the lever G, and at the
other, the galvanometer mirror H.   Both of these are carried on
short sleeves adjustable about the axis.   A counter-weight J serves
to balance the moving parts so that the system remains in any
position.   The mirror H reflects light from a circular scale into
the telescope K fastened to the barrel of the microscope by the
rigid clamp L.   The eye-piece of this telescope is conveniently near
to that of the microscope.   The circular scale is attached to the
wall and the microscope stand is fixed with reference to it by guides
or blocks, so as to be easily replaced in position.   This scale should

[1] Researches on Fungi, 1909, p. 158.

be so far from the mirror that tenths of scale divisions need not be estimated.

A calibration curve or table giving the nearly linear relation between scale reading and displacement of object can be made for one magnifying power by direct measurement of a standard, and used for any other power by changing the scale of lengths.

The advantages of the plate micrometer thus improved can be summarized as follows:

The device can be attached to any microscope tube threaded for standard objectives, without marring the instrument.
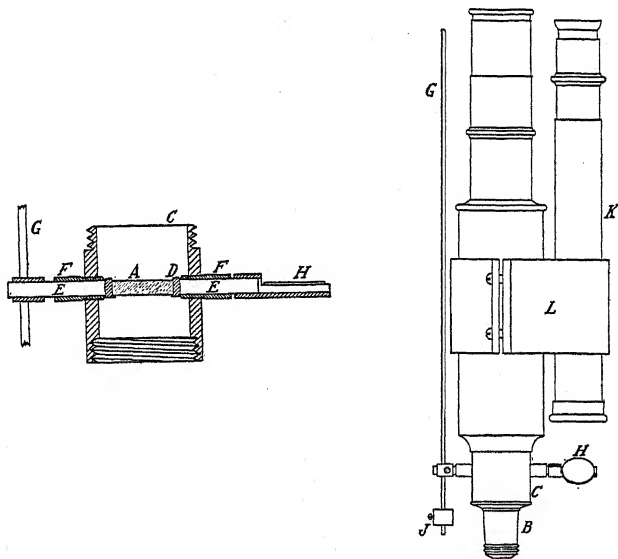
Fig. 1.

Any objective can be used with any eye-piece, giving a wide range of magnifications.

Different thicknesses of plate can readily be interchanged.

Back-lash, so troublesome in screw micrometers, is entirely absent.

The moving system is so light that its motion does not set up vibrations.

The use of a telescope avoids parallax and greatly increases the accuracy of scale reading. This would also be advantageous in the

ordinary form of plate micrometer where the plate is between object and objective.

Readings can be taken as rapidly as the accuracy of contact can be judged, since the plate can be tilted through its whole range in a fraction of a second, and the scale can be viewed by a slight motion of the head to one side.

The adjustment of telescope, mirror, and scale, is not affected by focusing the microscope.

Time is saved because it is not necessary to estimate tenths of scale divisions.

The instrument used in this laboratory has a glass plate 6 mm. thick, a circular scale of 131 cm. radius and 150 cm. long. Using Zeiss C objective and no. 5 eye-piece, giving a magnifying power of 325 diameters, the probable error in a single setting of the cross-hair on the edge of a small object was found to be 3 mm. on the circular scale, corresponding to .000003 cm. in the object. The maximum length measurable at this magnifying power is .0015 cm.

PHYSICAL LABORATORY,
    UNIVERSITY OF MINNESOTA,
        January 7, 1911.

THE

PHYSICAL REVIEW.

ON THE FREE VIBRATIONS OF A LECHER SYSTEM, USING A LECHER OSCILLATOR. I.[1]

BY F. C. BLAKE AND CHARLES SHEARD.

IT was Professor Rubens[2] who first quantitatively demonstrated the existence of the free vibrations of a Lecher[3] system. Since then a great deal of work has been done on the subject of stationary waves in parallel wires, so much so in fact that it would seem that at best one could only repeat the work of others if he undertook a research along this line. Moreover, many theoretical papers[4] have appeared on the subject and one has only to attempt to correlate the experimental and theoretical parts of the subject as exemplified in the literature to convince himself that there is something still to be desired in this field. But it was the work of Blake and Ruppersberg[5] that led us to repeat Rubens's experiments under the better working conditions now attainable. Their work had shown us that experimentally at least there ought to be found solutions of such vexing questions as these—(a) ought the bridge to be taken into account and to what extent?[6] (b) are the currents in the Lecher system transverse, or longitudinal, or partly one and partly the

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[1] Read before the American Physical Society at Chicago, November 26, 1910.

[2] Wied. Ann., 42, p. 154, 1891.

[3] Wied. Ann., 41, p. 850, 1890.

[4] Mention need be made of only three, viz., Cohn and Heerwagen, Wied. Ann., 43, p. 343, 1891; P. Drude, Wied. Ann., 61, p. 631, 1897; M. Abraham, "Theorie der Elektrizitat" (2d edition), Bd. 1, p. 347.

[5] Blake and Ruppersberg, PHYS. REV., Vol. XXXII., p. 445. 1911.

[6] See for instance Rubens, *l. c.*, p. 161.

other?[1]   Intimately related with each of these are two other more general questions, (c) are the various frequencies in a freely vibrating Lecher system related to each other? and (d) if so, what are the relative intensities of these harmonics?   It was hoped that further experimentation on this old problem would help to answer these questions.   To what extent this hope was justified will show further along in these papers.   A discussion of the work of previous investigators is accordingly reserved until after our own experimental results are presented.

APPARATUS.

The oscillator was of the Lecher type connected through two air-gaps and two water-resistances to the induction coil after the manner of Blake and Ruppersberg.[2]   The coil used was a 10-inch coil attached directly to a 110–volt, 60-cycle A.C. lighting circuit, the condenser and interrupter being removed.   As several different types of oscillators and primary circuits were used it seems best to describe them separately as the various phases of the work are touched upon.   The bolometer used was the same as that employed by Blake and Ruppersberg.   However, these writers did not make use of a check receiver and while their results cannot be materially in error on that account, nevertheless it was evident to us as we witnessed their work that the introduction of a check receiver would be a great improvement if one were to try for the greatest accuracy. Such a receiver was accordingly employed and of course two galvanometers were used.   The one was a duBois-Rubens iron-clad, the other a Broca instrument.   Their sensitiveness was 4.07×10^{-9} and 6.45×10^{-8} respectively for a period of 4 seconds.   After being once set up it was not necessary to adjust them throughout the work.

EFFECT OF VARYING THE DISTANCE BETWEEN THE LECHER WIRES.

Employing a check receiver, we tried at first to get the free oscillations of the parallel system.   Blake and Ruppersberg had found that the overtones of the free (?) system were not harmonic and after our results became known to them it appeared that the

[1] See Lecher, l. c., p. 854; also J. J. Thomson, "Recent Researches," p. 466.
[2] Blake and Ruppersberg, l. c.

reason for this was because of the closeness of the coupling employed by them.   Accordingly, we started with what we deemed a fairly loose coupling and hoped thereby to get the ideal free vibra-
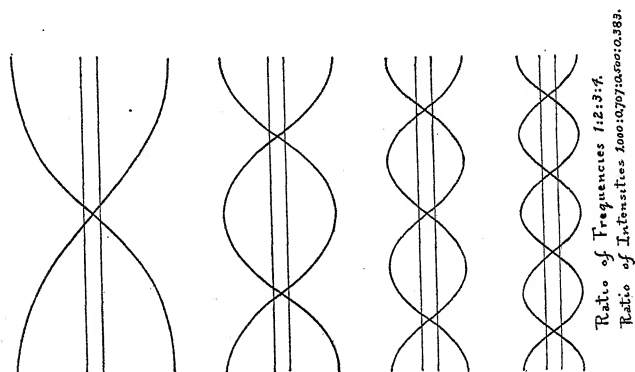
Fig. 1.

tions shown in Fig. 1.   For this the oscillator consisted of two brass balls each 3/8 in. in diameter, bored out and filled in with steel
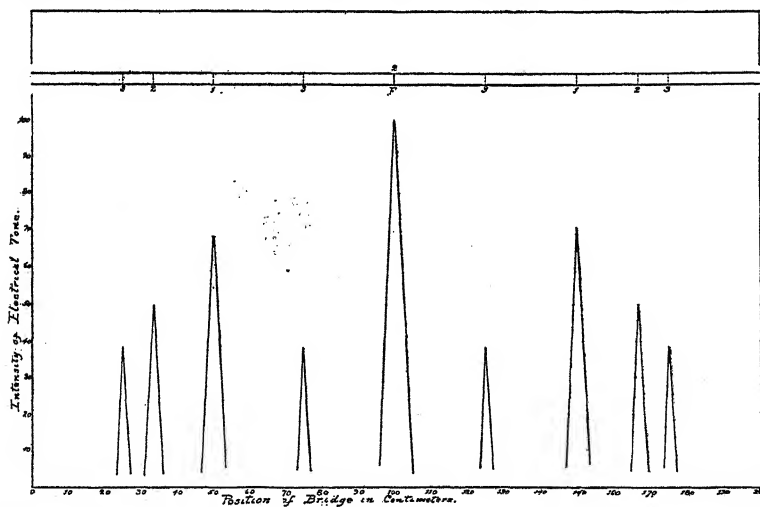
Fig. 1a.

cylinders tapered off to a truncated cone[1] at the spark-gap in oil. Soldered to these cylinders were copper wires attached to primary tin plates 5 cm. square.   This made the total length of the oscillator

[1] This was the oscillator used by Blake, Phys. Rev., Vol. XXX., p. 678, 1910.

circuit up to the plates 28 cm. Opposite these plates, at a distance of 2 cm., stood the two secondary plates each 3 cm. square. The whole arrangement of the oscillator and Lecher circuits is shown in Fig. 2. The air-gaps were each 7 mm. long. The check receiver
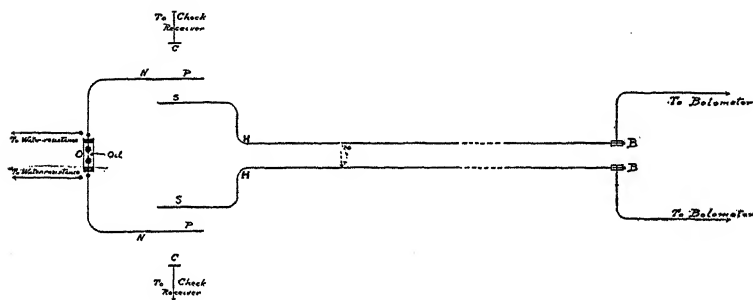
Fig. 2.

was a thermal couple of iron and constantan (diameter of wire 0.0208 mm.). It proved to be quite a problem properly to locate the check receiver with respect to the rest of the apparatus so as to accomplish the sole purpose of correcting the oscillator deterioration. Trial revealed the fact that it was readily influenced by stray radiation when only a short distance from the oscillator, so it was symmetrically placed about 50 cm. from the oscillator and two long equal lead wires led from it to small capacity plates each 1 cm. square placed 3 cm. away from the primary plates of the oscillator though parallel to them, as shown in the figure. The total length of the check receiver circuit was thus 3 meters. With this arrangement a Lecher curve was taken by moving a single bridge along the parallel wires. Plotting the check receiver readings against the bridge position showed a curve of sinusoidal nature and it became at once apparent that the check receiver was influenced by what might be called the mutual interaction of the oscillator and Lecher circuits, or better the back-action of the Lecher circuit upon the oscillator. This *back-action* will be spoken of again. A test experiment showed that this mutual interaction was present even though the check receiver was metallically protected from any stray radiation effects. The back-action upon the check receiver plates, *CC*, of Fig. 2 must have come from the secondary plates *SS through*

the primary plates *PP*. As we could not use the check receiver readings the curve was plotted with the oscillator deterioration uncorrected for and is shown in Fig. 3.
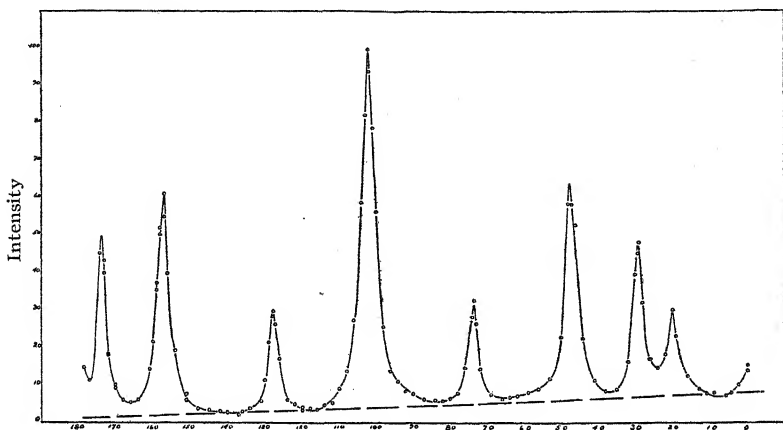
Fig. 3.

When it is remembered from Fig. 2 that the detecting system is at the free ends of the Lecher wires it is plain that the free vibrations shown in Fig. 1 will appear as Fig. 1a when properly interpreted. After comparing Fig. 3 with Fig. 1a no one will question the statement that we have in Fig. 3 a first approximation to the ideal case depicted in Figs. 1 and 1a. The following calculation shows, however, that Fig. 3 will not in all parts bear close theoretical scrutiny. The zero end of the 2-meter stick was placed vertically below the last bridge position obtainable on the Lecher wires at the bolometer end. Beyond this the wooden supports for the bridge and for the means of keeping the wires under tension occupied a linear space of 1.8 cm. The Leyden jars were themselves 1 cm. long and the small disc with its protruding knob of solder added 2 mm. The actual end of the wire was thus at − 3 cm. Just as Blake and Ruppersberg found a decidedly better agreement between theory and experiment by considering that the wave is reflected only after it reaches *the end of the wire*, so do we find this true also. That this interpretation is justified will appear from the work using cylindrical capacities reported later in this work. Table I. shows that the mean fundamental wave-length λ_0 is 434

cm., of which one fourth is 108.5 cm.   From the end of the wire to the bridge reading for the fundamental tone is 105.0 cm. giving as the equivalent value of the end-capacity 3.5 cm. of wire.   The rest

TABLE I.

Distribution of Energy. [Plate Capacities.] *Potential Difference* $= \Phi_1 - \Phi_2 =$ $2\dfrac{|A|}{K}\cos\left(\dfrac{vx}{w} - \gamma\right) \cdot \cos(vt) \propto \sin\left[\dfrac{\pi}{2}\left(\dfrac{1}{n+1}\right) - 2\gamma\right]$. *Phase Change* $= 2\gamma$ $\Big($where $\tan\gamma = \dfrac{v}{w} \cdot \dfrac{K_0}{K} = \dfrac{2\pi K_0}{\lambda K}\Big)$. ($n =$ no. of overtone.)

K_0/K (cms.).	l cms.	λ cms.	2γ	Tone.	Intensity.		Difference.
					Theory (Per Cent.).	Experimental (Per Cent.).	
3.5	105	434	5° 47′ 54″	Fund........	100	100	——
				1st har.	62.7	62.7	0.00
				2d har......	41.0	49.0	+8.00
				3d har......	29.0	31.7	+2.70

of the calculation is collected into the table and it is seen that the agreement between theory and experiment is very satisfactory except for the second harmonic where the difference is almost 8 per cent.   It will be shown later that undoubtedly an oscillator vibration swings in here *along with* the free second harmonic, thereby increasing its intensity.

Other curves were taken for distances between the wires as follows: 1 cm., 3 cm., 5 cm., 11.5 cm.   Two of these are shown in Fig. 4.   Curve *I* is for 3 cm., Curve *II* for 11.5 cm.   For each of these curves the oscillator deterioration was corrected for by employing the check receiver readings in the following manner. Plotting the check receiver readings against the bridge position the curves of Fig. 5 were obtained.   For 11.5 cm. distance between the parallel wires the sinusoidal nature of the curve was clearly shown, the oscillator evidently being in first class condition.   Although the oscillator for the 3 cm. curve was not in such good condition, the curve shown cannot be much in error.   The beginning of the 1 cm. curve is also shown, the oscillator being in unusually good condition, though the last half of the curve (not plotted) shows that the oscillator "went badly to pieces."   The figures on

the right of the 3 cm. curve for the bridge position 10 cm. represent
check readings at that position taken immediately after the bridge
position indicated by the number written opposite the points.   For
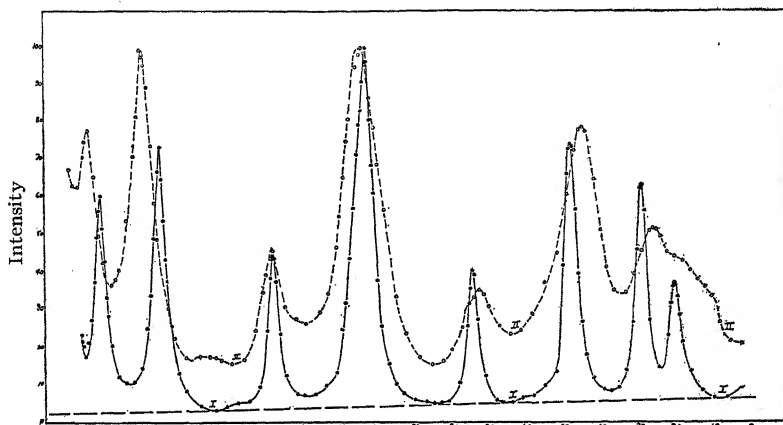
Fig. 4.
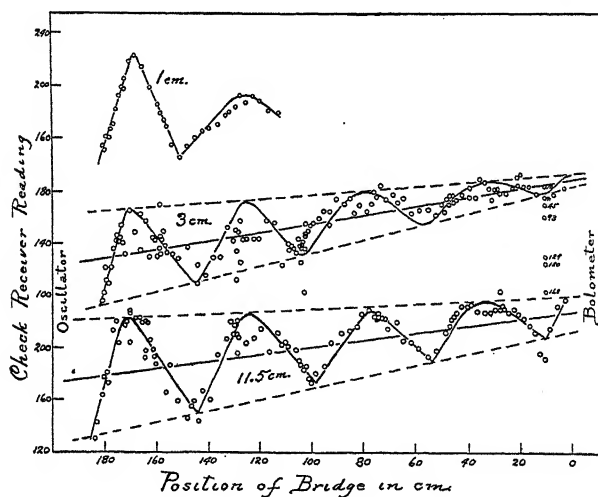
Fig. 5.

all the curves shown in Fig. 5 the bridge was moved toward the
oscillator and the downward slope of the axes of the curves, as well
as the 10-cm. check readings of the 3-cm. curve, shows the need of
correcting for the oscillator deterioration.   Since the check receiver

readings are themselves influenced by the bridge position a little thought will convince the reader that the oscillator deterioration is properly corrected for by multiplying the bolometer reading for a given bridge position by the ordinate for the sine curve of Fig. 5 and dividing by the ordinate for the axis of such curve. To show the order of magnitude of this correction it should be stated that it served to even up the intensities of any particular harmonic at the two ends of the Lecher system but in no case did it displace a maximum more than 3 mm.

It is seen from the curves of Fig. 4 that for increasing distance between the Lecher wires the fundamental maximum broadens out and the minima are less prominent. If one plots the ratio of the fundamental maximum to its own minimum (at 85 cm. say) for the different distances between the wires the curve shown in Fig. 6 is obtained. This curve shows a maximum at 3.4 cm. It would appear that, starting with this distance, as one decreases the distance between the wires the magnetic fields surrounding the two parallel wires nullify each other more and more and hence
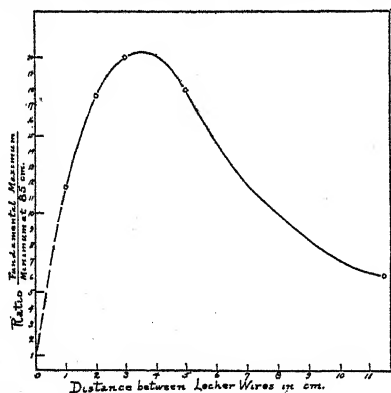
Fig. 6.

decrease the ratio of maximum to minimum. For zero distance between the wires one should expect this ratio to become unity and we have so extrapolated it in the curve. On the other hand, if we increase the distance between the Lecher wires it is plain that there must be an upper limit beyond which any further increase will begin to lessen the above ratio. For, theoretically, the free vibrations are essentially *longitudinal* vibrations such that the bridge serves only the purpose of enabling us to detect the vibrations without distorting them as to intensity or position. It is to be noticed that in Curve *II* of Fig. 4 (for 11.5 cm.) the system of vibrations is not the regular symmetrical system that it is for the lesser distances. It will be shown later (as indeed the curve for 5 cm. distance with the arrange-

ment of Fig. 2 shows when plotted) that up to a distance of 5 cm. at least with a Lecher wire length of 175 cm. or more the presence of the bridge introduces no disturbing factor. That is, so long as the bridge length is not more than 3 per cent. of the Lecher length we may safely conclude that, although there is *necessarily always* current in the bridge wire whatever its position the two ends of the bridge wire may be said to have the same phase and hence the phase relations at the bolometer end of the Lecher system are the simple ones the theory demands. For a distance as great as 11.5 cm. this is not the case, as Curve *II*, Fig. 4, plainly shows. An inspection of this curve reveals an oscillator vibration at 9, 76.5 and 144 cm., giving 67.5 cm. for $\lambda/2$. Now it seems to us that the distance between two adjacent maxima of Fig. 5, viz., 45 cm., is the half wave-length of *some* vibration of the oscillator. These two oscillator vibrations of 45 cm. and 67.5 cm. half wave-length bear the ratio 1.50. In the light of the work of Blake and Ruppersberg and of evidence still to be spoken of we think these represent the two fundamental vibrations of the oscillator, due to the comparatively close coupling. This makes the true oscillator wave-length 115.8 cm. Now the distance *NON* (Fig. 2) was 28 cm. When one allows for the plates *PP* and for the balls at the spark-gap it is plain that the oscillator half wave-length could easily be 57.4 cm.

An attempt was made from the curves of Figs. 3 and 4 to test out the law that the potential difference between the Lecher wires is proportional to the logarithm of the ratio of the distance between the wires to their radius, but not enough attention had been paid to the equality of the other conditions to make this possible. To insure success in this effort one must have absolute control over the oil-gap and this we did not have at the time these curves were taken.

Modified Lecher System.

In the course of the work referred to above it became plain to us that two features of the experimental arrangement needed improvement. First, it was plain that the check receiver must be located so as to be free from the back-action of the Lecher system. And secondly, it became very desirable to have the degree of coupling readily alterable. Both of these conditions called for a modi-

fication of the apparatus. As to the first, it was clear that if one were able to free the check receiver from the back-action it would certainly be accomplished if at all by placing the check receiver as a high resistance shunt directly across the spark-gap in oil. Accordingly this was done and it proved at once the solution of the difficulty. Small platinum plates 1 cm. in diameter were arranged as electrodes of a high resistance, one on each side of the thermal couple. After considerable experimentation the length of each olive oil resistance was chosen as best at 0.5 cm. The notion was to steal as little energy from the oscillator circuit as possible for checking purposes and at the same time not to alter its mode of vibration or wave-length.

For changing the coupling readily the experimental arrangement shown in Fig. 7 was finally chosen. Cylindrical capacities 15 cm.
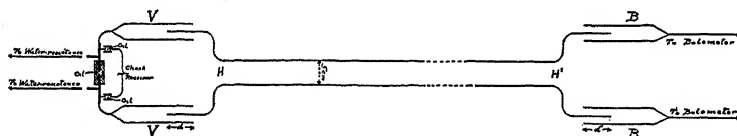
Fig. 7.

long by 3.5 cm. in diameter were attached both to the oscillator circuit and to the bolometer circuit. In this way the Lecher wires themselves were as free as possible at both ends. They were held in place at a distance of 5 cm. apart by supports of soft wood placed at *HH'* of the figure. To *H'* a tension of 4 kilograms was attached after the manner already described.

The degree of coupling was changed by changing the distance *d* to which the Lecher wires extended into the oscillator cylinders *VV*. Curves were taken with the Lecher wires protruding various distances into *VV* and *BB* but they served no useful purpose except to point out the best way of experimentation; for the type of curve materially altered as the distances *d* and *d'* increased much beyond 5 cm. Before describing further curves it seems well to illustrate the checking power of the check receiver and to show that in its new position it is uninfluenced by the location of the bridge. Fig. 8 demonstrates the latter point clearly enough, the check receiver readings being plotted against the bridge position for three different curves

chosen at random.   Although the need for a check receiver is clearly
seen one cannot possibly detect any sine nature of the curves such as
Fig. 5 shows.   The method of checking a bolometer reading was
as follows.   A preliminary experiment showed the location of the
fundamental maximum for a given set of conditions.   With the
bridge at the position of
this maximum and with
galvanometer shunts ad-
justed for large deflections
a set of from ten to twenty
readings both of the bolo-
meter and of the check re-
ceiver was taken.   These
readings were plotted
against each other with
the readings of the check
receiver as abscissæ; a
straight line was drawn
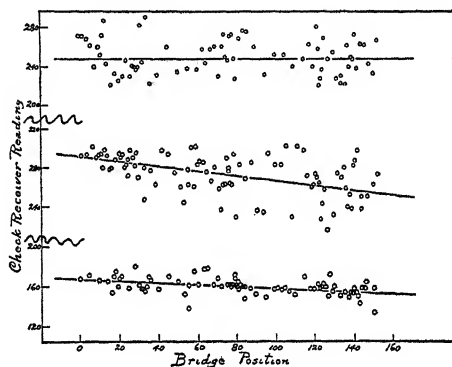
Fig. 8.

through the points thus obtained.   Then a regular curve was taken,
the bridge being moved along the wires, consisting usually of about
as many points as are shown in any of the curves of Fig. 8, both
galvanometers being read of course.   This usually took from one
and a half to three hours.   Then another set of ten or more readings
was taken with the bridge again at the fundamental maximum.
On account of temperature changes producing a slow unbalancing
of the bolometer (shown by the wandering of the zero of the gal-
vanometer) the straight line obtained from this last set of readings
never coincided with the straight line obtained at the beginning
of the curve.   It usually lay several per cent. above it, though
parallel to it.   Trial had shown us that taking the ratio between
the two galvanometer readings for any given bridge position would
not be accurate enough, although better than the bolometer readings
alone (see column 4, Table II.).   If, for two successive readings
with the bridge position constant the spark passed over a longer
distance one time than the other, the ratio of the galvanometer
readings should not necessarily be wholly constant.   By increasing
and decreasing the spark gap length we observed that the check re-

ceiver was always more strongly affected than the bolometer. If, however, we divided the bolometer reading for any given bridge position by its reading for the bridge at the position of the fundamental maximum for the same check receiver reading (making use of the straight line referred to above) we eliminated this error. The slow unbalancing of the bolometer was corrected for as follows. We added the per cent. of difference "x" between the two straight lines referred to above to 100 per cent. and plotted the two points 100, $100 + x$, with the mean zero of the bolometer galvanometer in each set as corresponding abscissa; these two points were joined by a straight line. For any given bridge position then we can get the

TABLE II.

Column 1.	Column 2.	Column 3.	Column 4.	Column 5.
Bridge Position.	Bolometer Reading.	Check Receiver Reading.	Ratio.	Relative Energy.
76	475	370	1.283	0.976
	449	341	1.318	0.972
79	201	287	0.700	0.485
	221	346	0.639	0.474
118	71	273	0.260	0.176
	77.5	320	0.242	0.175
149	32.8	380	0.086	0.066
	29.1	328	0.089	0.065
10	69	220	0.314	0.226
	75	260	0.289	0.224
88	51	240	0.212	0.158
	55	280	0.196	0.158
125	151	297	0.508	0.423
	135	248	0.554	0.423
149	111	232	0.478	0.351
	108	222	0.487	0.351

final correction factor by reading off the proper ordinates on this line. In this way we eliminate both oscillator deterioration and bolometer unbalancing entirely.[1] How nearly we have succeeded

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[1] This bolometer unbalancing seems to be due to several things; among others may be mentioned direct radiation from the bodies of the observers and from the electric lamps used for illuminating the galvanometer-scales and the heat that comes from the electrical oscillations which on account of the cotton-wool packing immediately around the instrument cannot be readily equalized. Moreover, it is, of course, a function of the resistance in the auxiliary battery circuit and of the value of the galvanometer shunt.

in doing this is shown in column 5 of Table II., where the relative energy was obtained in the manner just indicated. The table contains random duplications for two different curves. The great superiority of column 5 over column 4 or column 2 (unchecked) will readily be admitted. So far as the oscillator is concerned we say without hesitation that we are able, by the employment of a check receiver situated as a shunt of high resistance across the spark gap, to eliminate all errors except an occasional error due to a mistake in reading a large deflection on a quick swing. A glance at Table II. will convince the reader that in general it is possible to duplicate any reading to a degree well within one per cent. Even for a badly worn oscillator one can duplicate to within two per cent., though it pays to repolish the oscillator occasionally. The results of Table II. and the curves yet to be spoken of in this paper were taken with the oscillator shown in Fig. 7 where the spark passed between two cylindrical hard steel rods each 5 cm. long and 3 mm. in diameter. At first we tried aluminium rods, after the manner of Schaefer and Laugwitz.[1] They worked better than the steel rods for the first few readings (perhaps twenty) but beyond that their action became more and more irregular and capricious and so they had to be abandoned. For the steel oscillator, on the other hand, it was no uncommon thing to take as many as six curves (say 600 readings) without repolishing. The oscillator was so constructed that the spark gap could readily be polished and cleaned by simply removing the rods from their ebonite support.

Effect of Changing the Coupling.

Using the steel oscillator and applying both corrections referred to above, the effect of changing the coupling between the oscillator and Lecher circuits was studied. To preserve the fundamental maximum fixed at a certain bridge position any change in the coupling was made at *both* ends of the Lecher system, though for some reasons it would have been better not to have changed the bolometer ends of the wires. Curves were taken for the following couplings, expressed as the distance in centimeters the Lecher wires

[1] Schaefer and Laugwitz, Ann. d. Phys., 23, p. 953, 1907. See also Laugwitz, Phys. Zeit., 8, p. 378, 1907.

were inserted into the cylindrical capacities at both ends: $+$ 5 cm., $+$ 3 cm., $+$ 2 cm., $+$1 cm., 0 cm., $-$ 1 cm., $-$ 2 cm., $-$ 3 cm.   For the latter distance the coupling was so loose that the energy detected by the bolometer was very small, too small to make the results quite as reliable as for the other distances.   Three of these curves are shown in Fig. 9, viz., $+$ 2 cm., 0 cm., $-$ 2 cm.   One sees at a glance
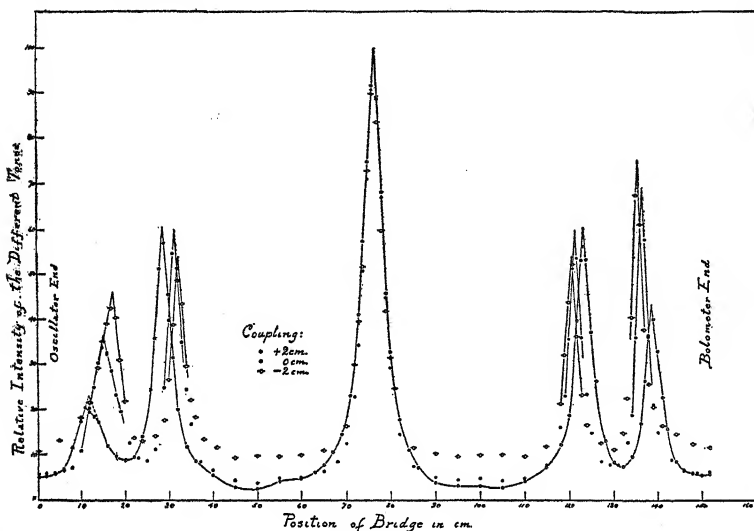
Fig. 9.

in contradistinction to Fig. 3, the practical absence of all third harmonics.   The maxima are all drawn as sharp peaks in the figure, whereas in fact they are all rounding as shown in Fig. 10, where for a given case the fundamental maximum and the first harmonics are drawn to a larger scale.   One sees at once from Fig. 10 that it makes no difference whether one chooses the peaks or the true maxima when comparing the various tones as to their relative intensities.[1]   It should be noted also from the figure that the center

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[1] For this reason we drew the curves of Fig. 9 as peaks for it was easier in this way to locate the maxima accurately.   In fact for the data collected together in Tables III. and IV. each maximum was carefully drawn to a larger scale after all the corrections had been applied and, following Rubens, the center of gravity lines were drawn for locating the exact position of a maximum.   When the center of gravity line was not vertical the position of the maximum was taken as the point of intersection of the actual curve with this center of gravity line drawn from the peak downwards.   In this way any maximum is located accurately to within 1 mm. in general, though in certain cases we do not claim closer than a 2 mm. accuracy.

of gravity line of each maximum is vertical. It will be shown later that when this is not the case it reveals two maxima in one, the extra one being due to the oscillator.

If we study Fig. 9 we note the following points. Figured from the fundamental at 76.4 cm. the semi-internodal spaces for the first harmonics are greater on the left than on the right of the figure, the disparity growing less as the coupling grows looser. The same
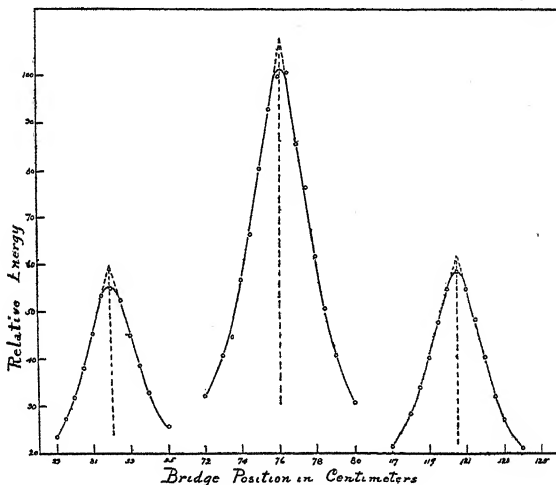
Fig. 10.

is true for the internodal distances for the second harmonics, only to a greater extent. This is more clearly seen in the two lower curves of Fig. 11, in which the data for several curves not shown in Fig. 9 are included. We see from these curves that the lack of symmetry in the internodal spaces disappears for the looser couplings, for practical purposes vanishing at the coupling — 1 cm. We see, moreover, from Fig. 9 that the two first harmonics decrease, but remain equal to each other as the coupling is changed, a thing they should do. For the second harmonics we notice that as the coupling gets looser, the two second harmonics (omitting the central second which is concealed in the fundamental and hence cannot be argued about) increase in intensity, but the one nearer the bolometer end is always the larger; for the looser couplings it overtops the first harmonics. If we examine closely the second harmonic near

the oscillator end we notice that it is much broader at the base in proportion to the other maxima than it ought to be. Moreover, its center of gravity line is usually distinctly out of the vertical.
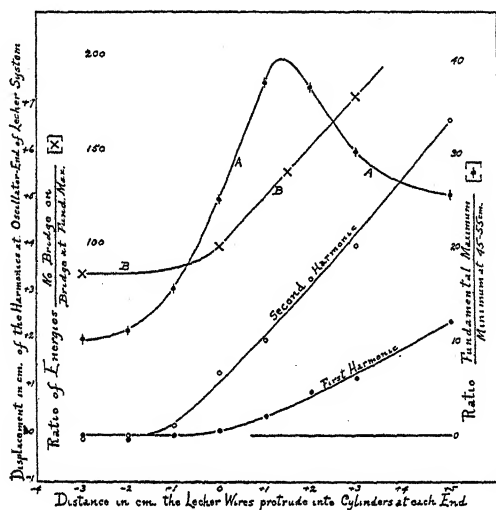
Fig. 11.

This shows two maxima run together into one and we found by experiment in certain cases that the true position of either one could not be determined accurately. This gives us a clue also to the very high value of the second harmonic near the bolometer end as the coupling gets looser. The fundamental vibration of the oscillator falls in more fully with this second harmonic and hence greatly increases its intensity. However, the oscillator vibration is still somewhat longer than this free second, for the slope of the fundamental on the oscillator side is less steep than on the bolometer side, while both the oscillator swing and the free second appear, as has been said, in the broad maximum near the oscillator end, the former lying closer to the oscillator by reason of its greater length. For this reason the internodal spaces for the seconds are even more unsymmetrical than for the firsts as Fig. 11 plainly shows. Moreover, the presence of two maxima in one makes it harder to determine the actual top of the double maximum, hence in Fig. 11 the points for the second harmonics lie further from the smooth

curve than for the firsts.   For the latter it is seen that the greattes deviation of any point from the curve is 1 mm.

Another thing should be said about Fig. 9.   As the coupling became looser we saw that the first harmonics were pulled down in intensity while the seconds were pulled up.   This is undoubtedly partly due to this oscillator vibration.   That this is so is shown clearly in Curve *I*, Fig. 12, where for a longer Lecher system than
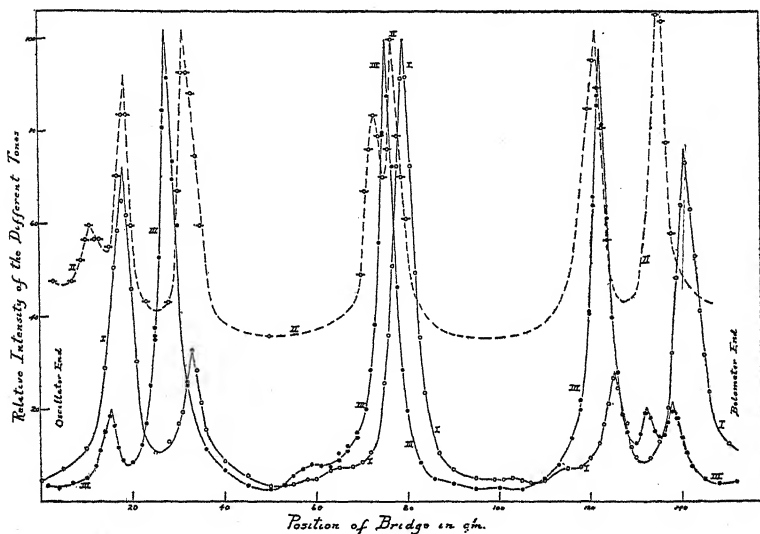
Fig. 12.

those illustrated in Fig. 9 the coupling was as follows: vibrator end − 1 cm., bolometer end + 3 cm.   Here the oscillator vibration was in exact unison with the free second harmonic, hence both of these harmonics are very high.   The first harmonics at 33 and 125 cm. are correspondingly low.   Without changing the actual length of the Lecher wires we then changed the coupling to − 1 cm. at the bolometer end leaving the coupling at the oscillator end as it was. In this way, since the effective length of the parallel wires was lessened, it ought to be possible to throw the free second out of tune with the oscillator vibration and hence they ought to stand out separately.   That this was accomplished is shown in Curve *II*, Fig. 12.   The oscillator swing comes in at 11, 73 and 135.5 cm.

with an internodal space of 62 cm. The free second appears at
18.5, 77 and 135.5 cm. giving an internodal space of 58.5 cm.
Both vibrations occur together at 135.5 cm. and make the maximum
very high (one and a half times the fundamental). For Curve *I*
both oscillations came in at 18.2, 79.5 and 140.8 cm. with an inter-
nodal space of 61.3 cm., in substantial agreement with the oscillator
internodal space of Curve *II*.[1]

That the oscillator vibration can be made to swing in with some
other harmonic is shown in Curve *III* of Fig. 12. This curve was
taken earlier with a different oscillator of somewhat greater length,
though the same cylindrical capacities were used as for the other
curves. The coupling was + 1 cm. at both ends. In this curve
there are two fundamental oscillations present, one at 27.8 and
122.3 cm., the other at 59 and 132.5 cm.,[2] the first one falling in
with the free first harmonic to make it of abnormal intensity. Here
we notice that when the firsts are very high the seconds at 15.4 and
137 are abnormally low.[3]

Referring again to Fig. 11 we saw that as the coupling at both
ends got looser and looser the internodal spaces on the two sides
of the fundamental became more nearly equal until for a very loose
coupling (— 1 cm. or less say) the equality is exact. If from the
two lower curves of Fig. 11 we were to calculate the fundamental
wave-length, we would obtain curves entirely similar in nature to
the curves of Fig. 6 of Blake and Ruppersberg. It is accordingly

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[1] In all the curves of Fig. 12 no corrections were applied. The bolometer readings,
reduced to 100 for the fundamental, alone are plotted. Hence the maxima are not
located with great accuracy.

[2] These internodal spaces of 73.5 cm. and 94.5 cm. for the oscillator vibrations
make the half wave-length of the oscillator when isolated 84.6 cm. These figures
are checked thus; adding 47.2 to 122.3 gives as the end of the wire including the end
capacity 169.5 cm.; subtracting 36.8 from 169.5 gives 132.7 cm., in agreement with
132.5 cm. Now the length of this oscillator up to the cylindrical capacities was 35.6
cm. This makes the equivalent length of wire for each cylindrical capacity 24.5
cm. or roughly 1.5 times its natural length.

[3] We are not entirely clear why apparently the intensifying of one harmonic by the
oscillator vibration should lower the intensity of a neighboring harmonic. The fact
that merely changing the coupling at the bolometer end from + 3 cm. to — 1 cm.
changed Curve *I*, Fig. 12, into Curve *II* of the same figure, seems to point to the
fact that the nature of this coupling plays an important part in determining the relative
intensities of neighboring harmonics.

plain that the closeness of the coupling in their work was the cause of their overtones not being harmonic.

A glance at Fig. 9 shows that as the coupling gets looser the ratio of maximum to minimum decreases. However, for some of the curves not shown (viz., + 3 cm., + 5 cm.) as the coupling gets closer this ratio also decreases. All the data are collected together into Curve A of Fig. 11. We see that this ratio attains a maximum for the coupling + 1.4 cm. Its value is 40 to 1. This is roughly four times as good as Rubens got and one third as good as the value obtained by Blake and Ruppersberg.

Curve B of Fig. 11 was obtained by carefully comparing the bolometer readings with no bridge on the Lecher system against those with a single bridge placed at the fundamental maximum, the coupling being changed symmetrically at both ends. For close coupling there is more energy detected at the bolometer for no bridge on the parallel wires than for a bridge placed at the funda-mental maximum and the closer the coupling the greater this energy. For couplings less than 0 cm. the reverse is true. It is interesting to note that for this particular coupling of 0 cm. the readings with and without a bridge are identical. It seems, perhaps, unprofitable at this time to speculate as to the meaning of this curve B. One might expect that all the tones ought to be present on the parallel wires when there is no bridge on them. If this were true the reading for no bridge ought to be roughly 2.3 times the reading for a bridge at the fundamental maximum. It should be borne in mind, however, that the free vibrations are only main-tained in the Lecher system (so far as our detecting devices register) by a steady in-pour of energy at the oscillator end of the system. Very likely this plays a part that must yet be taken into account. In this respect the case is different from that of pulling a pendulum back and then letting go.

COMPARISON OF THEORY WITH EXPERIMENT.

An attempt was made, from the curves of Fig. 9 and from the five other curves not there shown but taken with equal care, to compare the results with the theory outlined by Kirchhoff and fully developed by Abraham.[1] How well the attempt succeeded is shown

[1] Abraham, Theorie der Elektrizitat, Vol. 1, 2d ed., page 347.

TABLE III.

Coupling cms., Insert of Wires.	Tone.	Position on Wire.	Actual End of Lecher Wire.	End of Wire including K_0/K.	K_0/K (cms.).	l	Method for λ.	λ	$a = \lambda/K_0$	$a = \xi \tan \xi$	Per Cent. Variation.	$\xi = 2\pi \frac{\gamma}{\lambda}$	γ ($\tan \gamma = 2\pi K_0/\lambda K$)	$\xi + \gamma = \pi/2$.	Total Actual Length of Wire, l_0.	Capacity at Oscillator End.	$\lambda_0/l_0 = \frac{\text{wave-length}}{\text{wire-length}}$
+5	Fund.	76.3	168.4	174.2	5.8	92.1	A	391.8	15.88	15.63	1.6	84° 36'	5° 19'	89° 54'	181.3	8.8	2.16
	1st har.	125.4				43.0	A	196.4	74.20	69.32	6.0	78° 46' 54"	10° 29'	88° 47'			
	1st har.	25.0				143.4	B	205.2	24.72	13.15	47.0	71° 32' 24"	10° 4'	81° 36'			
	2d har.	141.6				26.8	B	130.6	4.64	4.46	4.0	73° 51' 15"	15° 36' 30"	90° 17'			
	2d har.	4.2				164.2	B	144.2	28.31	12.64	50.0	60° 30'	11° 58'	72° 28'			
+3	Fund.	76.6	166.4	171.2	4.8	89.8	A	378.3	18.71	18.66	1.5	85° 25' 48"	4° 33' 30"	90° 0'	177.4	6.8	2.14
	1st har.	124.1				42.3	A	190.0	8.82	7.92	11.0	79° 56'	9° 0' 30"	88° 57'			
	1st har.	27.8				138.6	B	195.2	28.87	17.66	40.0	75° 48' 36"	8° 47' 30"	82° 37'			
	2d har.	139.6				26.8	A	126.0	5.58	5.58	0.0	76° 32' 50"	13° 29' 20"	90° 1'			
	2d har.	9.7				156.7	B	133.8	32.65	12.98	62.0	60° 31' 12"	12° 22' 30"	72° 45'			
+2	Fund.	76.4	165.4	170.2	4.8	89.0	A	374.4	18.54	19.20	−3.5	85° 34' 40"	4° 13' 30"	89° 48'	175.5	6.4	2.14
	1st har.	28.7				136.7	C	189.6	28.48	24.33	14.0	79° 28' 20"	9° 2' 20"	88° 31'			
	1st har.	28.7				136.7	B	190.8	28.48	20.64	24+	77° 48' "	11° 12' 20"	89° 0'			
	2d har.	123.5				41.9	D	186.4	8.72	8.81	−1.1	80° 54'	9° 12' 40"	90° 7'			
	2d har.	11.0				154.4	B	130.4	37.16	16.82	55.0	66° 9'	13° 1' 20"	79° 22'			
	2d har.	76.4				89.0	A	124.8	18.54	18.94	2.5	76° 42'	13° 35'	90° 17'			
		138.8				26.6	A	124.8	5.54	5.65	2.0	76° 42' 30"	13° 35'	90° 17'			
+1	Fund.	76.4	164.4	168.5	4.1	88.0	A	368.8	21.47	21.26	1.0	85° 57' 40"	4° 0'	89° 58'	173.6	6.8	2.12
	1st har.	29.9				134.5	C	185.2	32.80	30.04	9.0	81° 21' 24"	7° 51'	89° 15'			
	2d har.	76.4				88.0	A	122.8	21.47	20.94	2.5	77° 52'	11° 52'	89° 42'			

0	Fund.	76.45	163.4	166.5	3.10	87+	A	360.4	28.06	27.79	1.4	86° 53′	3° 5′ 40″	89° 59′	171.6	5.5	2.10
	1st har.	31.45				132.0	A	180.1	42.58	42.44	0.3	83° 48′ 10″	6° 10′ 20″	89° 58′			
	1st har.	31.45				132.0	C	180.0	42.58	43.11	−1.3	83° 54′	6° 10′ 30″	90° 4′			
	1st har.	121.5				41.9	D	180.2	13.51	13.42	0.6	83° 46′ 48″	6° 10′ 30″	89° 58′			
	2d har.	15.4				148.0	B	122.0	47.74	31.99	32+	76° 36′	9° 4′ 10″	85° 40′			
	2d har.	76.45				87+	A	120.1	28.06	28.00	0.25	80° 46′	9° 12′ 20″	89° 58′			
	2d har.	136.50				26.9	D	120.1	8.67	8.49	2.1	80° 36′	9° 12′ 40″	89° 48′			
−1	Fund.	76.4	162.4	165.6	3.20	86.3	A	356.8	26.25	26.15	0.4	86° 41′	3° 13′ 30″	89° 56′	169.5	5.7	2.10
	1st har.	31.8				130.6	B	178.4	40.75	40.40	0.85	83° 28′	6° 25′ 40″	89° 54′			
	2d har.	76.4				86.3	A	118.8	26.25	26.64	−1.4	80° 20′	9° 36′ 20″	89° 57′			
	2d har.	16.8				145.6	B	119.0	45.50	45.10	0.9	80° 21′	9° 35′ 20″	89° 57′			
−2	Fund.	67.4	162.4	164.9	2.5	86.0	A	354.0	34.40	34.42	−0.01	87° 27′ 45″	2° 32′ 30″	90° 10′	169.5	5.0	2.09
	1st har.	32.2				130.2	B	176.8	52.08	51.93	0.3	84° 54′ 44″	5° 4′ 20″	89° 59′			
	1st har.	120.5				41.9	C	176.6	18.38	18.58	−1.1	85° 24′ 54″	5° 3′	90° 27′			
	2d har.	‡117.4				145.0	B	118.0	58.02	58.08	−0.1	80° 26′	7° 34′	90° 0′			
	2d har.	135.4				27.0	D	118.0	10.80	10.74	0.6	82° 22′ 40″	7° 35′	89° 58′			
−3	Fund.	76.1	161.4	163.8	2.4	85.3	A	350.8	35.54	35.48	0.02	87° 32′ 6″	2° 27′ 40″	90° 0′	168.5	4.5	2.08
	1st har.	32.3				129.1	B	175.4	53.79	52.89	1.8	84° 59′ 40″	4° 54′ 50″	89° 55′			
	1st har.	120.0				41.4	C	175.6	17.20	16.60	3.6	84° 56′	4° 54′ 30″	89° 50′			
	2d har.	17.7				143.7	B	116.8	61.60	59.90	3.0	82° 51′	7° 21′ 20″	90° 11′			
	2d har.	134.6				26.8	D	117.0	11.16	10.90	3.5	82° 26′	7° 41′	90° 7′			

A = wave-length calculated from the interval between the fundamental and the harmonic at *bolometer* end.
B = wave-length calculated from the interval between the fundamental and the harmonic at *oscillator* end.
C = wave-length calculated from the *internodal* space for a given harmonic.
D = wave-length calculated from the *quarter wave* distance of the harmonic from the corrected end of Lecher wire.

in Tables III. and IV.   Both tables are self-explanatory.   In order to see whether the maxima of the curves fall at their theoretical positions columns 10 and 11 of Table III. are to be compared, the percentage variation being given in column 12.   It is seen from the

TABLE IV.

Distribution of Energy.   [*Cylindrical Capacities.*]   *Potential Difference* $= \Phi_1 - \Phi_2 =$

$$2\frac{|A|}{K}\cos\left(\frac{vx}{w} - \gamma\right) \cdot \cos{(vt)} \propto \sin\left[\frac{\pi}{2}\left(\frac{1}{n+1}\right) - 2\gamma\right].\quad Phase\ Change = 2\gamma\ \Big(\text{where}$$

$$\tan\gamma = \frac{v}{w}\cdot\frac{K_0}{K} = \frac{2\pi K_0}{\lambda K}\Big).\quad (n = \text{no. of overtone.})$$

Coupling (cms.).	$\frac{K_0}{K}$ (cms.)	l (cms.).	λ (cms.).	2γ	Tone.	Intensity.		Diff.
						Theory (Per Cent.)	Experimental (Per Cent.)	
−2	2.5	86	354	5° 5′	Fund.	100	100	——
					1st har.	64.1	58.1	− 6.0
					2d har.	42.1	79.0	+36.9
+1	4.1	88	368.8	8° 0′	Fund.	100	100	——
					1st har.	60.2	62.8	+ 2.6
					2d har.	37.5	62.8	+25.3
+2	4.8	89	374.4	8° 27′	Fund.	100	100	——
					1st har.	60.2	61	− 0.8
					2d har.	37.1	43.5	+ 6.4
+3	4.8	89.8	378.3	9° 7′	Fund.	100	100	——
					1st har.	60.35	60	−0.35
					2d har.	36.70	38	+1.30
+5	5.8	92.1	391.8	10° 28′	Fund.	100	100	——
					1st har.	57	69.2	+12.2
					2d har.	33.4	34.6	+ 1.2

table that here the agreement is entirely satisfactory for *loose couplings.*[1]   For the couplings − 1 cm. and − 2 cm. one can say that there is agreement to within 1 per cent.   As the coupling gets closer the agreement is less good until for the closest couplings (+ 3 cm. and + 5 cm.) the disagreement for the maxima near the oscillator reaches values around 50 per cent.   We regard this as a complete experimental verification of the truth of the Kirchhoff-Abraham generalization of the Thomson formula

[1] It should be borne in mind that for the −3 cm. coupling the energy available was too small to make accurate work possible.

$$\frac{2\pi l}{\lambda} \tan \frac{2\pi l}{\lambda} = \frac{Kl}{K_0}.$$

So far as we are aware this verification up till now has been lacking. It is to be said, of course, that the formula has been proven to hold only for the case where the end-capacity is small in comparison to the total capacity of the discharge wire. The reverse case, the Thomson formula, has been verified by many investigators. The general case, viz., when the capacities of the condenser and discharge wire are comparable in value, remains to be verified. We have assigned ourselves the task of attempting its verification and hope to be able to make a report in the near future.

It is of course not to be expected that there should be good agreement between theory and experiment for the *close* couplings. For the theory takes no account of the mutual interaction of the oscillator and Lecher circuits. In view of the importance of the subject both from the theoretical and practical standpoints some of the mathematical physicists might find it desirable to consider the effect of coupling upon the free vibrations of such a system. Furthermore, agreement between theory and experiment is not to be expected as to the relative intensities of the various maxima. For the theory assumes the entire absence of oscillator vibrations. In no case, so far, have we been able to avoid them entirely. Experiments are now in progress where by employing a very long oscillator we hope to free the Lecher system from the oscillator vibrations. For the case that seemed the most promising, viz., the right half of the + 3 cm. curve we have recorded the comparison in Table IV. The agreement is satisfactory. However, the figures for the − 2, + 1, + 2, and + 5 cm. curves attest that there are two causes present in our experiments tending to vary the relative intensities of the harmonics, neither of which the theory takes into account; namely, the presence of the oscillator vibrations and the effect of coupling at both ends of the Lecher wires. These two causes are to be differentiated from each other. For the figures given in the table, which are only for the harmonics near the bolometer end, the coupling at this end as it gets closer (*e. g.*, + 5 cm.) raises the first harmonic above its theoretical value. For the loose

couplings (*e. g.*, — 2 cm.) the reverse is true, but superimposed upon this effect is the swinging in of the oscillator vibration in tune with the second harmonic to raise it far above its normal value. The figures for all the curves not shown in the table bear out this idea.

Referring again to Table III. it should be remarked that since both α and $\xi \tan \xi$ involve l it is plain that the per cent. of variation between theory and experiment is but little affected by a mistake of a few millimeters in determining[1] the actual end of the wire. It is, of course, difficult to determine accurately the true reading of K_0/K. If we plot the values as given in column 6 against the
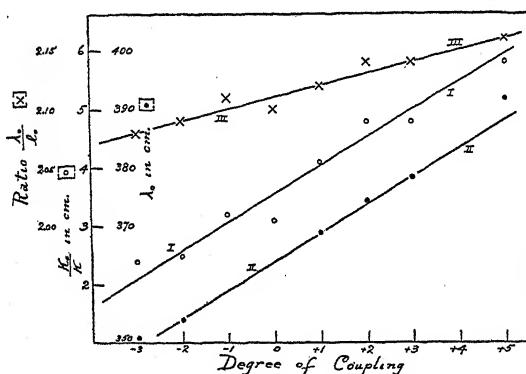
Fig. 13.

couplings we obtain Curve *I* of Fig. 13. Although the points do not fit the curve very closely the change in capacity seems to be a linear function of the extent to which the Lecher wires extend into the cylinders. Curve *II* of the same figure shows the change in wave-length as the wires were shortened. In both these curves it should be remembered that a change of one or two millimeters in locating a given maximum or the actual end of the wire could readily account for the variations from the curves shown.[2] We have seen

[1] The actual end of the wires was determined within 0.5 mm. for the coupling —3 cm., the other values being obtained by adding the amount clipped off of the bolometer end as we lessened the coupling. No great care was used in doing this cutting. The amount cut at the two ends was generally though not always the same.

[2] As a matter of fact the maxima were located more accurately than the measurements of the linear dimensions of the system would warrant. We were not aware of the need for the closest measurement until after these experiments were completed

that a close coupling displaces the harmonics near the oscillator toward the oscillator. For a coupling as close as $+ 5$ cm.[1] it is very probable that the fundamental itself is also somewhat displaced in the same direction. This would account for the fact that on the wave-length curve of Fig. 13 the $+ 5$ cm. point is too high.

The total length of the Lecher wires is given in column 16 of Table III. If we add to this length twice the value of K_0/K and multiply the result by 2, in every case we obtain a figure which is several centimeters short of the value for λ_0 given in column 9. Now physically the capacities at the two ends of the Lecher wires were always identical. The only possible interpretation of this discrepancy is this, it seems to us—electrically there is a real difference in value between the capacities at the two ends in spite of their identical geometrical configuration. The values of the end-capacity at the oscillator end are given in column 17 of the table. In general they are from 2 to 3 cm. higher than the corresponding capacities at the bolometer end of the system. Nor can this difference be materially altered by the "juggling" of the points of Curve I, Fig. 13, to fit the curve. In attempting an interpretation of this difference in the end capacities one should remember that there is a constant in-pour of energy at the oscillator end and an out-pour of energy at the bolometer end. This would necessarily introduce phase differences at the two ends. Indeed, the essential difference between the facts of our experiments and the theory lies here, it seems to us. Theory calls for a permanent train of waves continually reflected from end to end of the parallel wires.[2] In our experiments, to account for all the facts, one would have to calculate the phase relations between the incoming and reflected waves at the oscillator end of the Lecher system and between the outgoing and reflected waves at the bolometer end of the system. The original

and the comparison with theory was undertaken. The figures in column 4 are accurate only to within 2 mm. due to a lengthening of the Lecher wires (unfortunately they were soft copper wires) under tension. We have no means of correcting this, however, and it certainly can affect the arguments or the conclusions arrived at not at all.

[1] The terms "close" and "loose" couplings in this paper are relative terms only. It is probable that even the closest coupling we used would be called "loose" in wireless telegraphy.

[2] We are safe, we think, in assuming with MacDonald (Electric Waves, p. 82) that the train of waves is permanent except for radiation at the ends.

notion of Professor Rubens was to have the detecting instrument steal but an inappreciable fraction of the energy from the swinging Lecher system. This is undoubtedly correct. However, we ought to expect that as the coupling gets looser the difference between the two end capacities would tend to vanish. It seems from comparing columns 6 and 17 of Table III. that this is the case but more accurate measurement would be necessary to determine this.

If we plot the ratio between the fundamental wave-lengths, λ_0, to the total actual wire lengths, l_0, for the different couplings we obtain Curve *III*, Fig. 13. For the loosest coupling the ratio is 2.08, increasing in a linear ratio with the coupling. Extrapolated backward this curve cuts the horizontal axis 2.00 for the coupling − 11.5 cm. Now for the coupling − 3 cm. the energy available at the bolometer end was as small as our apparatus would detect. Great increases in the sensibility of the detecting instruments would have to be attained before this limiting ratio 2.00 could ever be reached. The question of the true value of this ratio in a given case and the factors that control it is very important. We regard it as unsettled. It demands a special investigation. It will be conceded we think that a greater degree of care has never been used in work on electric waves than we have used in these experiments. Curve *III* shows that the ratio can vary with the conditions. We mention this in the hope that other experimenters in this field will stop wasting time working on this question using the earlier crude methods.

When in the work of Blake and Ruppersberg it was stated that our experiments would show that there was always an "end-correction" of four per cent., it was thought that Curve *I* of Fig. 13 sloped off asymptotically toward the X-axis for looser couplings. Careful revision of the data shows that a straight line would fit the points as well as a curve however. We still have the feeling that such an end-correction is present even in an absolutely isolated Lecher system. But we are not able at present to insist on this. We must investigate further. Nor would the presence of this end-correction mean necessarily radiation taking place from the free ends. We can conceive that some of the energy might swing past the free end before reflection occurs and yet get back again instead of being lost to the system by radiation.

Summary.

1. We have shown by these experiments how it is possible to get the free vibrations of a Lecher system by using a Lecher oscillator with suitable water resistances and air-gaps between the induction coil and the vibrator spark-gap. It was proven that in this case the frequencies were in the ratio of the natural numbers 1, 2, 3, 4, etc., while in the case of the Lecher circuit closed at the end (Blondlot type) the frequencies present bear the ratios 1 : 3 : 5 : 7, etc.[1]

2. The effect of changing the coupling between the various circuits was studied and it was found that for a loose coupling the tones present were strictly harmonic. When this was the case it was found possible to verify the Kirchhoff-Abraham generalization of the Thomson formula for a discharge circuit to within one per cent. This was accomplished only by employing a check receiver and introducing all possible refinements of experimental methods.

3. The effect of changing the distance between the Lecher wires was studied and it was found to introduce complications as this distance and hence the bridge lengths became appreciable in amount in comparison to the length of the Lecher circuit, a result to be expected. The best set of free vibrations was obtained when the distance between the Lecher wires was 1.6 per cent. of their own length.

4. Our experiments were compared with the theory of free vibrations completely developed by Abraham and they were found to differ from the theory because of two conditions which the theory does not allow for, viz., the presence of oscillator vibrations on the Lecher circuit and the necessary proximity of the source of energy supply and of the means of detection of the vibrations.

5. When the coupling between the oscillator and Lecher circuits was close two fundamental oscillator vibrations were usually present. It was found that the oscillator vibration could be made to superpose itself upon any given harmonic to make it of abnormal intensity. No overtone of the oscillator seemed to be present.

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[1] The consistency (?) of the usual notation is here clearly shown. It is customary in alternating current work to say that only the odd harmonics are present, or in other words to *include* the fundamental as one of the harmonics. The same is true for closed organ pipes. For open organ pipes on the other hand we *exclude* the fundamental from the list of harmonics, for we say the octave of the fundamental is the first harmonic.

6. For the best set of free vibrations the ratio of maximum to minimum was 40 to 1. This was obtained when the coupling at both ends of the Lecher circuit was $+ 1.4$ cm. in arbitrary units for an unusual type of coupling.

7. The ratio of wave-length to wire length was found to vary with the coupling, ranging from 2.16 to 2.08, the last figure in agreement with the result of Blake and Ruppersberg.

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A COMPARISON OF THEORETICAL AND OBSERVED EQUILIBRIUM TEMPERATURES IN FOG CHAMBERS.

By Edgar K. Chapman.

THE first attempt to make a direct determination of the temperature existing the instant after expansion in a Wilson fog chamber seems to have been made by R. A. Millikan[1] in connection with work on the determination of *e*.

He found that if the temperature fell at all to the theoretical value it returned almost instantly to room temperature.

Further experiments by Millikan, Chapman and Moody[2] showed that this exceedingly rapid rise in temperature after the expansion was due to the rapid inflow of heat from the metal electrodes.

The present experiments were undertaken at the suggestion of Professor Millikan to determine whether, in a fog chamber in which such an inflow of heat was not possible, the actual equilibrium temperature reached in an expansion would agree with the value given by C. T. R. Wilson's method of computing this temperature.[3]

The experimental determination was made as follows: one junction of a Cu-Fe thermo-couple of mil wire was introduced into the middle of a large carboy of 100 liters capacity containing in addition to air, the liquid and its saturated vapor. This couple was connected in series with a sensitive galvanometer. The other junction was kept at a constant temperature by means of a large water bath in which the carboy was also immersed. A battery of constant E.M.F. was connected in series with two resistances, one of approximately 100,000 ohms, the other of a value which varied from one to ten ohms. The thermo-couple was connected to the terminals of this latter resistance so as to form a potentiometer system. The pressure within the carboy was then increased to any desired amount

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[1] Phil. Mag., Vol. 19, pp. 214, 1910.

[2] Phys. Rev., Vol. 30, pp. 286.

[3] Phil. Trans., A, pp. 209, 1907, also J. J. Thomson, Phil. Mag., Vol. XLVI., pp. 538, 1898, and "Conduction of Electricity through Gases," 1906.

by means of a bicycle pump and the system was left standing until the temperature of the bath had been attained. A large valve was then opened so as to allow the pressure to fall to that of the room. When the keys in both circuits were closed simultaneously the thermo-electric current was manifested by the deflection of the galvanometer. The value of the variable resistance was then changed and the operation repeated until the E.M.F. of the couple was balanced by the P.D. across this resistance.

The couple was then graduated and a *temperature-P.D.* curve drawn so that the temperature after expansion could be read directly from this curve.

The final form of the theoretical equations used by J. J. Thomson for computing these temperatures is

$$\rho = \frac{\rho'}{x} - \frac{C_v}{L} \frac{.00129}{x} \frac{273}{273 + T} \frac{p}{760} (t - t_2).$$

T is the temperature of the saturated vapor before expansion.
ρ' is the vapor density of the saturated vapor before expansion.
x is the ratio of the final volume of the gas to its original volume.
p is the original pressure.
t_2 is the temperature to which the gas falls by adiabatic expansion.
t is the equilibrium temperature of the gas, *i. e.*, the temperature to which it rises from t_2 because of the condensation of the vapor, and ρ is the vapor density at the temperature t. Thus ρ is a known function of t, and the solution consists in substituting values for t until one is found which gives ρ its known value.

After considerable work with this equation, it was found that the value given by the thermo-couple did not agree with the computed value, the former being constantly higher than the latter.

A consideration of the origin of the above theoretical equation convinced the author that the value of x given by Thomson in the form

$$x = \frac{V_2}{V_1} = \frac{P_1}{P_2} = \frac{B + p - \pi}{B - \pi}$$

should instead take the form

$$x = \frac{V_2}{V_1} = \frac{P_1}{P_2} = \frac{B + p - \pi}{B - \pi_{t_2}},$$

in which B = barometric pressure, p = difference in levels of manometer before and after expansion, π = vapor density at the temperature of the experiment and π_{t_2} = vapor density at the temperature t_2.

Since the pressure of the air after expansion is $B - \pi_{t_2}$ instead of $B - \pi$ the latter form is obviously correct and the difference between π and π_{t_2} in experiments of this kind is so great that there is a considerable difference between the results of the two formulæ.

When this correction in Thomson's formula was made the experimental and theoretical results were found to be in complete accord as the following table shows:

T	P_1	P_2	t (computed).	t' (observed).	$t-t'$
21.9	90.15	73.35	13.91	13.96	−.05
21.9	93.95	73.35	11.91	11.96	−.05
21.9	98.05	73.35	9.	9.98	−.98
21.9	102.1	73.35	7.5	8.	−.5
22.2	89.2	72.55	14.26	14.44	−.18
22.2	84.85	72.55	16.5	16.73	−.27
22.1	80.97	72.51	18.01	18.12	−.11
22.	76.73	72.58	20.31	20.01	+.30
20.95	78.15	73.65	18.82	18.99	−.17
21.85	82.46	73.56	17.65	17.76	−.22
21.85	86.06	73.56	16.	15.88	+.12
21.85	90.26	73.52	13.84	13.93	−.09
28.7	80.17	71.62	25.25	24.72	+.53
28.5	84.59	71.65	23.1	22.53	+.57
28.5	90.05	71.65	20.85	20.54	+.31
28.25	94.94	71.69	18.21	18.31	−.10
28.15	99.31	71.71	16.20	16.23	−.03
20.5	88.60	71.75	11.80	12.26	−.46

Average difference between the thermo-couple and computed values is .05° C.

The air was then saturated with alcohol vapor with the following results.

T	P_1	P_2	t (computed).	t' (observed).	$t-t'$
20.8	75.18	71.1	19.03	18.81	.22
20.45	81.66	72.61	16.65	16.47	.18
20.45	87.65	73.31	14.50	14.48	.02
20.6	94.16	74.	12.16	12.64	−.48

The results show that the thermo-couple takes up the temperature completely, the slight error of .05° C. being in the direction expected and practically nil.

Further in no case is there any evidence that the temperature falls below the equilibrium temperature. The heat of condensation counteracts the lowering of the temperature due to the adiabatic expansion of the gas. If this were not true, the thermo-couple would certainly indicate it, for the difference between the lowest temperature and the equilibrium temperature is in some cases more than 20° C.

RYERSON LABORATORY,
UNIVERSITY OF CHICAGO,
June, 1910.

SPARK LENGTH, POTENTIAL AND FREQUENCY OF OSCILLATION: THE "LAG EFFECT" IN ELECTRIC DISCHARGE.

By J. C. Hubbard.

Introduction.

THE spark potential for a given spark length is ordinarily defined as the greatest potential which can be applied to the spark gap for an indefinitely long time without causing the discharge to take place. The usual method of measuring spark potentials has accordingly been that of slowly increasing the potential, keeping the spark length constant, or of slowly diminishing the latter while the potential remains constant, until the discharge occurs. Extensive work has been done on the relation between spark length and potential for steady potentials, an admirable summary of which may be found in Sir J. J. Thomson's Conduction of Electricity through Gases (second edition). The study of short sparks has been especially prominent in recent years owing to the bearing of the phenomena upon the theory of ionization.[1]

It has long been known that a potential greater than the spark potential as defined above may be applied for a short time without producing discharge, especially if the gas be dry and free of dust. This effect has been called the "lag" of the spark, or the lag effect, and on it there has been much experimental work and discussion. Jaumann[2] pointed out some time ago that the time of lag is the interval necessary for some agency to convert the gas from an insulator into a conductor. It has been found that the presence of moisture, or the effect of such agents as ultra-violet light and Roentgen rays, in short, that the introduction by any agency of ions into the field between the electrodes, reduces the time of lag; though the ultimate discharge potential is not much affected.

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[1] Conduction of Electricity through Gases, pp. 455–460. See also E. H. Williams, Phys. Rev., 31, pp. 216–240, 1910.

[2] Jaumann, Wied. Ann., 55, p. 656.

No very definite attempt seems so far to have been made to study quantitatively the lag effect. A promising method consists in applying electric oscillations of known amplitude and frequency to the spark gap. In a previous communication of mine to this journal[1] very positive indication was found that for sparks produced by an oscillating potential the spark length is a function not only of the maximum potential but also of the frequency of oscillation. The relation found was such that for very slow oscillations the spark potential approached the value for steady potentials, while with increasing frequency and constant spark gap the spark potentials became systematically higher. The work was not undertaken primarily with the object of studying the lag effect, and, as the spark gap in this case was at the break of an inductive circuit where complications due to the motions of parts might arise, in so far as it relates to the lag effect it is not to be considered of final value. There appeared almost simultaneously some interesting experiments by J. Algermissen[2] on the relation of spark length and potential for rapid oscillations of potential leading to similar results. The oscillations were produced in a system containing a spark gap by induction from a neighboring circuit. Sparks up to 2 cm. in length were studied, and complications arising from the length of spark being large in comparison to the radius of spark electrodes were introduced. The result was unmistakable, however, that as the frequency is increased the spark potential is increased for a given spark length.

The present study of alternating spark potentials makes use of a new method designed to give values of the potential and frequency under perfect control, and presents results of observation of a large number of cases together with some general conclusions.

The Source of Potential.

As a source of potential in the present work is used the oscillations set up in a circuit containing inductance and capacity when the initial current is interrupted. The conditions for breaking such a circuit without producing a spark at the break have been given in

[1] J. C. Hubbard, Phys. Rev., 22, pp. 129–158, March, 1906.
[2] J. Algermissen, Ann. der Phys. (4), 19, p. 1016, April, 1906.

my paper referred to above. In brief, if the speed of separation of the contacts exceed a certain critical value, called the velocity of break, no spark will be produced on breaking the circuit. This critical velocity was found to be proportional to the frequency and to increase almost linearly with the maximum potential, the effect of damping being negligible. Making use of a circuit breaker operated under the proper conditions it is possible, within the limits set by the attainable velocity of break, to secure oscillations of any desired frequency and initial amplitude.

Method and Apparatus.

The electrical system is shown diagrammatically in Fig. 1. The principal parts consist of the inductance L having the resistance R, a condenser K, battery E, interrupter U, an aux-iliary resistance R_b for varying the initial current, and a micrometer spark gap S in parallel with the inductance and capacity. Let the current $i_0 = E/(R + R_b)$ be established, then provided the circuit at U is broken with sufficient sudden-ness to prevent a spark forming there, we shall have oscillations of potential at the electrodes of the micrometer spark gap, which, in view of the small values of K and R and the large values

Fig. 1.

of L (see measurements below) may be accurately expressed by
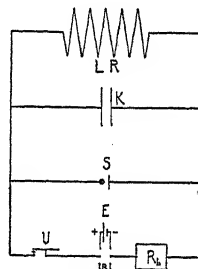
$$V = i_0 \sqrt{\frac{L}{K}}\, e^{-\frac{R}{2L}t} \sin\left(\frac{t}{\sqrt{KL}}\right)$$

and which have the period $T = 2\pi\sqrt{KL}$.[1]

The method of observation consists in slowly lessening the width of the spark gap while interruptions of the current are being made until a spark takes place. The mean of several spark lengths so found is taken as the spark length corresponding to the initial current in the expression for V given above. The spark observa-tions are made visually.

The condenser and inductance have been very fully described in previous communications from this laboratory and will be only briefly noticed here.

[1] Phys. Rev., 22, p. 131, 1906.

The *condenser*[1] consists of two thick steel plates of 25 cm. radius held apart by sets of three small cylinders cut from plane parallel plates of glass.   The capacities were calculated from the formula of Kirchhoff.   The range used in these experiments was between 200 and 600 cm.
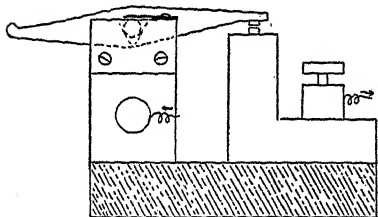
Fig. 2.

The *inductance*[2] consists of a coil of 3,600 turns of no. 18 (B. & S.) double cotton-covered copper wire wound in a large wooden spool.   The channel filled with windings has a square cross-section 3 inches on a side, and its internal diameter is 8 in.   The coil is divided into six independent sections of ten layers each; with all in series the inductance is 3.01 henries and the resistance 62 ohms. By using various combinations of sections a great range of inductance is possible.
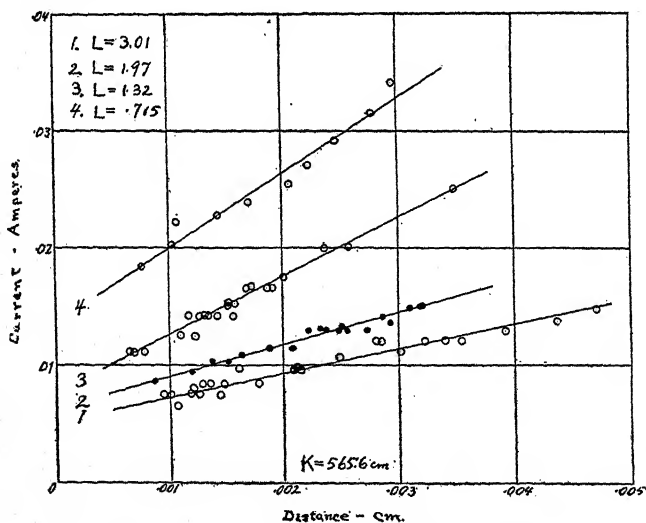
Fig. 3.

Besides this coil there were used two large coils from the Worcester Polytechnic Institute kindly loaned by Professor Duff.   These had

[1] A. G. Webster, Phys. Rev., 6, p. 300, 1898.
[2] J. E. Ives, Phys. Rev., 14, p. 298, 1902.

an inductance when used in series of nearly a henry. It was found, however, that the insulation of these coils was not sufficient for work with any but the smallest sparks.

The *interrupter*, one of several designed by Professor Webster for use with the drop chronograph (*loc. cit.*), may be readily understood by a glance at Fig. 2. The circuit is broken by the falling of a weight on the end of a lever. The requirements are perfect rigidity and freedom from vibration when the lever arm is struck,
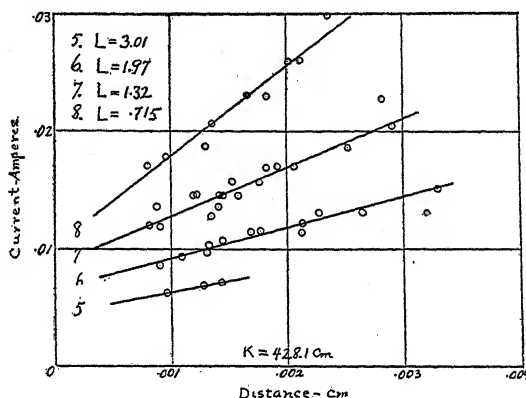
Fig. 4.

and a return to negligible resistance of contacts as soon as the circuit is closed. The lever should be of as small moment of inertia as is consistent with these requirements. The lever is of steel, shrunk on a steel axis which turns in V-shaped grooves in the tops of two upright projections of a brass frame. The latter is mounted on a block of ebonite which also carries the mounting for the other contact. A pair of stiff steel springs pressing the upper side of the axis keep it firmly in the grooves, and a stiff band of steel pressing against one end of the axis holds the other end against a metal stop mounted on the outside of the opposite post. Perfect and permanent adjustment is thus secured, the arrangement constituting a Kelvin so-called geometrical slide. The contacts are of silver and rubber bands (not shown) cause the contacts to press together as soon as the weight which operates the lever has done its work. This arrangement has given perfect satisfaction and stands the

roughest usage. The resistance of the interrupter with the circuit closed was invariably found to be a small fraction of an ohm.

The *circuit breaker* first tried was the projectile of the drop chronograph. Several seconds were required to produce a single interruption and many minutes to find a single spark length. In its place was substituted a bicycle wheel operated by an electric motor. The wheel carried a two pound mass of iron on its rim for striking the lever arm of the break, suitable counterpoise being added to the inside of the rim on the opposite side. The wheel was usually rotated so as to produce about two breaks per second.
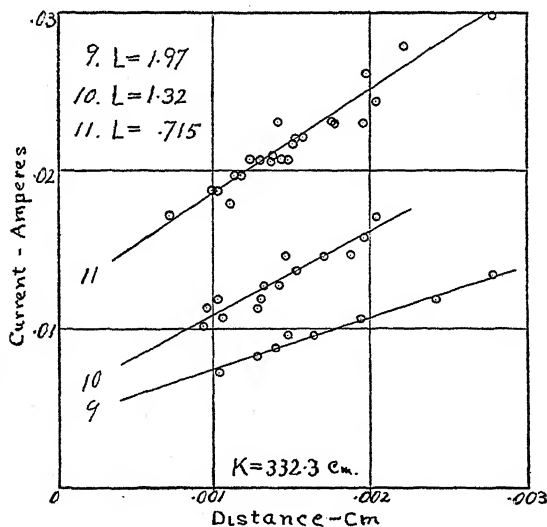
Fig. 5.

The *micrometer spark gap* is very similar in principle to many which have been described before.[1] The spark electrodes were of steel, one of them being a plane 10×12 mm., the other a ball 6.25 mm. in diameter mounted on the end of a brass cylinder of nearly the same diameter. These were ground with several pourings of flour of emery then given a high polish with diamantine on soft leather. The amount of labor involved in producing surfaces of high polish, free from scratches, on steel is extraordinary, and I am greatly indebted to Mr. H. F. Stimson, Scholar in Physics

[1] G. M. Hobbs, Phil. Mag. (6), 10, p. 620, December, 1905.

in Clark University, who undertook a careful study of the matter. Cleaning of the surfaces was effected by rubbing with soft, clean cloths, then with chamois skin and chalk or jeweller's rouge. Small particles of dust were removed by the use of an air jet. The plane was mounted in parallel ways and the ball with its axis of rotation at an acute angle with the plane. After the passage of a spark the plane was given a small translation and the ball a small rotaton so as to move the near surfaces in opposite directions, thus presenting fresh surfaces for the next spark. During the final measurements a jet of air (see below) was directed into the space between the electrodes. A separate circuit, the leads of which could be attached to the spark electrodes, containing a battery, resistance, and millivoltmeter was used to determine the position of contact of the electrodes. This zero was found immediately after the passage of each spark. It could be determined to one-tenth division of the drumhead of the micrometer screw, corresponding to .00002 cm. The results given here are mostly for sparks from .005 to .0005 cm. in length. The order of accuracy is therefore about one per cent. Difficulties of another sort discussed below render greater accuracy of reading useless for the present purpose.

Potentiometer and bridge circuits were arranged for frequently testing respectively the electromotive force E of the battery and the resistance of various parts of the system. For the battery E was used one or two storage cells.

CONTROL EXPERIMENTS.

Frequency of Interruption of the Circuit.—On changing from the drop chronograph to the bicycle-wheel break no change of result was observed, showing that it was immaterial whether the train of oscillations was applied to the spark gap once in many seconds or twice per second.

Surface of the Spark Electrodes.—Different methods of polishing and cleaning gave approximately the same results except for the higher frequencies where the passage of the spark is extremely sensitive to the state of the surfaces. Especial care had to be taken to use only the freshest of materials in cleaning the electrodes.

State of the Gas.—First experiments gave results which varied

by 50 per cent. among themselves. After much effort to get more uniform data it was found that a fine jet of air blown from a tapering glass tube into the discharge space reduced the necessary sparking potential to a very much lower value and enabled results to be repeated within a few per cent. of each other (curves 1–11, Figs.
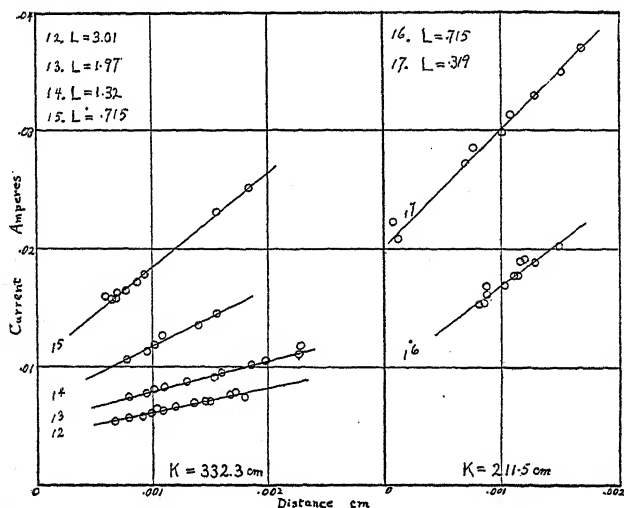
Fig. 6.

3–5).[1] Such discrepancies as then remained were found to be due to inefficient insulation in the windings of the coil, and before a last series of measurements was made were eliminated almost entirely by placing the coil in kerosene (curves 12–17, Fig. 6). Obviously, as the experiments go, there is no need to consider the effect of changes of atmospheric pressure. It may be remarked, however, that the most consistent results were obtained during rainy weather.

POTENTIAL AT THE TIME OF SPARKING.

While the damping in these experiments is small, it is of some

[1] The air coming from a filter-pump air-blast, the water in which was usually at about 6° C., passed through a long rubber tube and then through a plug of glass wool to filter out dust, before passing to the spark gap. The air was delivered at approximately the temperature of the room (20°) and was far from being saturated with moisture. The effect of the air was very striking, and seems to show that a part of the lag effect depends upon the number of ions initially present in the air. A tube containing a small quantity of radium placed under the spark gap produced similar results. A systematic study of the effect of various ionizing agents is to be made in later work.

interest to know whether the discharge begins during the first maximum of potential.   It is conceivable that if the effect of a maximum were just unable to produce a spark it might succeed in forming ions which would enable the spark to pass at some following maximum.   The first spark of condenser discharge has long
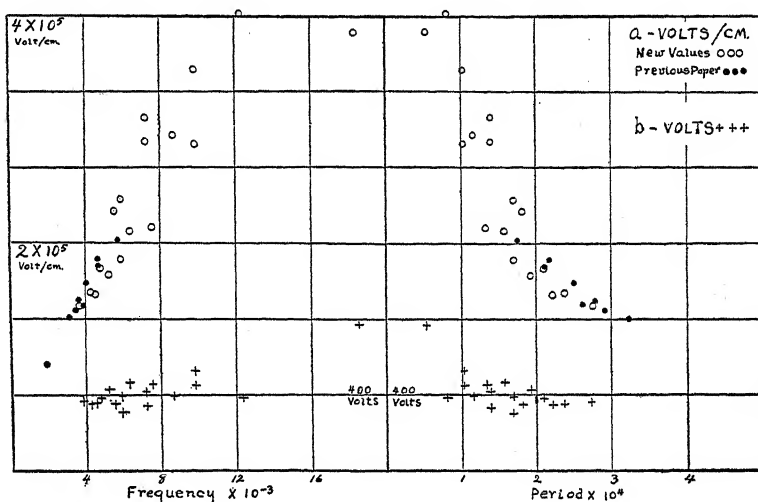
Fig. 7.

been known to be of different character from the others and has been given the name pilot spark.   I have found evidence to show that, given a spark length and the oscillating potential which will just produce a spark, but not greater, the spark will begin during the first maximum of induced potential.   A micrometer microscope was focused upon the space between the contacts of the break, and the velocity of break, at first high enough to prevent sparking altogether, was reduced until a very faint spark occurred.   The length of the spark together with the known velocity of break and the frequency of oscillation placed the time of occurrence of the spark very near the time of the first maximum of potential, *i. e.*, within one eighth of the period.   A small further reduction of the velocity of break caused a complete change in the character of the spark.   It was now thick, brilliant and longer and possessed a beaded appearance.   This is because the rapidly moving electrode was in

different positions at the times of successive oscillations. Measurement showed that the spark length for the first luminous condensation gave a time which invariably came in the second half period, again within one eighth period which was about the reading accuracy of the necessarily low power microscope. The pilot spark which must have preceded it was completely masked. In any event it would seem that the first spark takes place during the first maximum as was directly observed for the limiting case. For *constant* spark lengths at the micrometer spark gap no such changes were observed, which is easily understood as we do not there have the influence of the rapidly widening spark gap in preventing the second and successive discharges from passing. It seems probable, also, that unless the first maximum is great enough to produce a discharge, any ions that might be formed would be swept out of the field, making it less likely that the successive maxima would produce a discharge.

Further evidence on this question will be found in the discussion of results. Assuming that the spark takes place near the end of the first quarter period we have, in the most unfavorable case ($R = 27.1$ ohms, $L = .715$ henry, and $K = 6.28 \times 10^{-10}$ farads), as the value of the damping factor $e^{-\frac{R}{2L}\frac{\pi}{2}\sqrt{KL}} = .9987$, which is unity for our purpose. We therefore take

$$V = i_0 \sqrt{\frac{L}{K}} \sin\left(\frac{t}{\sqrt{KL}}\right)$$

and

$$V_m = i_0 \sqrt{\frac{L}{K}}. \tag{1}$$

This discussion ignores the fact that the resistance of the coil for rapid oscillations increases somewhat rapidly with the frequency.[1] Since we have a margin of safety of several hundred per cent., it is hardly likely that the effect of the damping need be considered.

RESULTS.

Several series of observations were made to show the relationship between spark length and initial current. The results of these

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[1] See, for example, A. Esau, Ann. der Phys. (4), 34, pp. 57 and 81, 1911.

measurements are shown graphically in Figs. 3–6. Abscissæ represent spark lengths in cm., and the ordinates represent the currents in amperes which were flowing at the various times of break and from which the potentials may be calculated from equation (1). Each point shown in the figures is a mean of several, usually five, readings.

The most important result of the measurements is at once seen from the figures; namely, that with given K and L and within the limits of the experiments the relation between initial current and spark length is a linear one. Lines best representing the results are drawn in the figures and are given numbers which correspond in general to the order in which the experiments were made. Series 1 and 2, however, contain observations weeks apart, some of the other series having been determined in the meantime. All the series obtained with the air jet in use are shown.

During most of the control experiments spark lengths were obtained up to .01 cm. Owing to the gradual deterioration of the insulation of the coils the available region of experiment became more restricted and by the time series 10 was reached it was almost impossible to get results at all, while such as were obtained scattered very badly. The coil was then immersed in kerosene for the remainder of the work, giving much more regular results (series 12–17). Immersion in kerosene did not, however, restore the available working region to its former value. Measurements taken with the two coils furnished by Professor Duff gave consistent readings for only the smallest initial currents which would produce visible sparks. These results indicate very clearly that for satisfactory work with these oscillations coils must be used of which there is no question as to high insulation. A coil is now being designed with which it is hoped to meet such objections in an extension of the work over a wider range of spark lengths and under varied conditions. It must be remarked, however, that no indication has been found of a change of slope of any of these curves due to the progressive deterioration of the insulation.

Treatment of the Observations.—Since the relationship between the initial current and the spark length is a linear one, and the slopes of the lines are affected by change in the inductance or

capacity only, it is at once evident that for oscillating potentials under such conditions the period alone and not the amplitude affects the spark constants. We may write

$$i_0 = \alpha x + \beta$$

as expressing the relation between the current i_0 and spark length x. The constants α and β depend only on K and L. Since

$$V_m = i_0 \sqrt{L/K}$$

(see equation (1) and discussion above), we may write

$$V_m = \alpha \sqrt{L/K} \cdot x + \beta \sqrt{L/K} = ax + b \qquad (2)$$

where

$$a = \alpha \sqrt{L/K} = f_1(\sqrt{KL}), \text{ etc.}$$

It now remains to examine a and b as functions of the period. In Table I. are given the values of α and β from which the curves in Figs. 3–6 are drawn, together with the corresponding values of a and b, and the periods $T = 2\pi\sqrt{KL}$ and frequencies n. The values of a and b so found are plotted in Fig. 7 as functions both of frequency and of period. The figure at once suggests that, within the errors of experiment, the constant a is a linear function of the frequency. It must be remembered that a few observations more or less in Figs. 3–6 would change the slopes of the lines as well as the intercepts, and at higher frequencies especially the values of a and b are enormously sensitive to errors of original observations. The values of a obtained in my previous paper (*loc. cit.*, p. 154) by an indirect method are given in Table II. and are shown in the figure as filled circles. These agree in a striking manner with the results of the present measurements. This astonished me, inasmuch as the former work was done under entirely different conditions. In the first place, the air jet was not used, though the experiments were carried on in very damp spring weather. Again, in the former case the spark gap was widening at a rapid rate at the instant the spark took place, the rate being such that the electric force was practically uniform between the electrodes from the start. The agreement of the two sets of results raises questions which can only be settled by further experiments. It will be ex-

plained if the spark takes place at the time at which the potential gradient has reached its first maximum in both sets of experiments. This is further justification for our use of equation (1) in the present case and of the treatment of the results in the former.

TABLE I.[1]

No.	K, cm.	L. henries.	$T \times 10^4$	$n \times 10^{-3}$	a	β	$a \times 10^{-5}$ $\frac{\text{volts}}{\text{cm.}}$	b, Volts.
1	565.6	3.01	2.73	3.66	2.10	.00523	1.45	362
2	565.6	1.97	2.21	4.52	2.77	628	1.55	352
3	565.6	1.32	1.81	5.52	4.98	764	2.28	350
4	565.6	.715	1.33	7.52	6.34	1360	2.14	458
5	428.1	3.01	2.37	4.22	1.97	440	1.57	350
6	428.1	1.97	1.92	5.21	2.66	662	1.72	426
7	428.1	1.32	1.58	6.35	4.05	878	2.11	462
8	428.1	.715	1.16	8.63	7.63	1026	2.96	397
9	332.3	1.97	1.69	5.90	3.27	415	2.38	303
10	332.3	1.32	1.39	7.21	5.19	565	3.10	338
11	332.3	.715	1.02	9.80	6.53	1202	2.87	529
12	332.3	3.01	2.10	4.76	1.96	422	1.77	382
13	332.3	1.97	1.69	5.90	2.54	540	1.86	394
14	332.3	1.32	1.39	7.21	4.84	696	2.89	416
15	332.3	.715	1.02	9.80	8.02	1024	3.53	451
16	211.5	.715	.814	12.29	7.28	964	4.01	384
17	211.5	.319	.545	18.35	10.46	2092	3.86	771

TABLE II.

From this journal, Vol. 22, p. 154, 1906.

$T \times 10^4$	$n \times 10^{-3}$	$a \times 10^{-5} \frac{\text{volts}}{\text{cm.}}$
1.75	5.72	2.03
2.13	4.69	1.80
2.16	4.63	1.86
2.50	4.00	1.65
2.61	3.83	1.46
2.78	3.60	1.50
2.91	3.44	1.41
3.23	3.10	1.34
5.05	1.98	.93

[1] The resistances of the coils $L = 3.01$, 1.97, 1.32, $.715$, and $.319$ henries are, respectively, $R = 62.5$, 51.3, 38.8, 27.1, and 17.6 ohms. No other appreciable resistance takes part in the oscillations.

It is interesting to notice the trend of a assuming it to be a linear function of the frequency n. The results in Tables I. and II. give, by least squares, for

$$a = pn + c$$

the values p, = 21.49, and c = 81,800 volts/cm., the constant c is the value for $n = 0$, that is, the value of a for steady potentials. This value is not far from the values of a which have been determined by others with spark lengths of the same order for constant potentials; the results of Earhart,[1] for instance, giving a as about 65,000 volts/cm. A still better agreement will be shown if the observations for the higher frequencies, which are very sensitive to the condition of the electrode surfaces, are left out of account.

It cannot be said whether the constant b depends on the frequency. The values of b are very sensitive to errors in a. The results for b show a trend to larger values with increasing values of n, and indicate a value of about 350 volts for zero frequency, again in agreement with steady potential work. Values of b from the previous paper are not shown for the reason that their absolute values are too largely affected by the residual errors of the method of approximation used in their calculation.

It is unfortunate that means were not available for the measurement of the inductance and resistance of the coils at the various frequencies used. The inductance diminishes slowly, while the resistance rises more rapidly with the frequency.[2] Both these effects, if taken into account, would diminish the calculated values of a and b by several per cent., but not nearly by enough to account for the changes observed in a. For example, for the moderate frequency of 4,000 vib. per sec., our value of a is about three times that for zero frequency. Since a involves the square root of L, the latter, in order to account for the whole change, would have to be diminished by eight ninths or 89 per cent. by the effect of the frequency, an amount which is of entirely different order of magnitude from that indicated by such experiments and theory as are available. These considerations will, however, be taken into account in the new experiments which are being planned.

[1] Earhart, Phil. Mag., VI., 1, p. 147, 1901.
[2] Esau, *loc. cit.;* see also J. G. Coffin, Proc. Amer. Acad., 41, p. 789, 1906, etc.

Summary and Conclusions.

This paper presents a study of the relation between spark length, potential, and frequency for oscillating potentials having frequencies from 4,000 to 18,000 vib. per sec., and for spark lengths ranging from .005 to .0005 cm. The potentials are produced by interrupting a known initial current in a circuit containing inductance and capacity. The interruption is made in such a manner that no spark is produced at the break, thus giving definite values of potential at the electrodes of an auxiliary spark gap.

The effect of different amplitudes and frequencies of potential upon the spark constants a and b in the equation $V = ax + b$, usually employed to relate steady potentials V and spark lengths x, is studied. The experiments relate to air at atmospheric pressure.

1. Evidence is presented showing that the discharge produced by a train of oscillations begins near the time of the first maximum of the potential (control experiments, etc.).

2. It is found that the spark lengths are linearly related to the strengths of initial cu rent (Figs. 3–6), the inductance and capacity in the circuit remaining the same; *i. e.*, the spark constants in the equation $V = ax + b$ are independent of the amplitude alone of the oscillating potential.

3. The spark constants vary with the frequency. The constant a in the equation is, within the limits of the experiments, a rapidly increasing linear function of the frequency. When the frequency is zero, that is, for steady potentials, the value of a reduces to its normal value which for these short sparks is about 6.5×10^4 volts/cm. At a frequency of 4,000 vib./sec., for instance, the value of a is nearly three times normal. This statement means that for an oscillating potential of this frequency the maximum potential of an oscillation necessary to produce a spark must be about three times as great as the sparking value of the steady potential, and so on. The value of b is practically the same as for steady potentials, *i. e.*, about 350 volts, except that there is a slight tendency to increase with the frequency.

4. It may be concluded finally that the lag effect of the electric spark depends upon at least two independent factors: (1) Upon the amount of ionization initially present in the spark gap. This

factor is of an accidental nature and is subject to regulation and control (see control experiments). (2) The lag effect depends also upon some process which takes place in the gas and which takes time for its completion such that the shorter the time the greater is the necessary sparking potential. The latter factor will probably be found to depend upon the nature and condition of the gas (pressure, etc.).

These facts accord in a general way with the theory of the spark discharge as developed by Sir J. J. Thomson.

Apparatus for a further study of this phenomenon is under construction, it being evident (Fig. 6) that it will be possible to obtain results of considerable precision. With such results it may be possible by a study of different gases at different pressures to obtain much greater insight into the problem of ionization by an electric field.

CLARK UNIVERSITY,
January, 1911.

A LECTURE ELECTROSCOPE FOR RADIOACTIVITY AND OTHER IONIZATION EXPERIMENTS.

By John Zeleny.

IN the electroscope to be described, the usual method of measuring ionization currents by the rate of motion of gold leaves or of fibers is replaced by one where, instead, the period of pendular vibration of a leaf is measured, the leaf recharging itself automatically after it has lost a definite portion of its charge.

As the period may be made quite short for ionizations as small as that due to uranium, the method has some advantages and is especially adapted for lecture demonstrations. The automatic charging of the leaf makes the action continuous so that the electroscope requires no attention, and various effects may readily be compared in an impressive way by simply noting the changes in the period of the leaf's motion.

A convenient form of the instrument is represented in section in the accompanying figure. The cylindrical case M, with glass ends, is 6.2 cm. in diameter and 4.5 cm. wide, the attached bulb L containing calcium carbide for drying the air. The rectangular plate H is 4.5 cm. long and 8 mm. wide, and can be moved in and out by sliding the supporting rod through the ebonite plug shown. The front face of this plate is coated with india ink, and the plate is connected to a constant source of potential of about 100 volts.

The gold leaf G, about 6 cm. long and 5 mm. wide, is hung in front of the plate H from the rod which supports the horizontal disc D. An amber plug inside of an ebonite one insulates this system from the case.

The disc B forms a condenser with D, the air in which is to be subjected to the ionizing agency studied. The rod O, supporting B, permits of motion up and down, and is insulated from the case by the ebonite stem Q.

The electroscope is used in the following way. B is connected to

the case of the instrument which is earthed, and H is charged to a potential of about 100 volts. The gold leaf is attracted to the plate H and then repelled, the disc D becoming charged to the potential of H, nearly. The electrical field between B and D will cause an ionization current to flow when, for instance, the shallow tray C placed on top of D contains some radioactive material; and
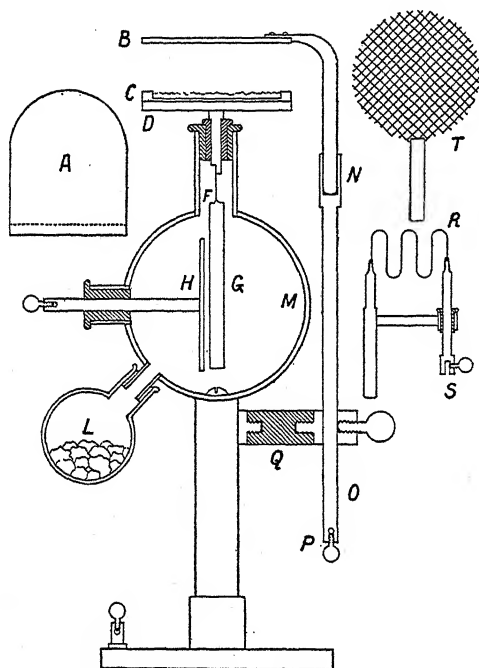
Fig. 1.

the gold leaf system will gradually lose its charge. After the leaf has lost a definite part, which need be but a small part, of its charge, it will be attracted over against H, again repelled and so on, the period of vibration of the leaf being a measure of the current flowing between B and D.

Absorption effects may be studied by placing sheets of material on top of the tray C.

The ionizing powers of γ rays and Röntgen rays may be shown by directing these rays at the electroscope from the side, while the conductivity of flame gases may be illustrated by blowing some of the air above a burner against the plates of the electroscope.

For showing the photo-electric effect, the disc B is removed, by pulling the rod out of the split sleeve N, and the wire gauze disc T is put in its place. Opposite this gauze there is placed on D the piece A, which consists of two similar metal plates joined at right angles, one being of zinc and the other of brass. By changing the position of the piece A, ultra-violet light may be projected through the gauze upon either a zinc or a brass surface. The effect of polarity is noted by reversing the charge on the plate H.

To illustrate the conduction produced by glowing bodies, the gauze T is replaced by the piece carrying the platinum wire R, which may be heated to incandescence by means of a current.

It will be noticed that the gold leaf G is placed with its plane at right angles to the plate H. To permit of motion in this plane, the leaf is cut part way across near the top at F, the upper portion being turned at right angles to the lower. When the plate and leaf are charged the electrical forces tend to keep the leaf in this edge on position. The chief advantage of this disposition is that the leaf, owing to the small resistance experienced when moving in its own plane, is repelled from the plate, after each contact, with an abrupt "kick," giving a sharp beginning and ending to each period. Any tendency of a leaf to adherence is also minimized by this arrangement.

Much difficulty was experienced at first, when the brass plate H was used with its natural surface, because the leaf would almost invariably stick to the plate. This adherence is doubtless due to a cohering effect caused by the rush of electricity across the contact-point when the leaf touches the plate. The smooth coating of india writing ink placed on the plate, besides making coherence more difficult, introduces considerable resistance at the point of contact and effectively overcomes the trouble.

The sensibility of the electroscope may be changed at will by altering the distance between the plate and leaf when these are charged. The potential to which the plate should be charged may vary from about 40 volts to 150 volts or more, the sensibility of the instrument not being greatly different between these limits provided the plate after being charged is pushed close up to the leaf. The potential for the greatest sensibility depends upon the

way each leaf hangs, and is that for which the leaf and plate are parallel when charged and close together. In general a potential of about 100 volts gives the best results, and a convenient way of obtaining this is from one of Kruger's boxes of minute cadmium cells.

The following data are given for illustration. Using uranium oxide in the tray C as the source of ionization, the distance between B and C being 2 cm. and H being at 100 volts, the period of the leaf was changed from 1.5 sec. to 40 sec. as the maximum distance between the leaf and plate was changed from less than 1 mm. to 8 mm. The loss of potential by the leaf between contacts with the plate was about 1.5 volts for each second of the period of the motion; so that for the shorter periods the strength of the electrical field between B and C was nearly constant throughout. The strength of this field may be varied, when desired, by disconnecting B from the case and charging it to any potential.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA,
January 28, 1911.

SPARK DISCHARGE AT VERY SMALL DISTANCES.

By E. H. Williams.

IN a previous article[1] the author investigated the nature of spark discharge at very small distances, especially studying the effect of the material of which the electrodes are made upon the discharge potential.   It was shown that not only does the material of which the electrodes are made have no effect upon the discharge potential, but that the nature of the discharge at very short distances is the same as for longer distances.   It was also shown that the discharge potential for a distance between the electrodes of one wave-length of sodium light is the same as for five wave-lengths, and that for distances less than five wave-lengths the path of the discharge is not along the shortest distance.   Results obtained in air of different degrees of dryness indicated that the nature of the dielectric affects the discharge potential in that this potential is lowered by the presence of moisture in the air between the electrodes.   It is the purpose of this article to extend the above investigations, especially with reference to the effect of the dielectric and pressure upon the discharge potential.
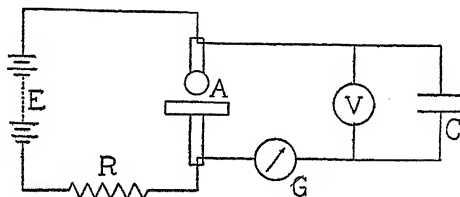
Fig. 1.

The apparatus and method of procedure were the same as in the work referred to above.   The method of noting the passage of the spark, which was the same as in the previous work, was as follows. In Fig. 1, let E be a variable E.M.F., R a high resistance, two meg-

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[1] E. H. Williams, Phys. Rev., 31, p. 216, 1910.

ohms or more, A the electrodes between which the spark passes, V an electrostatic voltmeter, C a small capacity in parallel with the voltmeter, and G a sensitive ballistic galvanometer. Now, when a discharge takes place between the electrodes at A, the potential between the electrodes is lowered for an instant and the condenser system is allowed to discharge through the ballistic galvanometer G, producing a deflection. It was noted, by watching the spark gap and galvanometer at the same time for a large number of readings, that the galvanometer invariably indicated the passage of a spark. It was also observed that the pointer of the voltmeter gave a slight deflection at the instant that the spark passed. The magnitude of the deflection of the galvanometer, or of the voltmeter, could be increased by increasing the resistance R. A telephone receiver, inserted in place of the galvanometer G, is an equally good indicator. By using the capacity C, the quantity of electricity passing through the galvanometer is greater and the time necessary for this quantity to be replaced is correspondingly increased.

NATURE OF THE GAS: EFFECT OF VARIOUS GASES.

In all of the previous work, air was used as the dielectric. It was now proposed to use hydrogen and carbon dioxide. After the box had been washed out with the gas, it was allowed to stand for some time in order that any moisture remaining in the gas might be taken up by the sulphuric acid in the box. The results, which are shown in Tables I. and II., confirm the former conclusion that the nature of the dielectric does affect the discharge potential. According to these results the minimum potential for hydrogen is about 248 volts, while that for carbon dioxide is about 425 volts. The minimum potential for air under the same conditions was previously found to be about 372 volts. In the above tables, which include two or three series taken on different days, distances in wave-lengths are given in the column under λ and potential differences in volts in the column under V.

Effect of Pressure.—The pressure of the gas surrounding the electrodes was varied from one atmosphere to one centimeter of mercury and the effects studied. The results are shown in Tables III., IV. and V. The general trend of these results indicates that

TABLE I.

Hydrogen. Platinum+ Brass−

λ	V	λ	V	λ	V
1.5	247	2.	243	3.	248
1.5	244	2.	249	3.	248
1.5	227	2.	247	3.	245
1.5	240	2.	248	3.	248
1.5	240	2.	247	3.	249
1.5	232	2.	247	5.	249
1.5	246	2.	240	5.	247
1.5	238	2.	249	5.	243
1.5	246	3.	249	5.	252
1.5	237	3.	248	5.	251
2.	246	3.	246	5.	248
2.	247	3.	241	5.	249
2.	247	3.	244	5.	250
2.	244	3.	248	5.	253
2.	241	3.	251	5.	248
2.	247	3.	249	5.	247

the minimum potential is not affected by pressure, although some of the readings point to the contrary, in that they are much higher than the minimum potential for air at atmospheric pressure.   Now,

TABLE II.

Carbon Dioxide. Platinum+ Brass−

λ	V	λ	V	λ	V
2.	362	3.	427	5.	427
2.	410	3.	425	5.	427
2.	380	3.	422	5.	426
2.	422	3.	424	5.	425
2.	414	3.	421	5.	424
2.	423	3.	423	5.	423
2.	407	3.	423	5.	425
2.	423	3.	423	5.	426
2.	422	3.	425	5.	423
2.	425	3.	423	5.	427
3.	424	3.	423	5.	425
3.	427	3.	424	5.	424

if the conditions can be made such that few ions are present in the box which surrounds the apparatus, it is possible that there will

be no ions present between the electrodes. If such is the case, the potential may be raised above the minimum potential without producing a spark. Evidently the chances for such conditions are augmented as the pressure decreases. As soon as an ion is drawn into the field between the electrodes a spark will pass. It is to be expected that there will be no regularity in the discharge potential when these exceed the minimum potential.

TABLE III.

Pressure = 30 cm. of Hg. Platinum+ Brass −

λ	V	λ	V	λ	V
1.5	376	2.	372	3.	372
1.5	372	2.	369	3.	372
1.5	372	2.	373	3.	415
1.5	380	2.	372	3.	373
1.5	362	2.	370	5.	369
1.5	372	2.	371	5.	420
1.5	371	2.	371	5.	372
1.5	372	3.	372	5.	373
2.	368	3.	368	5.	369
2.	372	3.	370	5.	372
2.	370	3.	371	5.	372
2.	369	3.	373	5.	373

TABLE IV.

Pressure = 10 cm. of Hg. Platinum+ Brass −

λ	V	λ	V	λ	V
1.5	393	2.	425	3.	398
1.5	370	2.	372	5.	372
1.5	311	2.	369	5.	372
1.5	373	2.	375	5.	435
1.5	372	2.	372	5.	371
1.5	373	3.	370	5.	372
1.5	328	3.	371	5.	380
1.5	371	3.	374	5.	382
2.	370	3.	372	5.	373
2.	371	3.	445	5.	372
2.	368	3.	372	5.	372
2.	412	3.	372	5.	371
2.	372	3.	373	5.	385

TABLE V.

Pressure =1 cm. of Hg.   Platinum +   Brass −

λ	V	λ	V	λ	V
1.5	329	2.	372	3.	370
1.5	370	2.	372	3.	394
1.5	369	2.	370	3.	370
1.5	342	2.	373	3.	371
1.5	371	2.	371	5.	445
2.	372	3.	445	5.	370
2.	378	3.	373	5.	405
2.	370	3.	370	5.	390
2.	413	3.	374	5.	443
2.	373	3.	372	5.	372
2.	409	3.	465	5.	406
2.	372	3.	445	5.	370
2.	463	3.	372	5.	373

Effect of Ultra-violet Light and Electric Waves upon the Discharge Potential.—In the article referred to above, the effect of ultra-violet light was found to be very large. However, the results for 1.5λ, 2λ and 3λ were very irregular, and since the ultra-violet light was produced by an electric spark between two aluminum electrodes inclosed in a quartz tube, it seemed possible that the large effect might not all be due to the ultra-violet light but that some of it might be due to electric waves or to other external effects due to the presence of high potentials in the neighborhood of the electrodes. In order to decide this point an electric arc between electrodes of iron and of mercury inclosed in a quartz tube was used as the source of ultra-violet light. This arrangement possessed the advantage that only low external potentials were applied in the neighborhood of the electrodes (14 volts across the arc), and that no electric waves were present. Therefore, any effect that might be produced would be entirely due to ultra-violet light. After a large number of observations, the conclusion was reached that the effect of ultra-violet light is very small, and that the large effect in the previous results is due to other causes.

That electric waves have an effect upon the discharge potential is evident from the difference in the results obtained with the two

sources of ultra-violet light—spark discharge and the electric arc. It is also apparent that the magnitude of the effect of the electric waves will depend upon the tuning of the system containing the electrodes to the particular wave-lengths used.

LABORATORY OF PHYSICS,
    UNIVERSITY OF ILLINOIS,
        February, 1911.

A CHARACTERISTIC OF SPECTRAL ENERGY CURVES.
—A CORRECTION.

By W. W. Coblentz.

UNDER the above title[1] the writer gave the complete solution of Planck's spectral energy equation in order to obtain the wave-length of the maximum of emission, "λ_{max}" which is useful in determining the constants entering into this equation.

The object of the present communication is to call attention to several errors in the published solution for "λ_{max}." The value of "α" as it was used in the computations is $\alpha = 4.9651$, and in order to avoid confusion with $\alpha = 5$ (Wien equation) should have been written α'. The first terms should contain the factor α/α'. The complete solution is

$$\lambda_{max} = \frac{\alpha(\log \lambda_2 - \log \lambda_1)\lambda_1\lambda_2}{\alpha'(\lambda_2 - \lambda_1) \log e}$$

(1)

$$+ \frac{\lambda_1\lambda_2 \left[\log \left(1 - e^{-\frac{\alpha'\lambda_m}{\lambda_2}}\right) - \log \left(1 - e^{-\frac{\alpha'\lambda_m}{\lambda_1}}\right)\right]}{\alpha'(\lambda_2 - \lambda_1) \log e}.$$

This follows directly by equating the Planck equation for $E_{\lambda_1} = E_{\lambda_2}$, writing $c_2 = \alpha'\lambda_m T$, taking logarithms, and solving for λ_{max}.

The more cumbersome (second) term previously published was obtained from this equation. In the computations the second term, which enters as a correction factor, is not so complicated as it may seem, for usually when λ_1 is small, this term may be neglected. Furthermore the correction factor need be computed for only three to five points on the long wave-length side of the spectral energy curve, through which a curve is drawn. From this curve the correction factors for other values of λ_2 may be read with a greater precision than demanded by the experiments.[2]

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[1] Phys. Rev., 29, p. 553, 1909.

[2] See Table I., Jahrb. Radioaktivität und Elektronik, 8, p. 1, 1911.

By expanding[1] log $(1 - e^{-k})$ and neglecting higher powers of e^{-k} equation (1) reduces to

$$(2) \qquad \lambda_{max} = \frac{\alpha(\log \lambda_2 - \log \lambda_1)\lambda_1\lambda_2}{\alpha'(\lambda_2 - \lambda_1)\log e} + \frac{\lambda_1\lambda_2[e^{-\frac{\alpha'\lambda_m}{\lambda_1}} - e^{-\frac{\alpha'\lambda_m}{\lambda_2}}]}{\alpha'(\lambda_2 - \lambda_1)\log e}.$$

This is sufficiently accurate[1] for the experimental data now available, but with each renewed effort a higher accuracy in the observational work is attained, which requires greater refinement in the computations. For example the equations kindly given me by Dr. Buckingham and used in the data published a year ago,[2] were applicable over only a narrow spectral region and hence too arbitrary. However, by computing a series of correction curves for the region of 3 to 5μ and extrapolating in both directions, it was possible to obtain fairly accurate data as shown[2] in Table I. of the aforesaid paper.

During the past three years more than 80 spectral energy curves have been obtained, under all sorts of conditions. It is hoped that the complete discussion of the instruments and methods used in obtaining these data and of the aforesaid equations employed in the computations, also the applications of the results may be given in the near future. It will be sufficient to add that the Planck equation appears to be valid for the temperature range from 450° to 1530° C. and for the spectral region to 6.5μ, provided the radiating enclosure is "black."

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[1] Mr. Dellinger has found that but small errors are introduced by dropping all factors in this expansion except $e^{-\frac{\alpha'\lambda_m}{\lambda_2}}$ which further simplifies the second term of eq. (2). In actual practice, however, the writer prefers to retain the factor $e^{-\frac{\alpha'\lambda_m}{\lambda_1}}$ as given in equation 2, but using it only when λ_1 and λ_2 are close together, *i. e.*, near the maximum of the spectral energy curve where the correction due to this term may amount to .6 per cent.

[2] PHYS. REV., 31, p. 317, 1910.

PROCEEDINGS

OF THE

American Physical Society.

Minutes of the Fifty-Seventh Meeting.

THE regular spring meeting of the Physical Society was held in Washington, D. C., on Friday, April 21, and Saturday, April 22, 1911, the Friday session being held in the lecture room of the Cosmos Club and the Saturday sessions at the Bureau of Standards. President Magie presided at the Friday session and Vice-President Peirce on Saturday.

A portion of the Friday session was devoted to a discussion of the relations between the Society and the Physical Review, and to the general question of the publication of the results of investigations by American physicists. In introducing the discussion the president presented to the meeting, on behalf of the council, the following communication from the editors of the Physical Review:

To the Members of Council of the American Physical Society:

The conference held in New York on March 3, for the purpose of discussing the relations between the Physical Society and the Physical Review and the question of the establishment by the society of one or more new journals, was unfortunately attended by only four members, and there was thus no opportunity for a really adequate discussion of the matter. Since the members of the council are so widely separated, it seems unlikely that a well attended conference can be had in the immediate future. It has seemed to us, therefore, that the discussion of the whole question would be facilitated by a statement to the council of the attitude of the editors of the Rexiew toward the different propositions that have been advanced.

In connection with the recent mail ballot on the question of the relations between the society and *Science Abstracts* and the *Physical Review*, the secretary has received numerous comments and suggestions and a number of long letters dealing with the matter. Aside from the discussion of methods of economy in conducting the Review, the sug-

gestions fall under two heads: (*a*) that the board of editors be enlarged so as to be more representative of the whole country, thus removing the objection that exists in the minds of many to the present local character of the board; (*b*) that the Physical Society assume full responsibility for the Review. A recent proposition of Professor Ames for the society to establish two new journals adds another to the plans suggested. In what follows, we take up each plan in turn and give a brief statement of the position of the editors of the Review in reference to it.

1. The proposition of Professor Ames that the society establish one or more new journals.

The establishment of another new journal of the same scope as the Physical Review seems inadvisable. If, however, there is a desire to establish a journal radically different from the Review in character or policy, the editors of the Review can see no objection. Whether or not members of the society would continue to receive the Review at a reduced price could be decided later.

2. The proposition that the Physical Society assume control of the Physical Review.

The chief factors in determining the attitude of the editors of the Review toward such a proposition are first the sentiment of the members of the Physical Society, and second the question of the practicability of the society's carrying out the plan at the present time. Whatever degree of success has been attained by the Review represents a considerable investment both of money and effort, and this fact places upon the editors a certain responsibility for the future which they cannot lightly avoid.

It has been the policy of the Review to offer a channel for the publication of all reputable research work in physics carried on in the United States and Canada. Articles are accepted from members of the Physical Society, and from physicists not belonging to the society, on the same terms. The publication of an article in the Review is not an indication that the editors regard the article as of preëminent excellence or that they agree with its conclusions; it simply means that, in the opinion of the editors, the article contains enough of value to make it deserve a hearing. Briefly put, the policy of the Review has been democracy in science as opposed to aristocracy.

It would be an essential condition for the transfer of the Review to the society that in regard to these points, and in all other important respects, the policy of the Review should remain the same as in the past, until it is shown by a vote of the society that a change is desired by at least two-thirds of the regular members. We should further stipulate that the name Physical Review be retained.

In the event of the society's assuming control of the Review we

should expect it also to assume full financial responsibility. This would of course mean that the cost of the Review to the society would be considerably greater than under either of the plans proposed in the recent mail ballot. We should not feel justified in turning over the control and responsibility until it has been demonstrated that a majority of the members are willing to assume this added burden.

We should regard it as essential that the first board of editors be so constituted as to make reasonably certain that its members are in sympathy with the policy of the Review and will work in harmony with the present editors during the period of transition. We should suggest therefore that the selection of the editorial board and of the active editors be made in such manner as will insure the approval both of the society and of the present editors.

If the transfer should be made there will doubtless be other matters of minor importance where it will be necessary for the society and the present editors of the Review to reach an agreement. We should not anticipate any difficulty in the arrangement of such details.

If a change should be made with the cordial support of the members of the society we feel that the success of the undertaking is reasonably certain. But to provide for the possibility of a different result, the editors make the final stipulation that if difficulty should be experienced in properly maintaining the Review, the society shall not accept financial aid, under circumstances which place the society under obligations to any individual or institution, until the present editors have been offered the option of resuming the control and financial responsibility which they now have.

3. The proposition, which has come in a variety of forms, that a representative board of editors be selected to coöperate with the present editors by giving their assistance and advice.

If this plan should meet with approval it is our intention to suggest as the members of the first board the eight elected members of the council, namely, Messrs. Guthe, McLennan, Millikan, E. F. Nichols, Sabine, Skinner, Stratton, and Zeleny. The exact functions of this board, and any changes which might be made in the method of conducting the Review, could be decided upon after consultation between the board and the editors of the Review. Our desire would be to have the connection between this board and the editorial control of the Review an active one and not merely nominal. We should suggest that the members of this board retire two each year and that their successors be elected by the society. We think, however, that the council should not constitute the board *ex-officio*. (This is the plan which the editors had decided upon several months ago, and which they had intended to suggest as soon as the question of the financial relations between the society and the

Review had been settled. If the council should feel any hesitation in regard to the proposed selection of the eight elected members of that body as members of the first board, it would doubtless be possible to find a different board which would be mutually satisfactory. The editors feel, however, that the proposed board,—owing to its representative character, geographically and otherwise,—is likely to be more satisfactory to the society, as well as to the editors of the Review, than any other that we have considered.)

The difference between plans 2 and 3 is largely, although of course not wholly, a difference in the location of financial responsibility. Under plan 2 full financial responsibility would be assumed by the society, in which case the discontinuance of *Science Abstracts*, or an increase of the dues to $8.50, seems to us unavoidable. Under plan 3 the Review would be compelled for a time to receive partial financial support from some other source than the society. This plan, in our opinion, is to be regarded as one step toward the ultimate assumption by the society of the complete responsibility called for in plan 2. While we should welcome the immediate relief from financial responsibility and editorial routine which would result from the adoption of plan 2, we feel that in some ways it would be safer for the society to proceed more slowly.

Of the three plans proposed, we feel that the first and second are mutually exclusive. If the society assumes responsibility for the Review, it would be inadvisable, from the financial point of view, for it to attempt at present to support another journal.

Plans 1 and 3, however, we regard as entirely independent, so that both may be adopted if the society so desires. The establishment of an enlarged editorial board for the Review would be fully as desirable if the Review is conducted without any financial connection with the society as under any other conditions.

When the council has reached a conclusion the next step, in our opinion, is to submit the whole matter to a vote of the society. In connection with this vote we should consider it essential that a statement of the whole situation be sent to each member, so that he may vote with a clear understanding of the responsibilities, financial and otherwise, which the society proposes to assume.

We shall be glad to place at the disposal of the council any information in regard to the Physical Review which they may need in discussing the situation. And we desire in conclusion to express our willingness to coöperate in whatever plan seems to the society to be best calculated to advance the interests of physics.

<div align="right">

(Signed)   Edw. L. Nichols,
Ernest Merritt,
Frederick Bedell.

</div>

March, 1911.

The discussion of the general question and the consideration of the above communication were carried on in committee of the whole, Mr. E. B. Rosa being elected chairman. Messrs. Webster, Magie, Arthur W. Smith, Humphreys, Merritt, Northrup, Hyde, Peirce, Carhart, W. P. White and D. C. Miller took part in the discussion. When the committee rose the chairman announced the adoption of the following two resolutions:

1. *Resolved*, that this meeting requests the council to present to the society a plan by which the society shall publish a journal.

2. *Resolved*, further, that the council be requested to enter into negotiations with the editors of the Physical Review to determine under what conditions the control of the Review may be transferred to the society.

The following papers were presented:

The Capacity of Phase Difference of Paraffin Paper Condensers as Functions of Temperature and Frequency. F. W. GROVER.

Resistance Coils for Alternating Current Work. H. L. CURTIS and F. W. GROVER.

A New Determination of the Selective Radiation from Tantalum. EDWARD P. HYDE.

Spectroscopic Evidence of Molecular Clustering. W. W. STRONG.

The Use of the Silicon Detector with Short Electric Waves and the Theory of Contact Detectors. ERNEST MERRITT.

The Theory of Dispersion and the Residual Rays. (By title.) O. W. RICHARDSON.

Note on Paschen's Method of Equal Ordinates for finding λ_m and C_2. (By title.) EDGAR BUCKINGHAM.

Recent Determinations of the Elementary Electrical Charge. WM. W. COBLENTZ.

The Detection of Small Heat Effects at High Temperatures. W. P. WHITE.

The Radiant Efficiency of Arc Lamps. H. P. GAGE.

The Physical Implication of the Principle of Relativity and the Definition of Mass. D. F. COMSTOCK.

Positive Thermions from Salts of Alkali Earths. C. J. DAVISON.

Photographs of Sound Waves from Various Sources. D. C. MILLER.

The Determination of the Form of Sound Waves by Means of a New Manometric Flame. J. G. BROWN.

The Elastic Properties of a Series of Iron-Carbon Alloys. (By title.) C. W. WAGGONER.

The Saturation Value of the Specific Intensity of Magnetization of Cobalt at Various Temperatures. (By title.) W. W. STIFLER.

Heat of Evaporation of Water at 100° C. ARTHUR W. SMITH.

Photoelectric Properties of Thin Cathode Films. P. H. DIKE.

The Difference in the Photoelectric Effect caused by Incident and Emergent Light. OTTO STUHLMANN, JR.

The Thermodynamics of Concentration Cells. H. S. CARHART.

On the Positive Potential of Alkali Metals as a Function of the Frequency of Ultra-violet Light, and the Determination of the Wave-length Equivalent of Roentgen Rays. (By title.) J. KUNZ.

The Enclosed Arc for Spectroscopic Work. JAMES BARNES.

On the Fusion of Carbon. O. P. WATTS and C. E. MENDENHALL.

Photographs of Vortex Rings in Liquids. E. F. NORTHRUP.

A Method for Eliminating the Effect of all Connecting Resistances in the Thomson Bridge. F. WENNER.

The Construction of Primary Mercurial Resistance Standards. F. A. WOLFF, M. P. SHOEMAKER, and C. A. BRIGGS.

A Tubular Electrodynamometer for Very Heavy Currents. P. G. AGNEW.

The Deflection Potentiometer Considered as a Generalized Null Instrument. H. B. BROOKS.

On Different Types of Potentiometer suitable for Thermo-electric and other Exacting Work. W. P. WHITE.

The Bureau of Standards' Current Balances. E. B. ROSA, N. E. DORSEY, and J. M. MILLER.

The Spectra of Aluminum, Copper, and Magnesium in the Arc under Reduced Pressure. JAMES BARNES.

A Heterochromatic Photometer. H. P. GAGE.

Ultra-violet Light from the Arc. H. P. GAGE.

The Measurement of Inductances with very small Time Constant. F. W. GROVER and H. L. CURTIS.

The Rôle Played by Gases in the Emission of Positive Thermions from Salts. C. J. DAVISON.

Methods of Increasing Calorimetric Accuracy. (By title.) W. P. WHITE.

At the close of the meeting it was voted to express the thanks of the Society to the Cosmos Club and to the scientific staff of the Bureau of Standards for the hospitality extended to the members of the society.

ERNEST MERRITT,
Secretary.

The Bureau of Standards' Current Balances.[1]

By E. B. Rosa, N. E. Dorsey and J. M. Miller.

THE current balances of the Bureau of Standards are of the type used by Lord Rayleigh, each consisting of a pair of fixed coils of equal radii and of square cross-section, and fixed at such a distance apart that the force upon the moving coil, also of square cross-section but of a smaller radius, suspended midway between the fixed coils and coaxial with them, is a maximum.

The balances were constructed in the shops of the bureau. The most novel features in their construction are the provision for water-cooling the fixed coils, the surrounding of the moving coil by a double-walled jacket maintained at a constant temperature by water circulation, and the making of the coils interchangeable so that many combinations of coils may be obtained.

All coils are wound bifilar of enamel insulated wire upon brass forms. To improve the winding, a layer of thin paper is placed over each layer before the next is wound. Over the outer layer of wire is placed a layer of paraffined paper, and two layers of cloth soaked in hot paraffin and well ironed on, with a hot block of brass, so as to effectually seal the coil from changes in atmospheric humidity. For further precaution, a layer of soft wax is melted on over the paraffin seal.

One moving coil (no. 4) had paraffin painted between the wires, and a strip of paraffined paper ironed down upon each layer as wound; it thus has its windings embedded in a practically solid block of paraffin.

Three pairs of fixed coils have been constructed, viz., one 40 cm. in diameter, each winding of 392 turns, forms of cast brass; one 50 cm. in diameter, each winding of 648 turns, forms of cast brass; one 50 cm. in diameter, each winding of 647 turns, forms built up of half-hard rolled brass.

Four moving coils have been constructed, viz., one 20 cm. in diameter, each winding of 72 turns, form of cast brass not sufficiently non-magnetic and consequently discarded; one 25 cm. in diameter, each winding of 72 turns, form of cast brass; one 20 cm. in diameter, each winding of 98 turns, form of cast brass; one 25 cm. in diameter, each winding of 71 turns, form cast from best rolled brass.

All portions of the balance have been tested by a very sensitive astatic magnetometer, and, excepting the discarded moving coil, found to be sufficiently non-magnetic.

All weighings having been made with the coils in the positions for maximum force, the only direct measurements of length needed are those

of the sectional dimensions of the coils, the main term in the expression for the force being completely determined by the *ratio* of the radii of the coils. This ratio was determined by a method identical in principle with that given by Bosscha, and used by Lord Rayleigh. The coils to be compared were placed concentric, coplanar, with their planes vertical and in the magnetic meridian. Currents were then passed in opposite directions through the two coils and adjusted until the resultant field, as indicated by a small magnetometer needle in the center of the coils, was zero; the two currents were at this instant simultaneously measured by two potentiometers. The ratio of these currents, corrected for the length of the magnetometer needle, is equal to the inverse ratio of the "galvanometer constants" of the two coils, and it gives at once the ratio of the two mean radii when the numbers of turns and the sectional dimensions of the coils are known.

With care it is not difficult to obtain the ratio of the galvanometer constants of two such coils with an accuracy of about 2 in a million. A careful study was made of the temperature coefficients of the coils and of the variations in the galvanometer constants with the load.

In assembling the balance proper, the correct position of the moving coil was determined electrically, and experimental methods were devised for determining the lack of coaxiality and the error in the spacing of the fixed coils, and for determining the effect, if any, of any fixed magnetic bodies that might be near the balance.

About two years and a half have been spent in strictly preliminary work, in the development of details of construction and of manipulation, and in the study of the various sources of error. The final result is based upon the observations of the next succeeding six months, and does not differ from the earlier observations by more than a few parts in 100,000.

The final weighings show that the electromotive force of the Weston normal reference cells of the Bureau of Standards is 1.01823 volts, in terms of the international ohm.

That is, the Weston cells have an electromotive force of 1.01823 absolute volts if the international ohm is equal to the absolute ohm. The value of the E.M.F. of these coils is 1.01830 international volts (in terms of the international ohm and the silver voltameter), showing an apparent difference between the absolute volt and the international volt of 0.00007 volt. However, there is perhaps a greater difference than this between the absolute ohm and the international ohm; hence we cannot say definitely until the absolute ohm is better determined whether the international volt or the absolute volt is the larger.

The following values have been found by other observers during the last four years:

These results seem to indicate that the electromotive force of the Weston normal cell at 20°.00 C. in terms of the international ohm and the absolute ampere is 1.01822 volts to within a few parts in 100,000.

A Method for Increasing Calorimetric Accuracy.[1]

By Walter P. White.

IN a series of articles on calorimetry, published in the previous volume of this Review, I tried to show that the cooling of calorimeters can easily be made to occur under conditions so definite and measurable that all errors due to irregularity or uncertainty of the cooling are quite imperceptible. There is, however, a source of error due indirectly to the cooling, namely, an increase in the number of accurate thermometric measurements, and therefore in the accidental thermometric error, due to the necessity of determining the cooling rate from which to calculate the correction. It was pointed out that this increase of accidental thermometric error could be avoided by keeping the cooling factor constant—repeated measurements of the cooling rate would then be unnecessary—and further that with such a procedure the error could again be decreased by choosing the initial temperatures so that the total correction was small; the absolute effect of a given proportional error in the value of the rate would thereby be diminished.

To decrease the total correction by suitable choice of initial temperature of course diminishes the accidental error in those cases, also, where the cooling rate is observed. This rather evident conclusion seems to deserve mention here for two reasons:

(1) It was, unfortunately, not clearly stated in the articles referred to; and (2) I have recently found, contrary to expectation, that it is not true to the same extent where the Pfaundler formula is used. More explicitly, the facts are these: If we use as a standard of comparison the accidental thermometric error obtained when the calorimeter is initially at the jacket temperature, then, if a single cooling period only is used, the reduction of the correction by choice of initial temperature reduces the maximum possible accidental thermometric error of a single determination by one

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21 and 22, 1911.

half, and the mean probable error of a series by about one third; with the Pfaundler method, and two cooling periods, the maximum error is not reduced at all, and the mean error only by one sixth, and this value occurs not when the correction is a minimum, but when the jacket temperature is half way between the initial and final temperatures of the calorimeter.

Since the accidental thermometric error is the principal error in ordinary calorimetry, the maximum accuracy will generally be found in those methods which most diminish it. From this fact and those stated above it follows that: (1) Maximum accuracy will be obtained only through a reduced cooling correction along with a cooling factor either constant or determined from a *single* cooling period; along with a reduced correction, or else through a constant factor, either with or without a reduced correction; in either case with some method calling for a measured jacket temperature and a knowledge of the heat of stirring and evaporation. (2) The Pfaundler formula is incompatible with maximum accuracy, though it may give something more than half the maximum. (3) The procedure often recommended, of reducing the total cooling correction by choice of initial temperature, is practically useless with the Pfaundler formula. (4) Study of cooling correction errors and formulæ has thus far failed to explain the experimental result reported by some observers, that such reduction did diminish the accidental error.

Of course an increase in the length of the cooling periods diminishes the proportional effect of accidental temperature errors, and thus resembles a reduction in the correction. It is, however, time wasting and likely to introduce other errors. It is, therefore, of little practical importance.

Of course, the greater error with the Pfaundler formula method is not serious, and will often be outweighed by other advantages to be derived from employing that method.

The Construction of Primary Mercurial Resistance Standards.[1]

By F. A. Wolff, M. P. Shoemaker and C. A. Briggs.

THE paper deals with the construction of four one-ohm mercury standards in accordance with the specifications adopted by the London International Electrical Conference.

The tubes are of Jena 59[III] glass and were calibrated over a length of 110 centimeters, the caliber factors of none of the tubes exceeding 1.0001. The tubes were cut at points giving a resistance for the finished tubes, including the end correction, of approximately one ohm. The ends were ground and optically polished to a plane (within one half

wave-length) very approximately at right angles to the axis. In addition, the tubes were tapered at the ends and ground to fit the end bulbs for electrical measurements and the glass cleaning and drying fittings.

The tubes were cleaned by dilute nitric acid, water, fuming sulphuric acid and water after which they were dried by drawing through them air which had passed through calcium chloride bulbs and by evacuation preliminary to filling.

The Reichsanstalt method was employed for determining the mass of mercury contained in the tube between the terminal planes. The lower end of the tube was closed by a plane glass plate clamped to the tube by means of a steel screw with an intermediate ball and socket joint, the screw being threaded through a yoke screwed to a steel collar cemented to the tube. After exhausting by means of a Geryk pump and with the end of the tube under mercury, mercury was admitted by rotating the receptacle containing it which was provided with a steel plate with a hexagonal hole to engage the hexagonal head of the screw. After admitting sufficient mercury to fill the tube, which was mounted on an inclined rack, its lower end was closed and the tube removed to a double wall ice bath. Suitable precautions were taken to prevent the water from coming into contact with the ends of the tube.

The excess of mercury was removed by "stroking off" with a plane glass plate in a gimbal mounting. Condensation of moisture was eliminated by a current of dried and cooled air directed at the end of the tube. Six fillings of each tube were made. The average deviation from the mean of the six fillings was approximately four parts per million with a probable error of approximately 1.5 parts per million.

The problem of comparing end standards was reduced to that of comparing line standards by means of platinum contact pieces carrying a line parallel to the plane of contact. These were held against the tube by means of a spring. The line standard thus produced was compared in an ice bath with corresponding known intervals on a nickel steel meter, observations being made for each of four positions of the tube 90° apart. The lengths found were corrected for the added length due to the end pieces which was determined by measuring the distance between the lines on them when the pieces were put in contact.

The values obtained from the tubes in the different positions agreed to within 1μ, the probable error of the set means not exceeding $.35\mu$.

Electrical Measurements.—Measurements were made by the Kelvin double bridge method, the primary standards and five sealed manganin standards being substituted in turn in the same bridge arm. By means of adjustable mercury resistances in the potential leads of the auxiliary one ohm comparison standard to compensate for the corresponding resistances in the leads to the mercury standard and an adjustable shunt

on the auxiliary standard, an exact balance of the bridge could readily be obtained.   The auxiliary standard, the ratio coils and their supports were contained in a thermostatically controlled oil bath.

The end bulbs were 4 cm. in diameter, all but one ground joint being eliminated and this was protected by a mercury seal, externally coated with celloidin.

The potential and current connections to the standard were made by sealed-in platinum wires protected from the ice bath by glass tubes, connections with the bridge being made by means of copper conductors and mercury which stood in the terminal tubes at a slightly lower level than the water in the ice bath.   Heat condition through the terminals was thereby practically eliminated.   The heating due to the test current employed was less than one part per million.   In filling the tubes for electrical measurements, an extension to one of the end bulbs was drawn out to a fine point which was broken under mercury after evacuation by means of a ground connection to the Geryk pump at the other end of the tube.

Preliminary Results.—Six fillings of each tube were made.   The calculated value of the tube minus its value in terms of the International ohm as agreed to by the International Scientific Committee in 1910 is given in the following table, in which the differnces are expressed in parts per million.

Tube.	Theoretical Minus Observed.	Average Deviation from Mean of Six Fillings.	Probable Error of Mean of Six Fillings.
1	+29	±1.9	±0.65
2	+33	±3.6	±1.22
3	+19	±1.4	±0.48
4	+21	±4.8	±1.50
Mean	+25	±2.9	±0.96

The above differences, expressed in terms of the ohm previously in use at the Bureau of Standards, would be 7 parts per million greater.

The final result is subject to various small corrections still to be determined.

The Detection of Small Heat Effects at High Temperatures.[1]

By Walter P. White.

IN calorimetric work at ordinary temperatures, stirring is generally necessary to equalize temperatures.   Although the need for it is enormously greater in the hot electric furnace, where the surrounding

temperature differences are much greater, and convection and radiation are far more active, yet the difficulty of providing for stirring by means of the few materials available at high temperatures is so great that stirring has almost never been tried, and observers have been satisfied to do as well as they could with the uneven temperatures which prevailed in its absence. This has been made easier because the work done has dealt largely with melting points, where quantitative measurement of heat is not required. Although the results are far less perfect that can be obtained at low temperatures, yet they have generally proved sufficient for the purpose in view. Some experimenters, however, have applied the melting point arrangements to the quantitative determination of heat effects, even claiming more than 1 per cent. accuracy. Others, on the contrary, have shown a disposition to reject almost all evidence as to heat quantity based on observations made with the electric furnace. A fair opinion seems to lie between these two views. Determinations in the electric furnace usually show an almost surprising reproducibility, which tends strongly to convey a false impression of accuracy. Large systematic errors, however, are usually present, especially where only one or two thermoelements are used, since the temperature at any given point affords a very imperfect indication of the average value of the temperature in either furnace or charge, while the distribution in both is liable to change both with time and with temperature. It follows, then, that where reproducibility is the main essential results of high accuracy can often be obtained, but for absolute results the very greatest patience and thoroughness are needed in varying the conditions and otherwise eliminating possible systematic errors.

In the detection of slight heat effects, high absolute accuracy is not expected. Hence, in this direction relatively great success can be obtained.

The general principle of the method is simple and fairly familiar. No moving bodies are present, and changes in the temperature of the charge are brought about solely by changing the furnace temperature. The heat effects in the charge are inferred from the changes in its temperature. The temperature of the charge thus depends upon two things, first, the furnace temperature, and second, the heat effects in the charge. The manipulation consists (after the furnace is once loaded) in observing (usually once a minute) the temperature rise, regulating the furnace current accordingly, and observing, either directly or indirectly, the temperature difference of furnace and charge.

The interpretation of the results may, in theory, be made very simple.

The temperature gradient, G, between charge and furnace, serves as the measure of heat flow to the charge. The flow actually is proportional to G, and to the heat-transmittance, F, of the space between furnace

and charge.   The exact values of both G and F are usually unknown, and, as already stated, may vary with time, temperature, rate of heating, etc., so that accurate heat determinations by means of them, though possible, are more difficult than has sometimes been realized.   But in the *detection* of a small heat effect we have merely to determine the change in G caused by the addition of the effect to the heat which is required to change the temperature of the charge.   For instance, a silicate charge (2 grams) of specific heat 0.3, heated 10° per minute, lagged 3° behind the furnace ($G=3°$).   Hence 3° in G corresponds to 3 calories per minute.   If an inversion absorbing 3 calories and extending over 100° should occur, G would be increased 0.1, or 0.3°, for 10 minutes; if the inversion should take place in one minute G would be doubled, or increased 3°, etc.   The detection of small heat effects is easier: (1) The larger G is per calorie per minute, (2) the freer G is from other variations, (3) the quicker the inversion occurs; it is only sluggish inversions whose detection gives any trouble.

G increases with the furnace rate, hence a rapid rate is of the first importance. G also increases with the diameter of the charge, but can be made steadier in the case of a small charge, and the advantage of the small charge appears to be greater on the whole.   Fluctuations in the furnace rate cause variations in G; these are partly eliminated by measuring G, not between charge and furnace, but between the charge and another body ("neutral body," or "dead body") closely resembling it.   For the best results the other body is used, and the furnace is regulated as carefully as possible, besides.   Runs are also made with both bodies alike, to eliminate effects due to the thermoelements or to lack of symmetry between the two bodies, and, of course, repetitions eliminate accidental irregularities.   The apparatus now used is small, two platinum crucibles holding 1 c.c. each, 3 mm. apart, and surrounded by a wider porcelain tube to increase uniformity of temperature.   A differential thermoelement is also used, which gives directly the temperature difference of the two bodies at any instant.   This not only increases accuracy, but saves much tedious computation, necessary when the temperatures are read separately.   A complete platinum inclosure shields the whole system from leakage currents out of the furnace coil.   In one set of determinations made on different days with the same set-up, conditions were reproduced over a 300° interval with a maximum variation of from 0.03° to 0.06° in the different determinations.   This was with a silicate charge and indicates that 1 calorie distributed over 100° could often be detected.   But this would be a more difficult case than has yet been found in practice.   In one case it was observed that the first heating in each day which, of course, occurred immediately after the furnace had been cold, gave results differing by 0.3° from later heatings, although the furnace was

cooled 300° between all the heatings. This condition reproduced itself at
0.1° on successive days and could, therefore, be largely eliminated, but
might prove decidedly deceptive if overlooked, and if the heat effect was,
as often happens, one occurring only in the first heating. This circum-
stance well illustrates the care which must be taken to avoid systematic
errors in this kind of work, and the danger of assuming too hastily that
the precautions in this direction have been sufficient.

The Capacity and Phase Difference of Paraffined Paper Condensers as Functions of Temperature and Frequency.[1]

By F. W. Grover.

IN an ideal condenser the charge is at every instant proportional to
the applied difference of potential between its plates, and the capac-
ity is consequently independent of the length of charge or discharge.
With alternating current, the current through the condenser is 90°
in phase ahead of the impressed electromotive force, and the capacity
is independent of the frequency. These conditions presuppose, not only
a perfect dielectric, but also a negligible resistance in the leads and plates
of the condenser.

Actually, condensers show anomalies which are usually included under
the name of absorption effects, and may be summarized as follows: (1)
with a steady applied voltage the capacity depends on the length of charge,
length of discharge, the interval between charge and discharge, or be-
tween discharge and recharge, that is, it depends on the previous history
of the condenser. After being discharged, the condenser, if allowed
to stand insulated, will yield residual changes. (2) With alternating
current the phase difference between the current and the impressed elec-
tromotive force falls short of 90° by an angle, usually small, which may
be designated briefly as the *phase difference*. Both capacity and phase
difference are functions of the frequency.

All these effects vary in magnitude with the temperature, so that the
capacity changes to a greater extent with the temperature than can be
explained by changes in its dimensions.

A number of theories to account for the phenomena of absorption
have been proposed, but most of the existing experimental work has been
done with direct current, under conditions which make it difficult to
compare the results with theory. The present work was done on con-
densers having a dielectric of paraffined paper, in which the various
effects of absorption are larger and more easily measured than in mica
condensers, and to obtain definiteness in the conditions of experiment,

alternating current was used.   The capacity and phase difference were measured simultaneously by a method described in detail in a former paper.[1]

In this way curves of the capacity and phase difference of thirteen commercial paraffined paper condensers were obtained for frequencies ranging from 33 to 1,000 cycles per second, and for temperatures ranging between 10° and 35°.   From these curves may be drawn the following conclusions for the condensers studied.

1. The changes of capacity with the frequency and temperature are in every case considerably larger than those observed with mica condensers, and are in some cases from ten to fifty per cent. of the value of the observed capacity at the lowest temperatures and highest frequencies. In a general way, the larger the phase difference the greater these changes.

2. The phase differences observed range from about 6′ to the enormous value of 22°.   The phase difference of a good mica condenser may be as low as 15 seconds of arc and usually lies under 3′, although mica condensers are often met with which are considerably poorer.

3. The temperature coefficient of the capacity is usually negative with condensers having a small phase difference, and tends to become more and more positive as the phase difference is greater.   In the best of the condensers the coefficient is negative and nearly linear, with a tendency to become rapidly numerically larger as the temperature is raised above 30°.   In the case of the condensers having large phase differences (1° or more) the temperature coefficient is positive and becomes very rapidly larger as the temperature is increased and the frequency is decreased.   With some condensers having a moderate phase difference, the temperature coefficient is negative at the lower temperatures and higher frequencies, and positive at the higher temperatures and lower frequencies.

4. With increasing frequency, the capacity falls, at first rapidly and then more slowly, and finally approaches asymptotically the geometrical capacity (the capacity as unaffected by absorption).   In general, these changes become greater as the frequency is increased.

5. In the majority of cases the phase difference increases with rising temperature, the changes being particularly great with condensers having large phase differences.

6. The phase difference decreases with increasing frequency except in those cases where the I^2r loss in the connections and plates is considerable, when the phase difference may actually increase with the frequency. This effect is particularly noticeable in telephone condensers made by rolling up together sheets of tin foil and paper.   In one of the condensers studied, the I^2r loss in the plates and connections was about three times as large at 1,000 cycles as the loss in the dielectric.

[1] Bull. Bureau of Standards 3, p. 371, 1907.

7. The use of paper condensers as reference standards in work with *direct current* is not allowable, and should be permitted in *alternating current* work, only after a study of the behavior of the condenser under different conditions of temperature and frequency has shown that the performance of the condenser meets the requirements laid down by the work in question.

In the theoretical part of the paper the fundamental equations of the theories of Maxwell,[1] Houllevigue,[2] Pellat[3] and Hopkinson[4] are developed, and the observed curves are discussed in the light of these theories. It is shown that of these theories none are in accordance with the facts, but that if, as suggested by von Schweidler,[5] the Pellat equations be extended so as to read as a summation of terms, only a limited number of constants are necessary to represent the observations. Constants have been calculated for a number of the condensers, from which the geometrical capacity may be calculated, and it is shown that the physical interpretation of these equations, as given by von Schweidler, can be made to explain some apparent anomalies of the observed curves.

The Measurement of Inductances with very small Time Constant.[6]

By F. W. Grover and H. L. Curtis.

MEASUREMENTS of inductances with small time constants are becoming increasingly important. For example, such diverse cases may be cited as the measurement of the small residual inductances of so-called "non-inductive" resistance coils, the inductances of multipliers for voltmeters or other measuring instruments, and the determination of the small inductances used in wireless telegraph circuits.

The method of measurement in any given case depends on the value of the resistance which is associated with the inductance. Thus a method adopted to the measurement of a coil of a few microhenrys, or smaller, and having a resistance of the order of one hundredth of an ohm is not suitable for the determination of the inductance of a coil of the order of 10,000 ohms.

Measurements of coils of as low inductance as one microhenry have

[1] Elect. and Mag., I., §§ 328–330.

[2] Jour. de Phys., 6, pp. 113, 120, 1897.

[3] Ann. Chem. et Phys., 18, p. 150, 1899.

[4] Phil. Trans., 166, p. 490, 1876.

[5] Ann. der Phys., 24, p. 711, 1907.

[6] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21 and 22, 1911.

been made, among others, by Wien and Prerauer,[1] Giebe[2] and Orlich, [3] all of whom were able to detect differences of inductance as small as a few centimeters.

In the method here employed, the inductance to be determined is compared by direct substitution with a standard, whose inductance can be calculated from its dimensions, in an alternating current bridge, using a frequency of 1,500 cycles. The difference in inductance between the coil to be measured and the standard was measured by the change of capacity necessary to reëstablish balance in the Anderson's bridge, when the unknown was inserted in place of the standard.

The standards used were resistances consisting of two manganin wires stretched parallel to one another at a measured distance apart. In order that no error should be introduced by the small difference in resistance between the standard and the test coil, changes of resistance in steps of one tenth ohm were obtained by substituting for manganin links, resting in mercury cups, copper links of the same diameter, form and size. For continuous resistance changes of less than 0.1 ohm, a variable resistance of a form suggested by Dr. F. Wenner and Mr. J. H. Dellinger was employed. Two straight glass tubes of different diameters, mounted vertically and united at the bottom by a cross tube, are filled with mercury. Two copper wires of diameters closely approximating those of the cross sections of the tubes may be raised or lowered in the mercury, giving a continuous variation of resistance, free from any unsteadiness, as great as 0.00001 ohm. With this form of instrument the change of resistance is accompanied by a change of inductance almost negligibly small. In the present instance, the maximum change of inductance for a change of resistance of 0.1 ohm, as determined by the use of the manganin and copper links, was only about 10 cm.

The above described method of measurement was applied with success to the determination of the inductances of coils of resistances as high as 1,000 ohms. Parallel wire standards were in every case prepared of the same nominal resistance as the coil to be measured. For the larger values the difference in inductance between the coil and the standard was measured by the change in the setting of a calibrated variable inductance on substituting one for the other. For coils of 1,000 ohms, where a small capacity in parallel with the coil has an appreciable effect on the phase of the current, the balance may be obtained by varying a capacity shunted around one of the coils, as was done by Brown[4] and Giebe.[5]

[1] Wied. Ann., 53, pp. 772, 928, 1894.

[2] Ann. der Phys., 24, p. 941, 1907.

[3] Zs. f. Instrk., 29, p. 241, 1909.

[4] PHYS. REV., 29, p. 369, 1909.

[5] Verh. Deutsch. Phys. Gesell., 12, p. 949, 1910.

In coils of 100 ohms or more, the effect of the capacity between the different turns of the winding is usually greater than the inductance, so that the current through the coil leads the impressed electromotive force. We may then regard the effective inductance of the coil as negative and equivalent to a residual capacity c assumed to be joined between the terminals of the coil, and of such a value that $-l = cR^2$, where l is the effective inductance, and R is the resistance of the coil.

For resistances larger than 1,000 ohms, where it becomes difficult or impracticable to prepare standards, whose constants may be calculated, the inductance may be determined by placing the coil as a shunt on a standard of known effective inductance and of about 1,000 ohms resistance. The change of balance of the bridge on joining the coil to be tested may be measured either by means of a variable inductance in the main circuit, or by the change in a variable capacity, also shunted around the standard. In still a third method the coil to be measured is placed in one of the arms of a bridge of which the ratio arms are of very unequal resistance, the effective inductance of the remaining three arms being known.

In the measurements of one-ohm coils a change of inductance of one centimeter was appreciable, and the measurements made with parallel wire standards were consistent with those in which the circular form of reference standard was used to about this order of accuracy. Equally small time-constants of ten-ohm and one hundred-ohm coils could be measured with about the same precision as in the case of the one-ohm coils. With 1,000 and 10,000 coils the difficulties increase. Capacity effects must be very carefully guarded against. Metal surfaces associated with the coils as in the ordinary forms of resistance box are a source of indefiniteness in the constants of the coils. It is also noticeable that the capacity between the windings of coils of such high resistance is not constant, but changes with the time, when the measuring current is flowing, the capacity becoming greater. This suggests that the shellac acts as a dielectric with appreciable absorption, the temperature coefficient of the capacity being large and positive.

It is to be noticed that the apparent capacity effect of a coil of high resistance, when used in an alternating current bridge, is largely dependent on the absolute potential of the coil. To measure the true capacity of the coil, the potentials should be so adjusted that the middle point of the resistance of the coil is at zero potential. In the case of several high resistance coils in series, in a bridge, this condition cannot be realized for all the coils, and in general the measured capacity of the sum will not be the same as that calculated from the measured values for the separate coils.

RESISTANCE COILS FOR ALTERNATING CURRENT WORK.[1]

BY H. L. CURTIS AND F. W. GROVER.

THE measurement of alternating currents frequently requires the use of resistance coils having small reactances. A plain bifilar winding has usually been relied upon in the case of coils whose resistance lay between 1 and 1,000 ohms. Of such coils, those below 50 ohms have a positive reactance, while those above 100 ohms have a negative reactance. The authors have made experiments in the design of coils to make these effects as small as possible. The aim has been to reduce the time constant to not more than 3 or 4×10^{-8}, and at the same time to keep the dimensions of the coil such that when immersed in oil an expenditure of one watt will not raise its temperature more than one degree.

Coils of four different values have been experimented upon, viz., 1, 10, 100 and 1,000 ohms. A 1-ohm coil which meets the above requirements may be made from a strip of manganin having a width of 3 mm. and a thickness of 0.1 mm. This is folded upon itself, the two parts being separated by a thin sheet of mica or silk. A 10-ohm coil may be made either of strip manganin in the same manner as above, using a width of 1 mm. and a thickness of 0.1 mm. or of three 30-ohm coils in parallel, the 30-ohm coils being made of the same wire as is used in the 100-ohm coil to be described. The 100-ohm coil may be made of double silk insulated manganin wire, having a diameter of 0.24 mm. It is wound in a single layer bifilar winding, using a spool having a diameter of 2.5 cm. Using wire of this diameter, wound as indicated, the inductance and capacity effects are very nearly balanced. The 1,000-ohm coil is made of wire having a diameter of 0.1 mm. and is wound in five sections each of 200 ohms. Each of these sections is a single layer bifilar winding. It is important that a coil made of several sections in series shall be wound upon an insulating spool. For this purpose a porcelain spool is quite satisfactory. The coils are to be shellaced and baked in the usual manner.

In making coils it is not found that every coil of the same resistance, made according to the same specifications, has the same effective inductance when completed. However, the variation should not be large. The following table gives the worst values which were found when the above specifications were used.

Resistance in Ohms.	Inductance in Microhenrys.
1	+ 0.04
10	+ 0.3
100	− 1.6
1000	−16.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21 and 22, 1911.

Recent Determinations of the Elementary Electrical Charge.[1]

By Wm. W. Coblentz.

THE value of the elementary quantity of electricity, or the free charge of a single ion or electron, derived by Planck[2] from his radiation theory is $e = 4.69 \times 10^{-10}$ E.S. He calls attention to the fact that the formula is derived with absolute exactness, and that the numerical value of this constant will depend upon the accuracy of the experimental determination of the constants of radiation.

This numerical value of e is based upon Kurlbaum's value of the Stefan constant σ of total radiation, $\sigma = 7.061 \times 10^{-15}$ erg/cm² deg⁴, and upon Lummer and Pringsheim's value of the Wien constant, b, of spectral energy distribution, $b = 0.2940$ cm. deg.

Some preliminary experiments on the diffuse reflection of platinum black shows that Kurlbaum's value of σ requires a correction (increase in value) of 6 to 7 parts in 1,000. Applying this correction to Planck's value of e gives $e = 4.72 \times 10^{-10}$ E.S.

A year ago the writer presented to this society some preliminary data on the value of the constant, b, based on 14 spectral energy curves, from which the mean value was $b = .2945$ cm. deg., with the possibility of the final value being a trifle higher, say, $b = .2950$. This gives a value of $e = 4.7 \times 10^{-10}$ E.S.

The most recent and the most consistent value of e obtained by Millikan is $e = 4.89 \times 10^{-10}$ E.S., which is considerably higher than usual. To obtain such a value from Planck's radiation theory would require a value of $b = .2978$ (an increase of 1.3 per cent. over $b = .2940$) or an increase in σ of 3.6 per cent.; and $c_2 = 14,800$. These latter values of b and c_2 are all very much higher than seem possible from the present experiments on the black body. On the other hand from a recent inquiry into the instruments to be used in measuring radiation in absolute value (various observers differ by as much as 10 per cent.) preliminary to a redetermination of the value of σ it appears that the most reliable values of σ may be higher than the one now commonly accepted.

The high value of $b = .2945$ to $.2950$, and of $c_2 = 14,600$ to $14,800$ and the agreement of these values with those required by extrapolation of the optical temperature scale; also the close agreement ($\Delta e = 3$ per cent.) of the values of the elementary charge, e, obtained from electrical and radiation methods, seen to indicate the approach to a place where we can harmonize the various conflicting data and can feel that there is some basis for the underlying theories.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21 and 22, 1911.

[2] Vorlesungen ü. Wärmestrahlung, 1906.

A Method for Eliminating the Effect of all Connecting Resistances in the Thomson Bridge.[1]

By F. Wenner.

THE arrangement of conductors for a Thomson bridge is shown in Fig. 1. Here the low resistances under comparison are designated by X and N, the resistances for the main ratio by A and B, and of the auxiliary ratio by α and β. Farther, the connecting resistances from the points where the current and potential terminals divide to the terminals of the ratio sets are designated by x_1, x_2, n_1 and n_2. These letters are used both to designate the conductors and to represent the values of their resistances, C represents the value of the resistance between X and N from branch point to branch point while L designates the low resistance in parallel with the auxiliary ratio. If then the conductors are all *linear* and the bridge is balanced we have the following relation between the resistances:[2]

$$X = N\frac{A+x_1}{B+n_1} + \frac{A+x_1}{B+n_1}C\frac{\alpha+x_2}{\beta+n_2+\alpha+x_2} - C\frac{\beta+n_2}{\beta+n_2+\alpha+x_2}. \quad (1)$$

This equation may be put in a more convenient form if we substitute

$$\frac{A}{B}(1+a) \text{ for } \frac{A+x_1}{B+n_1}, \quad \frac{A}{B}(1+b) \text{ for } \frac{\alpha+x_2}{\beta+n_2},$$

and

$$D \text{ for } \frac{C}{N}\frac{\alpha+x_2}{\alpha+x_2+\beta+n_2}$$

which give

$$X = N\frac{A}{B}[1+a+D(a-b)]. \quad (2)$$

If by suitable adjustments a and b can be made so small as to have no appreciable effect this reduces to the equation for a simple bridge. In order to make these adjustments possible, low resistances, forming a part of x_1, x_2, n_1 and n_2, are connected between the potential terminals and the ends of the ratio sets. These resistances can be varied continuously, or in small steps, at least on one side of each ratio.

Starting with the ratio A/B and X/N approximately equal

1. With q and q' as branch points, the bridge is balanced by an adjustment of x_1 or n_1.

[2] Wm. Thompson, Phil. Mag., 24, p. 149, 1862.

2. With p and p' as branch points and the connection L between X and N open[1] the bridge is balanced by an adjustment of x_2 or n_2.

3. With p and p' as branch points and the connection L between X and N restored, the bridge is balanced by an adjustment of N or a proportional adjustment of the main and auxiliary ratios.

Where the connecting resistances are small in comparison with the ratio sets and starting with the three ratios X/N, A/B and α/β approxi-
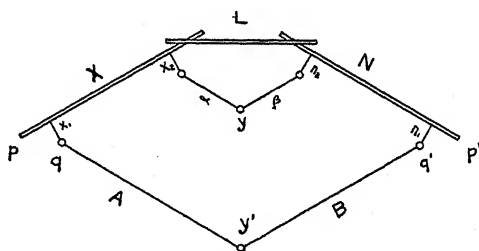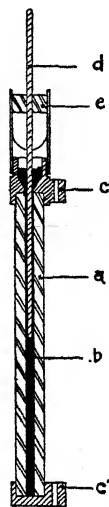
FIG. 1.   FIG. 2.

mately equal; the first operation makes a negligibly small, the second makes b equal to a, and the third fulfills the conditions of the fundamental equation. The three adjustments then give

$$X = \frac{A}{B} N,$$ (3)

which is the equation for the corresponding simple bridge.[2]

As the three adjustments are not entirely independent, any change in making the second or third disturbs the first slightly, it may be necessary in some cases to repeat them in the same order using those already made as first approximations. Where the measurements are made in this way the accuracy is as high or higher than where any of the connecting resistances are determined and the corrections applied, while the time and attention necessary for making the measurements and calculations are considerably less.

[1] J. H. Reeves, Proc. Phys. Soc. London, 14, p. 166, 1896.
[2] W. Jaeger and H. Diesselhorst, Wiss. Abh. d. P. T. R., 4, p. 120, 1906.

The conditions that must be fulfilled on order that a four terminal resistance may have a definite value are pointed out, Farther it is shown that there can be but three definite values and one of these is the sum of the other two.   If one of the three values is zero the effect is the same as if the conductors were in the form of a wire and we shall call it *linear* to distinguish it from the other case which we shall call *non-linear*.

In using the Thomson bridge with non-linear conductors the current through the potential connections to or from the ratio sets causes a change from the normal current distribution in such a way as to destroy the relations given by the fundamental equation.   A more extended theory is given showing that, under certain conditions easily realized, correct values are obtained whether the resistances are linear or non-linear and that, if the adjustments here outlined are carried out, equation (3) gives the relation between the resistances with which we are concerned even in those cases in which equations (1) and (2) are not valid.

As the success of the method depends upon having suitable variable low resistances, the form used which was developed jointly by J. H. Dellinger and the author, is shown in section in Fig. 2.   Here the letters have the following significance; *a* a hard rubber tube, *b* mercury, *c* and *c'* copper terminals amalgamated where they come in contact with the mercury, *d* an amalgamated copper rod, *e* a spring clamp for holding the copper rod in position.   Electrical connection is made through the terminal blocks *c* and *c'* and the resistance is varied by changing the position of the copper rod.

Where alternating current is used the conditions which a four terminal conductor must fulfill in order that it have a definite inductance as well as resistance are pointed out.   Farther, it is shown that where the measurements are carried out as outlined above, adjusting inductances as well as resistances, the same simple relations are obtained, both between the inductances, and the resistances.   Unless the inductances as well as the resistances are adjusted a balance can not be obtained except with non-inductive conductors.

If a direct current galvanometer is used and commutated synchronously with the current or the galvanometer is of the type having the field excited by a current from the same source as the test current it is possible by adjusting resistances only to get a zero deflection but this does not necessarily signify that the bridge is balanced.

An Enclosed Arc for Spectroscopic Work.[1]

By James Barnes.

DURING the past few years the author has been engaged in the study of the spectra of a number of substances in the arc burning in gases under reduced pressure. A number of vessels were made for holding the arc, but none gave perfect satisfaction until the one, which is briefly[2] described here, was built. It consists of a large bell-jar ground to fit an iron disc. Between the rim of the jar and the disc a small amount of vaseline or better, stopcock[3] lubricant is placed. The external air pressure upon the jar when the inside air is removed makes this joint almost air-tight, the small leakage is quickly removed by a Geryk pump. With a mercury seal around this joint it is possible to exhaust with a Gaede pump to very low pressures. The arc is made by moving the lower pole by means of an external electro-magnet. The positive pole and the bell-jar are cooled by water coils. The jar has a short side tube closed by a quartz plate which allows the radiations a free exit.

The arc under reduced pressure gives spectra which are not only purer than those obtained from the arc in air at atmospheric pressure but the lines are narrower and sharper, which is of much importance in the measurements of wave-lengths. The narrowness of the lines is best shown by the interference bands they produce when passed through an interferometer. A Fabry and Perot instrument was adjusted with about 5 mm. separation of plates and the interference rings focused on the slit of a Hilger constant-deviation spectroscope. It was found that the lines of the iron, copper and brass arcs *in vacuo* produce bright interference rings much more distinct than those one obtains with the same arcs in air, using the same voltage and current. In the case of substances like cadmium it is almost impossible at atmospheric pressure to get any interference fringes at all in any of the lines, but when one of the iron poles is bored out and filled with the metal and then used in the arc *in vacuo* the fringes come out very clearly, in fact almost as sharp as those obtained with the Hereus cadmium lamp. This arc has been run for three quarters of an hour with one filling and probably can be used much longer with proper adjustment of the current. Such an arc will be of use in interference methods such as is used for the determination of indices of refraction where a large number of narrow bright lines throughout the spectrum are required.

Observations were made on the drop of potential between the poles

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21 and 22, 1911.

[2] For details see an article soon to be published in the Astrophysical Journal.

[3] See Travers' Study of Gases, p. 24.

of the arc in air at a pressure of 1 cm. as the strength of the current was changed and the results plotted and compared with those of the arc in air at atmospheric pressure. They show that with small current, 3 amperes or less, the drop is about the same for both cases, but from this point up to the higher currents used, about 16 amperes, the curves gradually separate till in the case of copper the difference is about 10 volts, for carbon 20 volts.

Potential Drops Across Poles. Current, 16 Amperes.

	Cu	C	
In vacuo	23	40	The voltage of the line was 220.
In air	32	60	

At times the arc will leave one pole and end on a metal part of the vessel, often having a length of over 10 cm. with a current-strength between 2 and 3 amperes and potential fall about 130 volts. This discharge is very interesting, striations have been observed in it, particularly with carbon poles.

Bryn Mawr, Pa.,
    April, 1911.

The Spectra of Aluminium, Copper and Magnesium in the Arc under Reduced Pressure.[1]

By James Barnes.

THIS paper contains the results of some observations upon the spectra of aluminium, copper and magnesium in the arc where the pressure of the air was reduced to about 1 cm. The enclosed arc was described in the above paper and the rest of the apparatus used has for the most part been described in former[2] papers.

In aluminium there is an unmistakable difference in the relative intensities of many of the lines in the arc *in vacuo* compared with their intensities in the arc in air, all other conditions being kept constant. The lines showing the greatest charge are $\lambda\lambda 4663$, 3587, 2631 and 2816. The so-called oxide bands are just as intense as they are in air. A beautiful fluting with four heads makes its appearance in this arc. It was first found by Basquin in a rotating arc in hydrogen. The wave-lengths of these bands have been measured as accurately as possible using iron lines as standards from the new international system.

Copper has without doubt a larger number of lines affected by the reduction of pressure than any of the elements so far tried. Mention should be made of $\lambda\lambda 4675$, 4587, 4378, 4177 for they are just visible on

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21 and 22, 1911.

[2] Astrophysical Journal, 27, 152, 1908, and 30, 14, 1909.

the plates of longest exposure while they are among the strongest in the arc in air.

Besides the charges in the relative intensities of some of the magnesium lines recorded in a former[1] paper the following lines $\lambda\lambda 2790$, 2798, 2929 and 2937 are all affected in the same way. The so-called oxide and hydride bands appear clearly on the spectrum plates of the arc *in vacuo*.

The results of these observations may be divided into two classes:

1. The bands which have been attributed to compounds of oxygen and hydrogen with aluminium, magnesium and calcium all appear in the arc in air under reduced pressure. Whether or not they are true oxide or hydride bands one must admit is still an open question, for it can always be claimed that there is enough oxygen left in the vessel surrounding the arc or sufficient hydrogen occluded in the metal to produce these bands. It might be of interest to state in this connection that the hydrogen lines could not be found on the plates while the nitrogen bands showed very clearly.

2. With regard to the lines, the effect of the reduction of the pressure is in general to increase the intensity of the spark lines of aluminium and magnesium and to diminish many of the arc lines of copper. Whatever the explanation of this may be one can state that the arc *in vacuo* is a step between the arc in air and the spark. The lines which are affected do not belong to the principal or subordinate series in Kayser and Runge's arrangement.

Bryn Mawr, Pa.,
    April, 1911.

The Theory of Dispersion and the Residual Rays.[2]

By O. W. Richardson.

THIS paper discusses the reflection, at normal incidence, from the surface of an insulating medium, of light whose period is close to that of one of the principal periods of vibration of the medium.

It is shown that the well-known difference between the position of the energy maximum in the residual rays and the corresponding natural frequency is about what would be expected from the values of the constants entering into the usual dispersion formulæ. This is particularly true of fluorite which shows the greatest difference.

The case of two natural periods close together is considered and the results are compared with Nichols's experiments on the reflection of

infra-red rays from quartz. The agreement between the experimental results and the theory is satisfactory.

PALMER PHYSICAL LABORATORY,
    PRINCETON UNIVERSITY,
        April, 8 1911.

POSITIVE THERMIONS FROM SALTS OF ALKALI EARTHS.[1]

BY C. J. DAVISSON.

THE experiments constitute an extension of investigations begun by O. W. Richardson on the nature of positive thermions from salts in general. The nature of the thermions is inferred from determinations of their specific charge. These determinations depend on measurements of the magnetic deflections of the ions moving in a vacuum in a uniform magnetic field. The values obtained for salts of the alkaline earths have in all cases been found capable of classification under two heads:

1. Those corresponding to one half the specific charge of the metal ion of the salt in electrolysis.

2. Those due to some common impurity, probably potassium. The inference is reached that the positive thermions from salts of the alkaline earths are single atoms of the metal of the salt minus a single electron.

PALMER PHYSICAL LABORATORY,
    PRINCETON UNIVERSITY,
        April 3, 1911.

THE RÔLE PLAYED BY GASES IN THE EMISSION OF POSITIVE THERMIONS FROM SALTS.[2]

BY C. J. DAVISSON.

EXPERIMENTS have been carried out to determine if any appreciable part of the thermionic currents from surfaces of heated salts is carried by the gas in contact with these surfaces. Determinations of the specific charge of the thermions by means of their electro-magnetic deflections have been made at various low gas pressures in various gases. The results indicate that even when the gas pressure is sufficient to cause a marked increase in the thermionic current at given constant temperature no appreciable part of the current is carried by ions of the gas.

PALMER PHYSICAL LABORATORY,
    PRINCETON UNIVERSITY,
        April 3, 1911.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21 and 22, 1911.

[2] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21 and 22, 1911.

The Difference in the Photoelectric Effect Caused by Incident and Emergent Light.[1]

By Otto Stuhlmann, Jr.

TRANSPARENT metal films, made by sputtering in vacuo from a cathode were investigated for their photoelectric properties. A beam of ultra-violet light at normal incidence, showed that the photoelectric effect was greater in the emergent than in the incident side. The metals investigated, when films were used that were so thin that their absorption can be neglected, gave the following results for the ratio of emergent to incident effect.

Magnesium....................................1.03
Copper.......................................1.08
Zinc...1.11
Silver.......................................1.07
Tin..1.15
Platinum.....................................1.17
Lead...1.12

Selenium and carbon gave unsatisfactory results.

PALMER PHYSICAL LABORATORY,
    PRINCETON UNIVERSITY,
        March, 1911.

Spectroscopic Evidences of Molecular Clustering.[2]

By W. W. Strong.

1. *Solvates.*—Compounds of solvent (such as water or methyl alcohol) and dissolved salt (such as uranous chloride) are shown by the characteristic solvate bands of the salt. In a mixture of the solvents, the salt will show the two absorption spectra superimposed. For a given temperature, there is a definite percentage of each solvent required in order that each set of solvent bands should have the same intensity. This proportion of a given solvent may be defined as a measure of the persistency of the given solvent bands. The persistency of each band in a solvent spectrum seems to be the same. On account of the definiteness of some solvent spectra, it may be considered that the absorbing centers are definite compounds and that the "atmosphere" of solvent consists of a single kind of molecules and that the number of these is so considerable that atoms or molecules outside this "atmosphere" have no influence on the frequencies of the vibrations within.

The persistency of solvent bands for different salts varies. The alcohol and water bands of neodymium chloride (0.5 normal) are of the same intensity when there is 8 per cent. water and 92 per cent. methyl alcohol present. For samarium chloride a smaller proportion of water is required while in the case of uranous chloride and uranous bromide 40 per cent. water and about 60 per cent. alcohol is required, the uranous salts being about 0.2 normal. The presence of free acid or of other salts usually decreases the persistency of the water bands compared with the alcohol bands. In the above example of neodymium chloride as the concentration is decreased, the alcohol bands increase in persistency.

The effect of rise in temperature is to increase the relative persistency of the absolute bands (with reference to the water bands). A very marked instance of this can easily be shown with a pocket spectroscope in the case of uranous chloride dissolved in water and methyl alcohol. If the proportion of solvents is such as to show the water and alcohol bands of equal intensity at room temperature, then at 70° or 80° C. the water bands will be found to disappear almost completely.

Isomeric alcohols may show characteristic bands. For instance, the corresponding bands of neodymium chloride in isobutyl alcohol are of considerably greater wave-length than the bands in butyl alcohol and the bands for propyl alcohol are of greater wave-length than in isopropyl alcohol.

2. *Aggregates.*—The presence of free acids and salts has been shown to change the wave-lengths of uranyl, uranous and some neodymium bands. This change of wave-length is apparently a continuous one. In some cases as aqueous solutions of uranyl intrate and uranyl sulphate or neodymium chloride and acetate, the absorption of the two salts are quite different. In most cases when one of these salts is transformed to the other salt, the change from the one spectrum to the other is a continuous one. Thus the uranyl nitrate bands of a nitric acid solution can be gradually changed to the acid uranyl chloride bands by adding hydrochloric acid.

Uranyl Bands.

	a	b	c	d	f	g	h
Nitrate in nitric acid.	4790	4670	4510	4370	4125	4000	3900
Chloride in hydrochloric acid.	4950	4800	4635	4450	4280	4050	4015

If it is assumed that a characteristic absorption spectrum is due to a more or less stable compound, then the above changes indicate the existence of series of compounds and these will be called aggregates. There will be nitric acid uranyl aggregates, neutral nitrate uranyl aggregates, etc. In some cases the aggregates may have their own characteristic bands. For instance an acid solution of neodymium chloride in ethyl alcohol has strong and characteristic bands at $\lambda\lambda 3695$ and 3760.

Mixture of varying amounts of two salts in the same solvent seems to result in some cases in the formation of aggregates consisting of both salts, the frequencies of the resulting bands depending on the relative amounts of the two salts present.

Acid aggregates of uranous salts may be much more stable than neutral aggregates. For instance, uranous nitrate is very unstable but uranous chloride dissolved in nitric acid (the absorption spectra is very different from that of uranous chloride) will remain unoxidized for hours. Neutral uranous salts in water heated to 80° or 90° C. form a black precipitate, but if some free acid is present, the uranous salts are much more stable.

Aggregates form solvates that possess absorption bands that are as characteristic as those formed by the pure neutral salt.

The addition of hydrogen peroxide to a uranous salt solution oxidizes the salt to the uranyl conditions. Spectrophotographs of these oxidization reactions show the complete independence of the uranyl and uranous spectra.

In the reduction of the uranyl salts by the hydrogen process it was found that very concentrated solutions of uranous salts were obtained from ether solutions of the uranyl salts. The reduction of the uranyl salts does not seem to be due to the presence of nascent hydrogen for the addition of a small amount of nitric acid and zinc to an aqueous solution of uranyl chloride does not result in any uranous salt being formed although the amount of nitric acid added is not great enough to change the uranyl chloride bands to the uranyl nitrate bands.

Selective Action on Solvates.—The addition of hydrogen peroxide was found to oxidize the hydrate and alcoholate (both being present in approximately equal amounts in the solution) at the same rate. On the other hand, when potassium or calcium nitrate is added to such a solution the water bands become very weak or even disappear while the alcohol bands do not become weaker at all. Sometimes selective precipitation may take place and only the alcohol bands remain in the absorption spectrum. The addition of sodium chlorate to a very concentrated solution of uranous bromide in water and alcohol produces selective precipitation. In general the addition of an oxidizing agent shows a selective action on the uranous bands due to different solvates.

The Effect of Dilution.—As the uranyl sulphate and nitrate bands in aqueous solutions have different wave-lengths, it might be expected that for great dilutions, the uranyl bands would have the same wave-lengths for both salts. Changes of concentration from 1.5 to 0.003 normal of the uranyl nitrate and sulphate showed very little change of wave-length. This very small change consisted of a shift of the nitrate bands to the red and the sulphate bands to the violet. Similar experi-

ments should be carried out on the bands of uranyl chloride and nitrate in acetone as these have quite different wave-length.

The above results may be explained by assuming that the uranyl aggregates break up but little as the concentration is decreased and that these aggregates may act as ions. Since the intensity of the uranyl bands does not decrease greatly with dilution one may consider that this intensity depends upon the constitution of the aggregate.

The Elastic Properties of a Series of Iron-Carbon Alloys.[1]

By C. W. Waggoner.

THIS work is a continuation of a study[2] the writer is making on some of the physical properties of a special series of iron-carbon alloys in which the constituents, other than the carbon, are practically constant. This study was made to determine, if possible, any definite connection between the carbon content of these alloys and their elastic properties. The actual tests were carried out under the supervision of Professor C. R. Jones, by two senior students, Messrs. Crewson and Patterson, on a 20,000-pound Riehle standard testing machine in the testing laboratory of West Virginia University. Test samples were given the same heat treatment as was used in the magnetic tests,[3] namely annealing for two hours at 1000° C. to remove any hardening effect due to machining or to the previous rolling in the manufacture.

The results may be briefly summarized as follows:

1. The elastic properties, like the magnetic properties, show a critical point between .9 per cent. and 1 per cent. carbon. This is to be expected from the phase diagram for iron.

2. The maximum strength increased with the carbon content up to the critical percentage of carbon and then decreased. The maximum strength for pure iron being about 40,000 pounds per square inch.

3. The breaking strength follows the same law as the maximum strength, the slope of the curve plotted with breaking strengths as ordinates and carbon percentages as abscissas being greater throughout the lower ranges of carbon.

4. The methods of locating the elastic limit are such as to call in question any results; but there is some evidence in favor of a slight increase in the elastic limit up to the critical percentage of carbon and then a decrease as the carbon content is decreased.

5. The results on the modulus of elasticity need further study, but

present results indicate a slight decrease with increase in carbon through-out the range of carbon studied.

6. The total elongation varies almost exactly in an inverse proportion to the carbon content. Here, too, there is a slight change in the curve at the critical carbon percentage.

7. The reduction of area decreases with the increase of carbon content; a small change in carbon making relatively a large change in the reduction of area up to the critical percentage of carbon, but beyond this critical percentage the decrease in reduction of area is not so marked as for the lower range of carbon content.

WEST VIRGINIA UNIVERSITY.

THE DEFLECTION POTENTIOMETER CONSIDERED AS A GENERALIZED NULL INSTRUMENT.[1]

BY H. B. BROOKS.

Outline.—The potentiometer has until a few years ago been regarded only as a null instrument. The deflection potentiometer for voltage measurements, described by the writer about five years ago, showed the possibility of making precision voltage measurements with a potentiometer in which an appreciable part of the result is given by the reading of the galvanometer. More recently the question of measuring current in the same manner has been investigated. This brought up new problems, which have been solved, and the theory has been extended so that all measurements within the range of a given potentiometer may be made under the condition of current in the galvanometer circuit. The limitation, "galvanometer current equal to zero," is thus removed, and the potentiometer method is therefore generalized. The chief advantages secured by this combination of null and deflection methods are rapidity of working, and the ability to work with unsteady currents and voltages if necessary.

THE SATURATION VALUE OF THE SPECIFIC INTENSITY OF MAGNETIZATION OF COBALT AT VARIOUS TEMPERATURES.[2]

BY W. W. STIFLER.

THE saturation values of the specific intensity of magnetization of cobalt were determined at intervals of 100° to 150° from 22° C. up to 1000° C., and from that temperature up to 1150° C. at much

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21 and 22, 1911.

[2] Abstract of a paper presented at the Washington meeting of the Physical Society April 21 and 22, 1911.

smaller intervals. The method used was that described by Weiss[1] and consisted in observing the deflections produced when a small ellipsoid was jerked out of a helix placed in a strong longitudinal field. For the higher temperatures this helix was surrounded with heating coils. The sample was protected from oxidation by placing it in an exhausted tube of glass or quartz. The temperatures were measured by a thermojunction which was sealed into the tube and which was separated from the cobalt by a thin strip of mica.

External fields up to 6,900 gausses were used, giving fields in the ellipsoid up to 5,000 gausses. Below 350° C. this was not sufficient to produce saturation. Hence the saturation value was determined by plotting σ against $1/H$ and $1/H^2$, where σ is the intensity of magnetization per gram, and extrapolating. The extrapolated values are probably accurate to 1 per cent. Above 350° C. saturation was reached with the fields used.

The value of σ at room temperature is 160. It decreases gradually to 113 at 800° C. after which it falls off very rapidly with increase of temperature, the cobalt ceasing to be ferro-magnetic at about 1080° C. The curves plotted with σ and temperature as coördinates are identical for two samples tested and are of the form required by the electron theory of magnetism. The results are discussed from the point of view of this theory.

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The Radiant Efficiency of Arc Lamps.[2]

By H. P. Gage.

THE object of the present investigation is to determine the radiant efficiency of arc lamps and the mechanical equivalent of light. The definition here used of radiant efficiency is:

$$\frac{\text{Energy of the visible part of the spectrum}}{\text{Total energy radiated}} = \frac{L}{R}.$$

Using the symbols, R = total energy radiated,

W = energy passing through 8 cm. layer of water,
L = energy visible part of spectrum.

The ratio L/R or W/R was obtained by a slight modification of the method of Ångström. After dispersing the spectrum of a light source

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[1] Journ. de Phys., 9, pp. 373–393, May, 1910.

[2] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21 and 22, 1911.

the infra-red was removed by an opaque screen.   The light was then re-assembled by a mirror to a white spot.   The energy of this white light was compared by means of a radiomicrometer with white light of equal intensity and like color from the same source which had not been robbed of its infra-red.   In some cases owing to the small value of L compared with R, L was compared with the intermediate quantity W.

In a previous paper (October 15) the reasons were indicated for choosing the value of L as the energy of the spectrum included between the limits $.4\mu$ and $.68\mu$.   These are briefly: The small photometric effect of all energy of greater wave-length than $.68\mu$, and the greater certainty of setting the cut off screen at this than at greater wave-lengths.

The most important results follow.

Source.	Eff. L/R Per Cent.	Light Equivalent of Energy. Candles per Watt.
Positive crater, right angle carbon arc.	8 to 12 depending on current.	30
Right angle lamp alternating current.	6.5 to 8.5	21
Yellow flame arc.	15.4	39
Yellow flame, arc stream.	39.	
Brilliant white.	14.6	21
Hefner, recalculated from Angström for limit $.68\mu$.	.363	21.3
Nernst lamp viewed through $CuSO_4$ solution.	100	39.5

The greatest efficiency of 39 per cent. was found in the case of the arc stream between yellow flame electrodes, the tips being shaded.   The light equivalent of energy was found to be as high as 30 candles per watt with white light and almost 40 candles per watt with light distributed more in the visually intense part of the spectrum.

A Heterochromatic Photometer.[1]

By H. P. Gage.

A PHOTOMETER constructed on the same principle as the Lummer-Brodhun photometer except that in place of the usual cubical prism there was substituted a piece of ordinary looking glass, part of the silver being scratched off with a sharp knife.   It was found that if instead of removing large areas of the silver, small scratches were made, these scratches could be made to disappear with differently colored lights

when with large areas, a sharp match never could be made.   With this
apparatus it was possible to make fairly sharp settings with such differently colored lights as acetylene, the right angle arc, daylight, the yellow flame arc and the Hefner lamp.   When it comes to comparing red light with green light, however, the results are not at all satisfactory unless very low intensities are used.

Fig. 1.

The form of pattern adopted was as shown in the figure, the black representing the silver removed.

Ultra-Violet from the Arc.[1]

By H. P. Gage.

IN experimenting with different substances in the arc it was found that uranium gave a very nearly continuous spectrum from the visible to the ultra-violet as far as $.22\mu$.   The following electrodes will give a spectrum with rather weak lines on a continuous background.   Make a tube of thin sheet iron or "tin" $1/4$ inch inside diameter, fill with powdered uranium oxide (U_3O_8).   Use as lower electrode, connect to the positive terminal and pass 6 amperes.   The negative terminal may be carbon.   If the current is reversed a rich line spectrum will be obtained.

The amount of ultra-violet light in a source may be photometered by using a Lummer-Brodhun photometer with a screen towards the unknown source which will transmit only the wave-lengths desired.   The ultra-violet light is made apparent to the eye by using a screen coated with anthracene instead of white paper.   Examples of measuring the photographic brilliancy of several arc lamps.

Heat of Evaporation of Water at $100°$ C.[1]

By Arthur Whitmore Smith.

IT is well known that the vapor rising from boiling water carries with it a certain amount of water in a finely divided state, but nevertheless still liquid.   Even with a steam separator 1 per cent. or more of the total mass may be water, and unless precautions are taken the percentage of moisture in the stream from an ordinary boiler is greater than this.

These considerations lead to a feeling of very great uncertainty when one studies the methods which have been used for the determination of the heat of evaporation of water.   If the steam used contained 1 per

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21 and 22, 1911.

cent. moisture, the familiar number 537 is 1 per cent. too low. As far as I know no attempt has ever been made to determine the heat of evaporation from a quiet surface of water at 100° C. where there is small chance of spray being thrown into the vapor above.

This paper describes a new form of calorimeter in which the water is evaporated by a gentle current of air passing over its unruffled surface. Heat is supplied by an electric current, both E and I being measured in terms of a Weston cell. The temperature is maintained constant by regulating the air current to allow the evaporation to proceed just fast enough to use the heat as it is delivered by the current.

Each result is computed from two experiments, in one of which about 36 grams of water are evaporated and in the other only about 9 grams. The corresponding amounts of heat are supplied by the electric current plus small corrections. The *difference* between the two heats eliminates these corrections, and when divided by the *difference* between the corresponding amounts of water gives 541.3±0.2 calories per gram for the heat of evaporation of water at 100° C.

Check experiments made and computed in the same way and with the same apparatus, but allowing the water to boil, give the old value of about 538 calories per gram.

UNIVERSITY OF MICHIGAN.

A TUBULAR ELECTRODYNAMOMETER FOR VERY HEAVY CURRENTS.[1]

BY P. G. AGNEW.

TO obtain the same current distribution for both alternating and direct current, it is usual to strand the windings of the field coils of electrodynamometers. For very heavy currents, this becomes a matter of extreme difficulty. An entirely different method of obtaining the same end is employed in the present instrument. The field at an outside point due to a straight cylindrical or tubular conductor is independent of any skin effect if we have axial symmetry. The field "coil" consists merely of two concentric heavy copper tubes, thus giving a circular field between the tubes. There are two moving coils astatically placed between the tubes, one on either side of the inner tube, and suspended by a phosphor bronze strip. The common axis of the tubes is placed in a horizontal position in order to allow the vertical suspension. While the distribution is not the same an alternating as on direct current, the magnetic field in the space between the tubes and hence the torque is the same. For cooling it is arranged so that water may be passed through the inner tube. The instrument was designed to carry 5,000 amperes,

and it will carry 1,000 amperes continuously without water cooling. Magnetic impurities on the moving system are exceedingly troublesome, the requirements being even more exacting than in the moving coil galvanometer.

The Silicon Detector used with Short Electric Waves, and the Theory of Contact Rectifiers.[1]

By Ernest Merritt.

THE writer has used the silicon detector with waves ranging from 15 cm. to 50 cm. in demonstrating the properties of Hertz waves to a class, and finds it highly satisfactory. The detector is amply sensitive even when used without any E.M.F. and with a d'Arsonal galvanometer of only moderate sensibility. To damp out the low frequency oscillations that would otherwise be produced in the induction coil and connecting wires, the oscillator—of the Righi type—was connected to the coil through two Geissler tubes. Under these circumstances, and with a Wehnelt interrupter, the indications were remarkably steady and free from disturbances.

In connection with the explanation of the rectifying action of detectors of this type, attention is called to the probable influence of the sudden change in specific conductivity which occurs at the contact. If the metals used are of different conductivity, or if the contact is imperfect, there will be a layer of transition in which the specific conductivity changes rapidly. The equation of continuity requires that in this transition layer, as in all conductors, div. $i = 0$, where i is the current density. If the electric field is E and the conductivity C, we therefore have

$$\text{div. } i = \text{div. } CE = 0 = C \text{ div. } E + E \text{ grad. } C.$$

In homogeneous isotropic conductors the conductivity is constant at all points and grad. $C = 0$. In such cases therefore div. $E = 4\pi\rho = 0$ and there is no free charge on the interior. But in the transition layer C is not constant and grad. C, and in consequence ρ, may differ very appreciably from zero. The transition layer will therefore contain an excess of positive ions for one direction of current and an excess of negative ions when the current is reversed. If the negative ions only are free to move, and if the material is of such a nature that its molecules do not readily liberate electrons to make up for a deficiency, the conductivity of the transition layer will thus depend upon the direction of the current.

It can readily be shown that the change in conductivity with the direction and magnitude of the current will be such as to lead to a relation

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1911.

between current and E.M.F. of the same general form as that determined experimentally. For small currents it can also be shown that the rectified current through such a contact should be proportional to the square of the applied alternating E.M.F. This conclusion is in agreement with the experimental results of Austin.

PHOTO-ELECTRIC POTENTIALS OF THIN CATHODE FILMS.[1]

By PAUL H. DIKE.

IT has been found by Professor Millikan that aluminum, copper, silver and some other metals when exposed for considerable lengths of time to the action of ultra-violet radiation in very high vacua, gradually increase in photo-electric activity, giving values for the photo-electric potential which are unprecedentedly high, running up to 25 volts for copper. Von Baeyer and Gehrts, taking precautions to avoid the reflection of the electrons projected from the plate by the action of the light, obtained a maximum of 6.3 volts for aluminum, gold and copper. Values not exceding three volts have been found by other observers.

In the work here described an attempt was made to repeat Millikan's high values for the potential by preparing and testing the metal surface in the same vacuum without exposure to the air, so as to prevent as far as possible the formation of surface films. The first method tried was to use the plate to be tested as cathode during a long discharge, giving a fresh, clean surface. The potential obtainable could be increased to four volts at most with copper and iron, the metals tested, an increase over the usual values but not a great one. Another method of preparing the metallic surface is to form a cathode deposit of the metal on a plane surface, and test it without exposure to the air. As a result of three trials agreeing among themselves in their general features it may be said that a platinum cathode film on glass is photo-electrically active even when of the utmost tenuity—down to 10^{-8} cm. in thickness. The potential rises to a somewhat uncertain maximum of from 12 to 18 volts at a thickness of about 5×10^{-8} cm. and at 10^{-7} cm., drops suddenly to a minimum of about one volt, and from that value it increases very gradually with thickness of film to 1.5 or 2.0 volts. These values are approximate, and probably would have been much higher if electronic reflection had been avoided as was done in a later type of tube. In this the cathode could be turned out of the way while the film was being exposed to the light and the walls of the tube were much farther from the plate under test, while diffused light was also nearly eliminated. In this tube very thin films did not give as high potentials depositing on

quartz, but slightly thicker layers gave persistently high potentials, and an aging effect appeared. A freshly deposited film renders the plate inactive for several minutes but a gradual change takes place and the potential obtainable on exposure to ultra-violet light increases for several hours going up finally to 25 or 30 volts. A platinum plate cleaned with French chalk gave a potential of 6.2 volts when first placed in the tube. On being covered with a cathode deposit of platinum not more than 10^{-7} cm. in thickness and standing for several hours it gave a potential of 44 volts. A slight increase in the thickness of the film restored the original lower potential and the high values could not be regained till the film had been removed by using the plate as cathode and then covering it with a fresh deposit.

Very thin films of platinum cathodically deposited have been shown to possess extreme photo-electric activity as measured by the velocity with which the electrons are projected, giving potentials as high as 44 volts, which is about twenty times the potential ordinarily obtained and nearly double Millikan's highest reported value for copper.

A slight increase beyond the critical thickness restores the metal to its normal condition.

The same effect is obtained even when platinum is deposited on platinum.

Further experiments with thin films are in progress.

DEPARTMENT OF PHYSICS,
    UNIVERSITY OF WISCONSIN,
        April 17, 1911.

A NEW DETERMINATION OF THE SELECTIVE RADIATION FROM TANTALUM.[1]

BY EDW. P. HYDE.

IT has been shown elsewhere[2] that, if two incandescent bodies are at a color match and have different luminous efficiencies, then one of the bodies radiates selectively with respect to the other. The existence of a color match is a consequence of there being approximately the same energy distribution in the visible spectra. A difference in the luminous efficiencies indicates that, for the entire spectrum taken as a whole, there exists a difference in the distribution of the radiated energies from the two sources. If two bodies are not selective in their radiation with respect to one another,

it was shown that at a color match they must have the same luminous efficiency. The test for a difference in their radiating selectivities is then logically that which has been stated above. A color match may be determined with considerable accuracy with the aid of a photometer. The luminous efficiencies, in case the substances are mounted as incandescent lamp filaments, may be determined by standard methods. In the paper already noted there were presented results on various types of incandescent lamps when compared with a black body. Among other things it was found that untreated carbon was very closely non-selective. These results did not include corrections due to losses in efficiency resulting from the cooling effects of the supports for the filaments, to energy dissipated in the base and the leading-in and connecting wires, and to the absorption of the radiated luminous energy by the glass bulb and the deposits thereon. In some recent work[1] these losses have been investigated. It seemed worth while to redetermine the relative selectivity of tantalum and carbon taking into account the losses just mentioned. Two special lamps, one with an untreated carbon filament, the other with a tantalum filament, were used in which these losses could be carefully measured. The results are included in the following table:

Approx. Black Body Temp.	Measured Luminous Efficiencies in Lumens per Watt.		Corrected Luminous Efficiencies in Lumens per Watt.		Ratio of the Luminous Efficiency of Tantalum to that of Carbon.	
	Carbon.	Tantalum.	Carbon.	Tantalum.	Present Results.	Previous Results.
1420° C.	$.37_3$	$.48_1$	$.41_9$	$.51_9$	1.2_4	1.3
1610	1.4_1	1.7_0	1.5_9	1.8_2	1.1_4	1.15
1780	3.1_5	3.5_6	3.5_2	3.7_6	1.0_7	1.1

It is to be noted that the corrections for the losses mentioned above have changed but little the previous values for the ratio of the luminous efficiency of tantalum to that of untreated carbon, obtained by using ordinary lamps.

A Kinetic Theory of Gravitation.[2]

By Charles F. Brush.

THE author discusses at length the origin of the energy acquired by a falling body, and concludes that the gathered energy comes from the ether through which the body falls. Conversely, equivalent

[1] Hyde, Cady & Worthing, Elec. World, March 9, 1911, p. 624. The complete paper will appear shortly in the Trans. of the Ill. Eng. Soc., and in the London Illuminating Engineer.

[2] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 30, 1910.

energy is restored to the ether when the body is raised against gravitative attraction.

The ether is assumed to be endowed with vast intrinsic kinetic energy in wave form. The waves are of such low frequency, or otherwise of such character, that they pass through all bodies without obstruction other than that concerned in gravitation. They are propagated in straight lines in every conceivable direction, so that the wave energy is isotropic, being in this respect like radiation in the interior space of a furnace with uniformly heated walls. Distribution of the ether's intrinsic energy is uniform throughout the universe except as modified by the presence of matter. Any kind of ether waves capable of exerting motive action on the atoms or molecules of matter will fulfil the requirements.

Atoms are imagined to be continually buffeted in all directions by the ether waves in paths almost infinitesimally short, but without collision because neighboring atoms follow *very* nearly parallel paths. The moving atoms are likened to particles of a precipitate suspended in turbulent water.

Each atom or molecule is regarded as a center of activity due to its kinetic energy of translation derived initially from the ether. There is continual absorption and restitution of the ether's energy, normally equal in amount. But the ether is permanently robbed of as much of its energy as is represented by the mean kinetic energy of the atom. This energy deficiency in the ether is not wholly local, but extends indefinitely into space, diminishing as the square of the distance increases.

A body of matter is pictured as casting, in effect, a spherical energy-shadow consisting of the sum of the shadows of its constituent units, the depth or intensity of the shadow varying with the inverse square of the distance from the body's center of mass. Another body, at any distance, will cast a similar shadow, and the two shadows will intersect, each body being partially shielded by the other from waves coming from that direction; the extent of the shielding effect depending directly on the mass of the shielding body.

Of the several components into which the composite motion of each atom can be resolved, that one lying in the direction of an attracting body will be greatest, because the waves from that direction, being partially intercepted by the attracting body, are weakest; and the atom will be *pushed* in that direction by the superior waves behind it. If free to fall, the atom will continually absorb more energy from the stronger waves behind it than it restores to the weaker waves in front, and will thus acquire additional kinetic energy of translation in the line of fall, measured directly by the number of waves involved, *i. e.*, by the distance

moved. Conversely, if the atom be forced away from the attracting body, restitution of energy will exceed absorption, and the energy expended in moving the atom against attraction will be transferred to the ether.

It will be seen that gravitation is a *push* toward the attracting body, and not a pull. It is clear also, that the velocity which a falling body can acquire tends asymptotically to a limit, which is the velocity of the ether waves which push it—the velocity of light, if transverse waves are involved.

NEW BOOKS.

Electrical Nature of Matter and Radioactivity. Second edition. By HARRY C. JONES. New York: D. Van Nostrand Company, 1910. Pp. viii+210. Price, $2.00.

The Theory of Ionization of Gases by Collision. By JOHN S. TOWNSEND. New York: D. Van Nostrand Company, 1910. Pp. xi+88. Price, $1.25.

Lehrbuch der Kosmischen Physik. By WILHELM TRABERT. Leipzig: Teubner, 1911. Pp x+662. Price, M. 20.

Précis D'Optique. By PAUL DRUDE and MARCEL BOLL. Paris: Gauthier-Villars, 1911. Pp. x+375. Price, Fr. 12.

Précis De Télégraphie Sans Fil. By P. BLANCHIN, G. GUÉRARD, and E. PICOT. Paris: Gauthier-Villars, 1911. Pp. viii+385. Price, Fr. 12.

Elements of Mechanics. By GEORGE W. PARKER. New York: Longmans, Green and Co., 1911. Pp. ix+245.

Mathematical and Physical Papers. Vol. 5. By Sir WILLIAM THOMSON, BARON KELVIN. Cambridge: The University Press, 1911. Pp. xv+602.

The Grammar of Science. By KARL PEARSON. Third edition. New York: The Macmillan Company, 1911. Pp. xx+394. Price, $1.60.

Die groszen Physiker und ihre Leistungen. By F. A. SCHULZE. Leipzig: Teubner, 1910. Pp. i+108. Price, M. 1.25.

Werdegang der modernen Physik. By HANS KELLER. Leipzig: Teubner, 1911. Pp. i+113. Price, M. 1.25.

Die Optischen Instrumente. By MORITZ VON ROHR. Leipzig: Teubner, 1911. Pp. i+140. Price, M. 1.25.

Taschenbuch für Mathematiker und Physiker. By FELIX AUERBACH and RUDOLF ROTHE. Leipzig & Berlin: Teubner, 1911. Pp. ix+567. Price, 7 M.

INDEX TO VOLUME XXXII.

ERRATA.

Page 64.   Preparation and Properties of the Heusler Alloys. The photographs in the right hand column of Fig. 4 from top to bottom are 24, 36, 37, 54; in the left hand column from top to bottom are 44, 41, 38.

Page 324.   A New Method of Producing Ripples.   After publishing this note Dr. Hugo Schultze called my attention to the fact that he had already described the methods of ripple formation therein discussed.   (See *Zeitschrift für Instrumentenkunde*, p. 150, 1906, and p. 151, 1907; also *Tätigkeitsberichten der Phys. Techn. Reichsanstalt*, 1905 and 1906.)   I wish to take this opportunity of acknowledging priority to him and to express the regret that the former articles escaped my notice.

<div align="right">A. H. Pfund.</div>